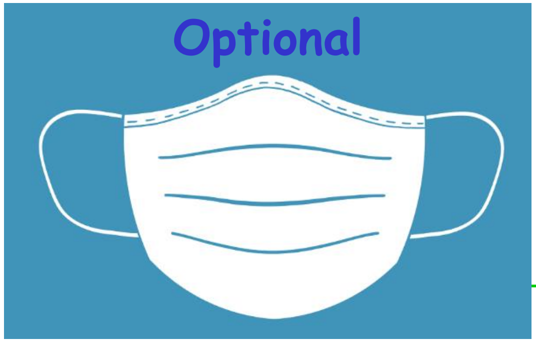
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

DESIGN OF MACHINE ELEMENTS
ME-3320, B'2023

Lecture 04

October 2023



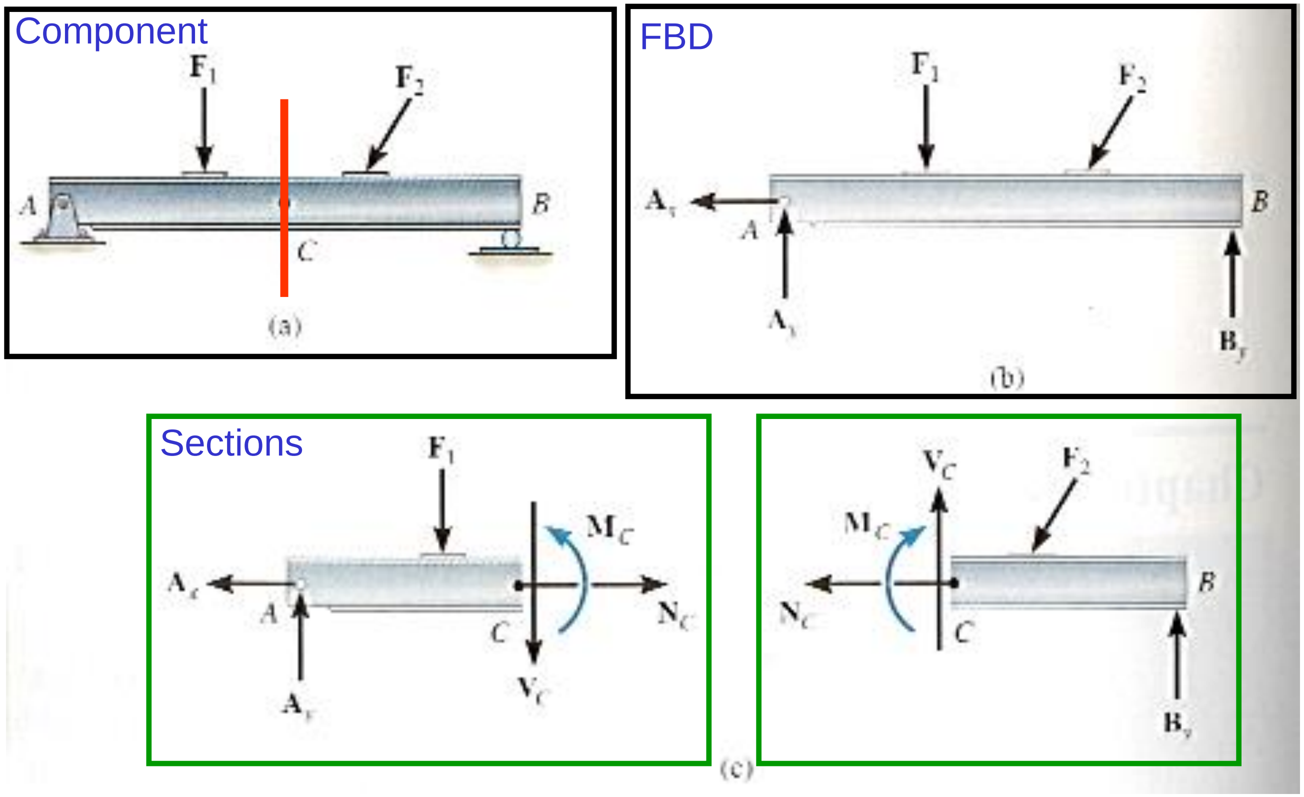
Topics for today

- **Introduction to MathCAD: step functions**
- **Shear, moment, torsion diagrams:** examples w/singularity functions

Internal forces and moments

Shear, Normal, and Bending moments

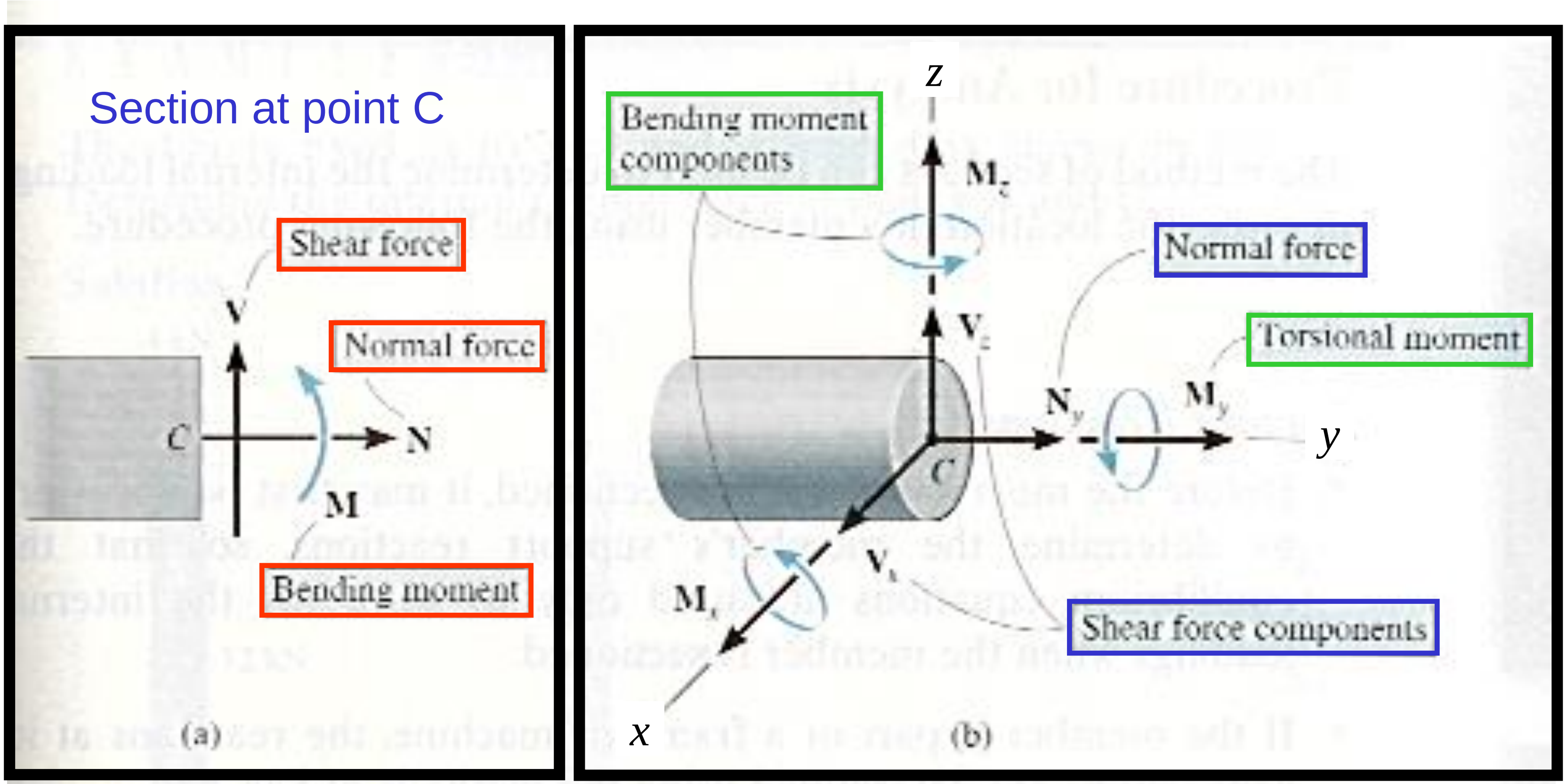
Internal forces (determination of shear and moment diagrams)



Internal: moments, shear, and normal forces at point C

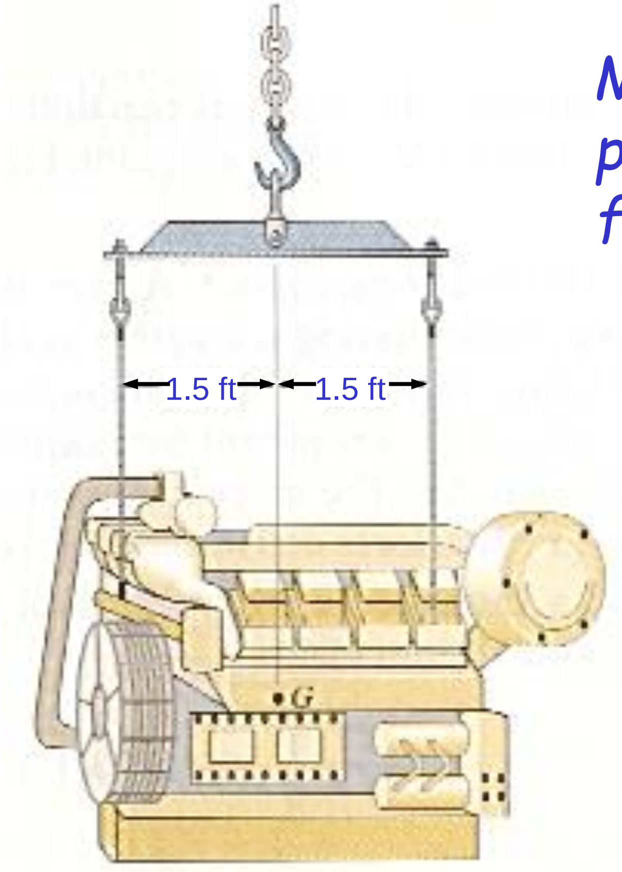
Internal forces and moments

Shear, Normal, and Bending moments



Shear and bending-moment diagrams

A suspended bar supports a 600-lb engine. Plot the shear and moment diagrams for the bar.



Method of sections:
plot using step functions + MathCad

Details on example that is provided in class and in notes

Shear and bending-moment diagrams

Method of sections: *plot using step functions + MathCad*

ME-3320: example in Mathcad

A suspended bar supports a 600-lb engine.
Plot the shear and moment diagrams for the bar.

Input:

$L := 3$

$a := 1.5$

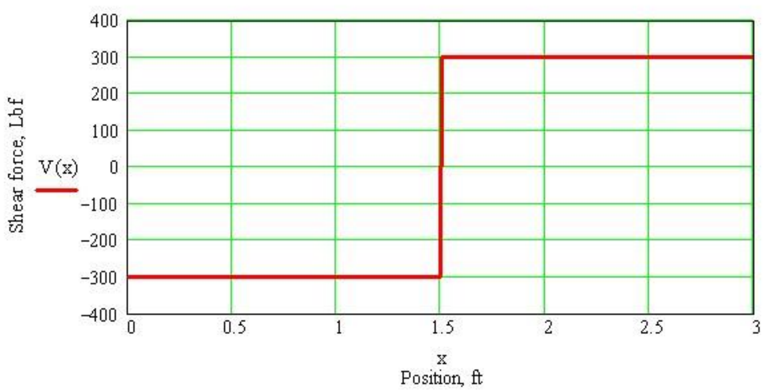
$x := 0, 0.001 \cdot L .. L$

Define unit step function:

$S(x,z) := \mathrm{if}(x \geq z, 1, 0)$

Define shear function:

$V(x) := -300 \cdot S(x,0) + 600 \cdot S(x,1.5)$

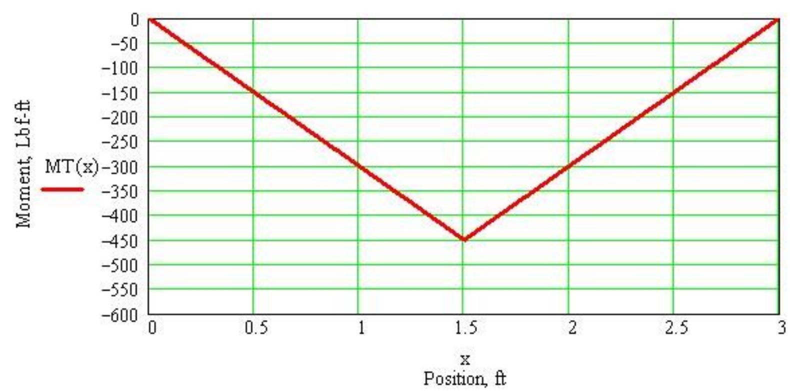


Define moments function:

$M1(x) := -300 \cdot x$

$M2(x) := 300 \cdot x - 900$

$MT(x) := S(x,0) \cdot M1(x) - S(x,1.5) \cdot M1(x) + S(x,1.5) \cdot M2(x)$



Shear and bending-moment diagrams

Singularity functions

Singularity functions:

•Definitions:

•$n<0^*$: $f_n(x) \equiv \langle x-a \rangle_n = \begin{cases} \infty & x = a \\ 0 & x \neq a \end{cases}$

•$n \geq 0$: $f_n(x) \equiv \langle x-a \rangle^n = \begin{cases} (x-a)^n & x \geq a \\ 0 & x < a \end{cases}$

•Integration rules:

•$n<0$: $\int_{-\infty}^{x} \langle x-a \rangle_n \, dx = \langle x-a \rangle_{n+1}$

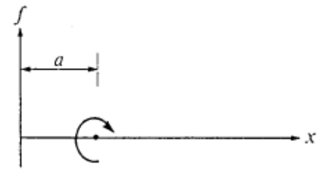
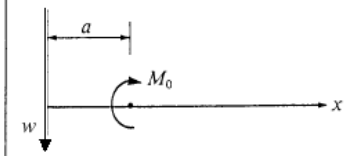
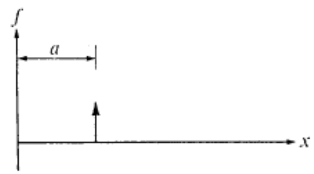
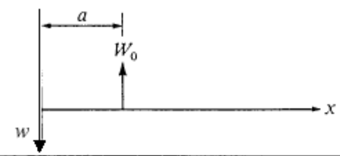
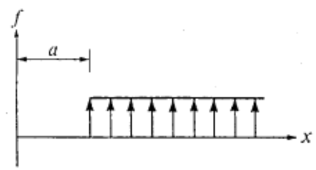
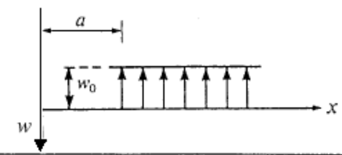
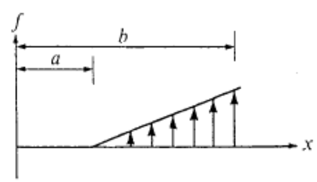
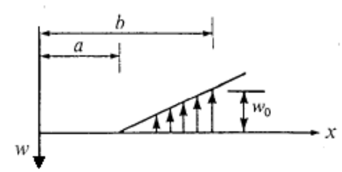
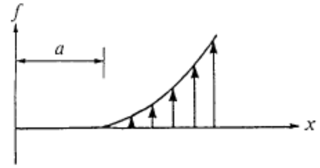
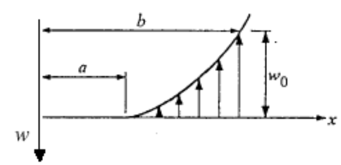
•$n \geq 0$: $\int_{-\infty}^{x} \langle x-a \rangle^n \, dx = \frac{1}{n+1} \langle x-a \rangle^{n+1}$

*Remark: the subscript positioning of n when $n < 0$ is sometimes used to emphasize the fact that the singularity function behaves differently from $n \geq 0$

Shear and bending-moment diagrams

Singularity functions

Main singularity functions and their use

Singularity function	Graphical representation	Loading	
$f_{-2}(x) = \langle x - a \rangle_{-2}$ (couple)		$w(x) = -M_0 \langle x - a \rangle_{-2}$	
$f_{-1}(x) = \langle x - a \rangle_{-1}$ (concentrated load)		$w(x) = -W_0 \langle x - a \rangle_{-1}$	
$f_0(x) = \langle x - a \rangle^0$ (uniformly distributed load)		$w(x) = -w_0 \langle x - a \rangle^0$	
$f_1(x) = \langle x - a \rangle^1$ (linearly distributed load)		$w(x) = -\frac{w_0}{b-a} \langle x - a \rangle^1$	
$f_2(x) = \langle x - a \rangle^2$ (quadratic distributed load)		$w(x) = -\frac{w_0}{(b-a)^2} \langle x - a \rangle^2$	

Shear and bending-moment diagrams
Singularity functions

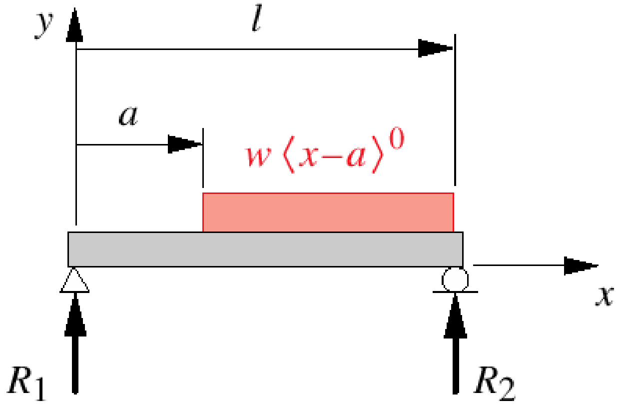
Loading function: $q(x)$

Shear function: $V(x) = \int q(x)\,dx$

Moment function: $M(x) = \int V(x)\,dx$

Shear and bending-moment diagrams

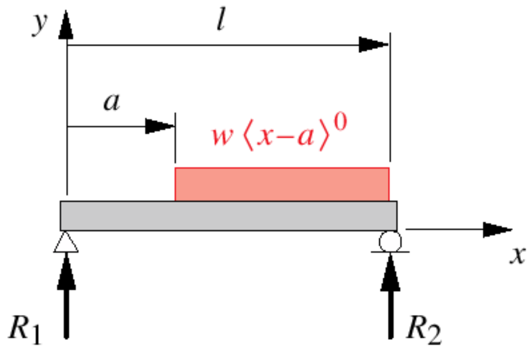
Singularity functions: in-class examples (loading functions)



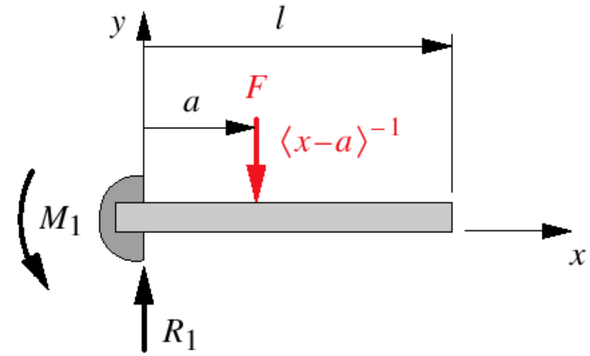
(a) Simply supported beam with uniformly distributed loading

Shear and bending-moment diagrams

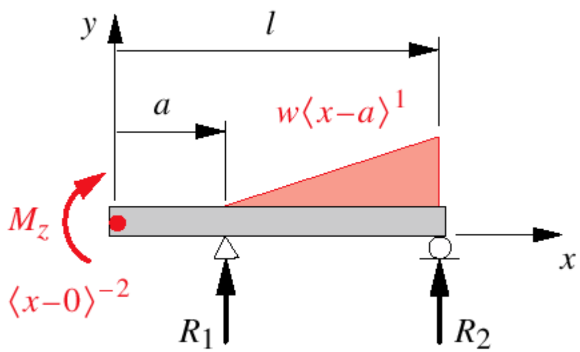
Singularity functions: in-class examples (loading functions)



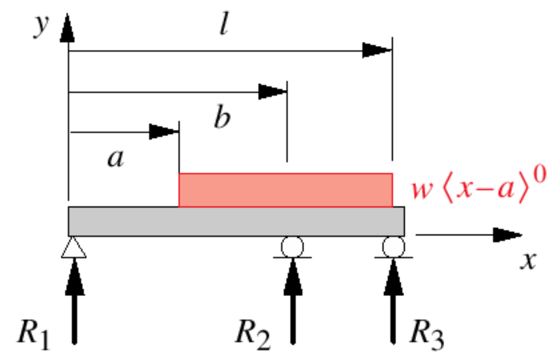
(*a*) Simply supported beam with uniformly distributed loading



(*b*) Cantilever beam with concentrated loading



(*c*) Overhung beam with moment and linearly distributed loading

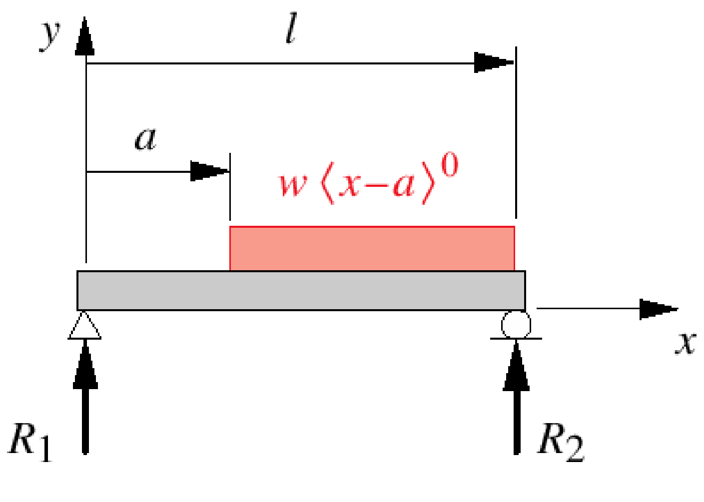


(*d*) Statically indeterminate beam with uniformly distributed loading

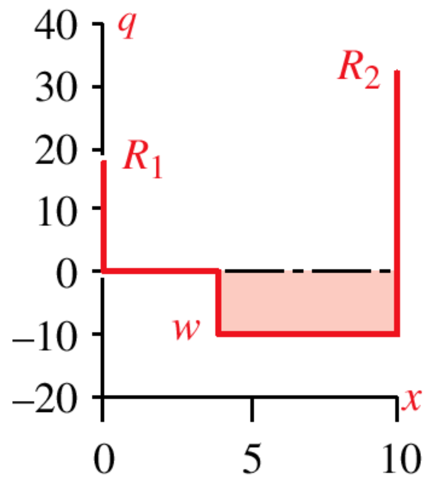
Shear and bending-moment diagrams

Singularity functions: example **E1**

Determine and plot the shear and moment functions for the simply supported beam shown:



(*a*) Simply supported beam with uniformly distributed loading



Shear and bending-moment diagrams

Singularity functions: example **E1** - MathCad

To generate the shear and moment functions over the length of the beam, equations (*b*) and (*c*) must be evaluated for a range of values of *x* from 0 to *l*, after substituting the above values of C_1, C_2, R_1, and R_2 in them. For a Mathcad solution, define a step function *S*. This function will have a value of zero when *x* is less than the dummy variable *z*, and a value of one when it is greater than or equal to *z*. It will have the same effect as the singularity function.

Range of *x* $\quad x := 0 \cdot in, 0.01 \cdot l .. l$

Unit step function $\quad S(x, z) := if(x \geq z, 1, 0)$

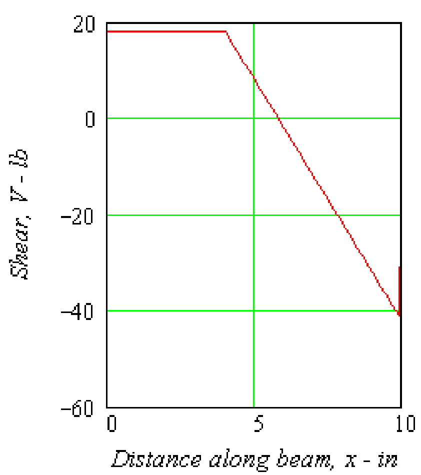
Write the shear and moment equations in Mathcad form, using the function *S* as a multiplying factor to get the effect of the singularity functions.

$$V(x) := R_1 \cdot S(x, 0 \cdot in) \cdot (x - 0)^0 - w \cdot S(x, a) \cdot (x - a)^1 + R_2 \cdot S(x, l) \cdot (x - l)^0$$

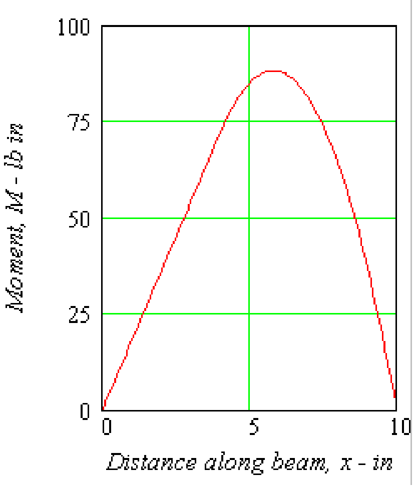
$$M(x) := R_1 \cdot S(x, 0 \cdot in) \cdot (x - 0)^1 - \frac{w}{2} \cdot S(x, a) \cdot (x - a)^2 + R_2 \cdot S(x, l) \cdot (x - l)^1$$

Plot the shear and moment diagrams.

(*b*) Shear Diagram



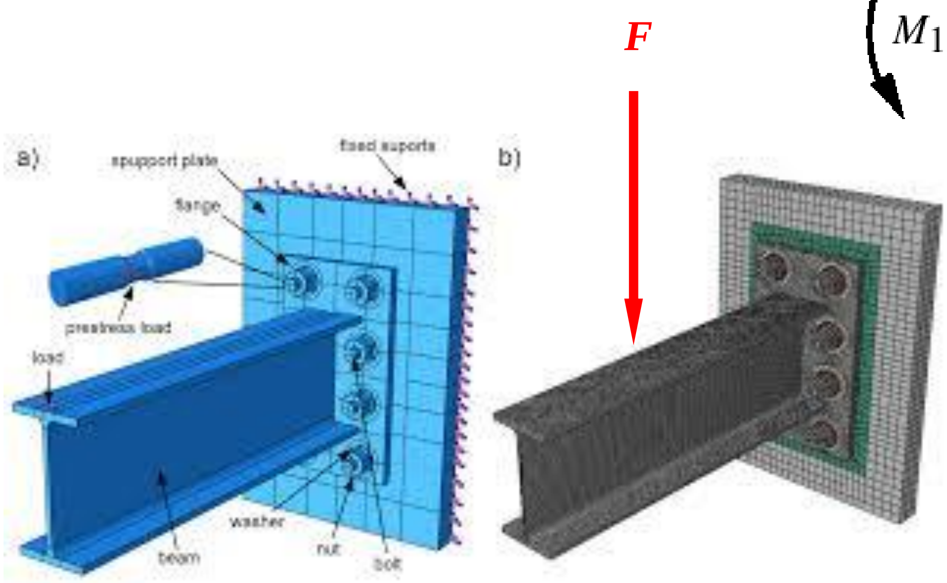
(*c*) Moment Diagram

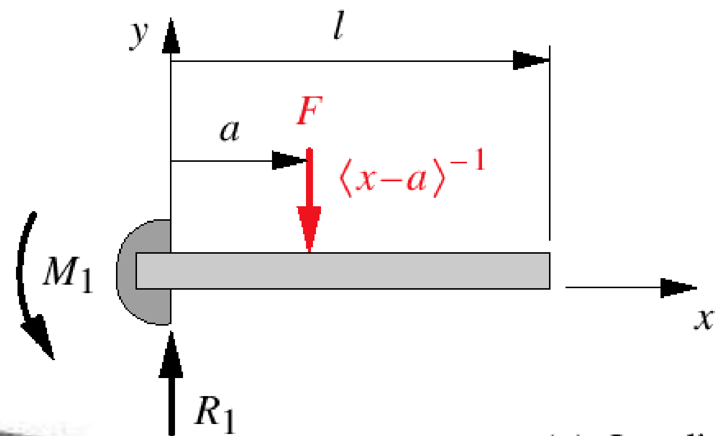


Shear and bending-moment diagrams

Singularity functions: example **E2**

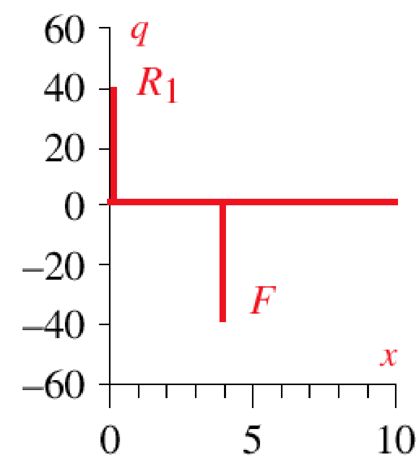
Determine and plot the shear and moment functions for the cantilever beam shown:





(*b*) Cantilever beam witl concentrated loadin

(*a*) Loading Diagram



Shear and bending-moment diagrams

Singularity functions: example **E2** - MathCad

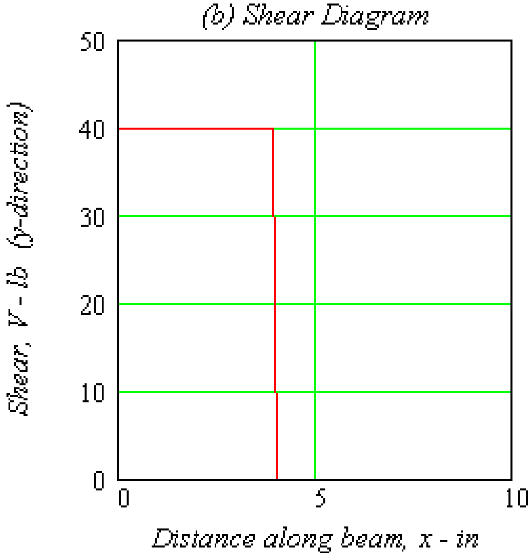
To generate the shear and moment functions over the length of the beam, equations (b) and (c) must be evaluated for a range of values of x from 0 to l, after substituting the above values of C_1, C_2, R_1, and M_1 in them. For a Mathcad solution, define a step function S. This function will have a value of zero when x is less than the dummy variable z, and a value of one when it is greater than or equal to z. It will have the same effect as the singularity function.

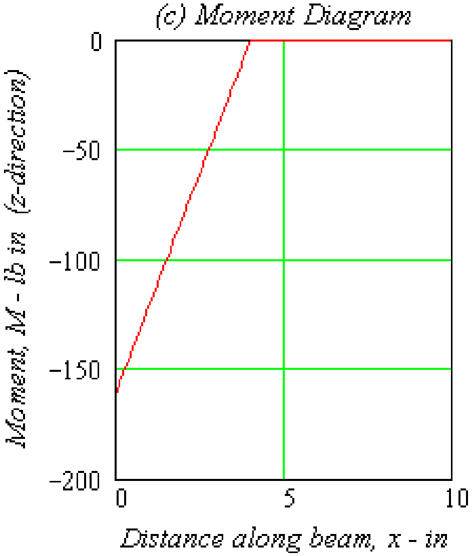
Range of x	$x := 0 \cdot in, 0.01 \cdot l .. l$
Unit step function	$S(x,z) := if(x \geq z, 1, 0)$

Write the shear and moment equations in Mathcad form, using the function S as a multiplying factor to get the effect of the singularity functions.

$$V(x) := R_1 \cdot S(x, 0 \cdot in) \cdot (x-0)^0 - F \cdot S(x,a) \cdot (x-a)^0$$

$$M(x) := -M_1 \cdot S(x, 0 \cdot in) \cdot (x-0)^0 + R_1 \cdot S(x, 0 \cdot in) \cdot (x-0)^1 - F \cdot S(x,a) \cdot (x-a)^1$$

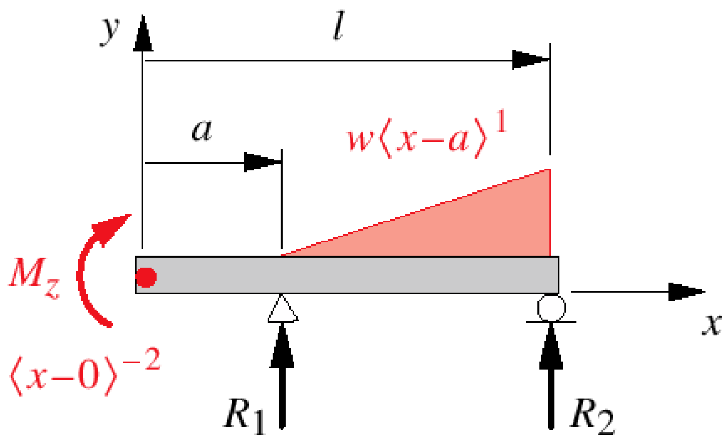




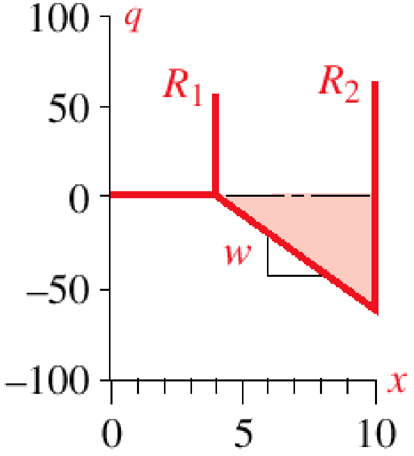
Shear and bending-moment diagrams

Singularity functions: example **E3**

Determine and plot the shear and moment functions for the beam shown:



(c) Overhung beam with moment and linearly distributed loading



Shear and bending-moment diagrams

Singularity functions: example **E3** - MathCad

To generate the shear and moment functions over the length of the beam, equations (*b*) and (*c*) must be evaluated for a range of values of *x* from 0 to *l*, after substituting the above values of C_1, C_2, R_1, and R_2 in them. For a Mathcad solution, define a step function *S*. This function will have a value of zero when *x* is less than the dummy variable *z*, and a value of one when it is greater than or equal to *z*. It will have the same effect as the singularity function.

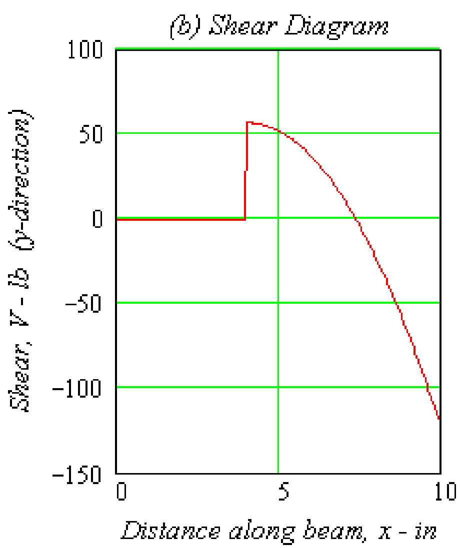
Range of *x* $\qquad x := 0\cdot in, 0.005\cdot l .. l$

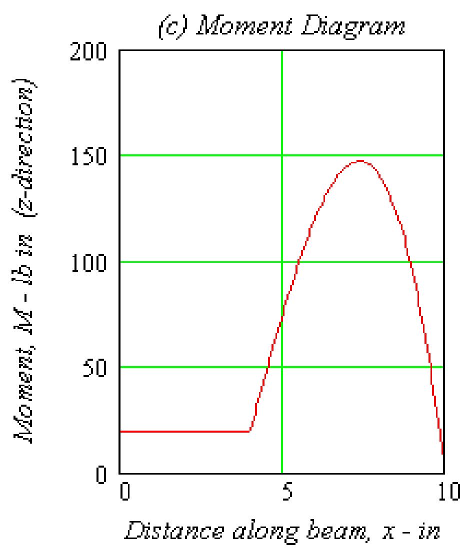
Unit step function $\qquad S(x,z) := if(x \geq z, 1, 0)$

Write the shear and moment equations in Mathcad form, using the function *S* as a multiplying factor to get the effect of the singularity functions.

$$V(x) := R_1\cdot S(x,a)\cdot(x-a)^0 - \frac{w}{2}\cdot S(x,a)\cdot(x-a)^2 + R_2\cdot S(x,l)\cdot(x-l)^0$$

$$M(x) := M_1\cdot S(x,0\cdot in)\cdot(x-0)^0 + R_1\cdot S(x,a)\cdot(x-a)^1 - \frac{w}{6}\cdot S(x,a)\cdot(x-a)^3 \ldots$$
$$+ R_2\cdot S(x,l)\cdot(x-l)^1$$





Reading assignment

- Chapters 1, 3, and 9 of textbook
- Review notes and text: ES-2501, ES-2502

Homework assignment

- **Author's:** posted in Website of our course
- **Solve:** posted in Website of our course