Worcester Polytechnic Institute
Mechanical Engineering Department

Optical Metrology and NDT
ME-593L, C'2018

Introduction: Fringe Skeletonization
February 2018
Quantitative analysis

Fringe skeletonization

• Recording of interferograms
• Identification of boundary area (AOI)
• Preprocessing, e.g., filtering
• Identification of fringe centers
• Numbering of interference fringes: fringe ordering
• Reconstruction of phase using interpolation
  ■ Review reference papers (list is on next page)
Quantitative analysis

Fringe skeletonization

Reference papers:

- J. Budzinski, “SNOP: a method for skeletonization of a fringe pattern along the fringe direction.”
Fringe skeletonization

Recording of interferograms

- Classic interferometry
  - Michelson
  - Mach-Zehnder
  - Sagnac
- Holographic interferometry
- Digital holographic interferometry
- Speckle pattern interferometry
- White-light interferometry
Fringe skeletonization

Identification of boundary area

- Identification of area of interest (AOI)
  - Area of operation
  - Selection of convolution Kernel
  - Minimization of power leakage
Fringe skeletonization

Pre-processing, e.g., filtering: convolution

Application of digital spatial convolution to the pixel located at the \((m, n)\) image plane position

Image convolution:

\[
Q_D(m, n) * H_{kl} = Q'_D(m, n)
\]

Original image  \(Q_D(m, n)\)

Kernel  \(H_{kl}\)

Convolved image  \(Q'_D(m, n)\)
Fringe skeletonization

Pre-processing, e.g., filtering: convolution

Digital spatial convolution by scanning the convolution kernel line by line over the entire image plane.

Kernel (3 x 3, in this case)
Fringe skeletonization

Pre-processing, e.g., filtering: convolution

Low-pass filtering

Image convolution:

\[ Q'_D(m,n) = \frac{1}{(2r + 1) \cdot (2r + 1)} \sum_{k=-r}^{r} \sum_{l=-r}^{r} Q_D(m+k, n+l) \]

Weighting factors:

\[ H_{kl} = \frac{1}{(2r + 1) \cdot (2r + 1)} \]
Fringe skeletonization

Pre-processing, e.g., filtering: convolution

Median filtering

Digital spatial convolution to median filtering the pixel value located at the \((m,n)\) image plane position

Original image

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Convolved image

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Sorted list

12 14 16 25 31 33 70 74 133

Kernel (3 x 3, in this case)
Fringe skeletonization

Pre-processing, e.g., filtering: convolution

Fourier filtering

Fourier transformation:

\[ Q_D(f_u, f_v) = \mathcal{F}\{Q_D(m,n)\} \]

Convolution in the frequency domain:

\[ Q_D^T(f_u, f_v) = Q_D(f_u, f_v) \cdot W_f(f_u, f_v) \]

\( W_f(f_u, f_v) \) is the weighting function

Inverse Fourier transformation:

\[ Q_D'(m,n) = \mathcal{F}^{-1}\{Q_D^T(f_u, f_v)\} \]
Fringe skeletonization

Pre-processing, e.g., filtering: convolution

Fourier filtering

Fourier transformation:

\[ Q_D(f_u, f_v) = \mathcal{F}\{Q_D(m,n)\} \]

Original interferogram

Original interferogram: frequency domain
Fringe skeletonization

Pre-processing, e.g., filtering: convolution

Fourier filtering

\[ W_f(f_u, f_v) \text{, weighting function: Gaussian function} \]

2D representation

3D representation
Fringe skeletonization

Pre-processing, e.g., filtering: convolution

Fourier filtering

Convolution in the frequency domain:

\[ Q_D^T(f_u, f_v) = Q_D(f_u, f_v) \cdot W_f(f_u, f_v) \]

Inverse Fourier transformation:

\[ Q_D'(m, n) = \mathcal{F}^{-1}\{Q_D^T(f_u, f_v)\} \]
Fringe skeletonization

Pre-processing: thinning

Multiple algorithms exist, e.g., contrast enhancement, edge detection, etc.
Fringe skeletonization
Identification of fringe centers

Fringe centers

Thinned fringes: contrast enhancement
Semi-quantitative analysis

1 fringe is \( \approx 2\pi/\lambda \): fringe ordering and counting

Interferogram of a turbine blade: contouring

Interferogram of a turbine blade: vibrations

Fringe-locus function (fringe localization):

\[
\Omega(x, y, z) = K \cdot L \\
\Omega(x, y, z) = 2\pi n \\
n = \text{is the fringe order}
\]

- A fringe represents a contour of constant phase
- \( n \) is the fringe order or “number of waves”
Fringe skeletonization

Sample: identification of fringe centers

Closed fringe pattern and continuous deformations:
Fringe skeletonization

Fringe ordering: $n$-th order

- Identify zero-order: understanding of the physical phenomena
- Use sequential ordering

Fringe orders: $n$

Zero order

-2 -1 0 1 2 3

-2 -1 0 1 2 3
Fringe skeletonization

Fringe ordering

Opened fringe pattern and continuous deformations:

Intensity line: A-A

Fringe ordering:
Fringe skeletonization

Phase reconstruction

Fringe interpolation, e.g., Lagrange, Zernike polynomials, etc.

Lagrange polynomials of degree $n-1$

\[
f(x_0) = \sum_{i=1}^{n} w_i(x_0)f(x_i)
\]

\[
w_i(x_0) = \frac{\prod_{j=1, j\neq i}^{n}(x_0-x_j)}{\prod_{j=1}^{n}(x_i-x_j)}
\]

\[
f(x_0) = w_1(x_0)f(x_1) + w_2(x_0)f(x_2) \\
+ w_3(x_0)f(x_3) + \ldots + w_n(x_0)f(x_n)
\]
Fringe skeletonization

Phase reconstruction

Fringe interpolation, e.g., Lagrange, Zernike polynomials, etc.

Lagrange polynomials of degree \( n-1 \)

\[ n = 2: \]

\[ w_1(x_0) = \frac{x_0 - x_2}{x_1 - x_2} \]

\[ w_2(x_0) = \frac{x_0 - x_1}{x_2 - x_1} \]

and

\[ f(x_0) = w_1(x_0) \cdot f(x_1) + w_2(x_0) \cdot f(x_2) \]

\[ = \frac{x_0 - x_2}{x_1 - x_2} \cdot f(x_1) + \frac{x_0 - x_1}{x_2 - x_1} \cdot f(x_2) \]
Fringe skeletonization

Phase reconstruction: \(1\)-fringe = \(\lambda/2\)

Reconstruction along specific lines:

Interpolation along specific lines: