STRESS ANALYSIS
ES-2502, D'2020

We will get started soon...

08 May 2020
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Lecture 24:
Unit 21: Bending of beams:
Deflection analysis (Ch.12, textbook)

08 May 2020
General information

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Deflection of beams and shafts

The elastic curve

We can study how beams and shafts deflect by knowing both:

a) Distribution of bending moments (\( V-M \) diagrams), and
b) Material & geometrical properties of components
Deflection of beams and shafts

(a) A B C D

(b) Moment diagram

(c) Elastic curve

Inflection point

$\Delta_A$ $\Delta_E$
Deflection of beams and shafts

(a) A

(b) M

(c) A

Elastic curve

Moment diagram

Inflection point

$\Delta_D$

$\Delta_C$
Bending deformation of straight beams

\[ \Delta s = \Delta x \]

longitudinal axis, \( x \)
Bending deformation of straight beams

Normal strain:

\[ \varepsilon_x = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s} \]
Bending deformation of straight beams

Normal strain:

\[
\varepsilon_x = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}
\]

\[
\varepsilon_x = \lim_{\Delta \theta \to 0} \frac{(\rho - y) \cdot \Delta \theta - \rho \cdot \Delta \theta}{\rho \cdot \Delta \theta}
\]

\[
\Delta s' = (\rho - y) \cdot \Delta \theta
\]

\[
\Delta x = \Delta s = \rho \Delta \theta
\]

\[
\rho = -\frac{y}{\varepsilon_x}
\]

\[
\frac{1}{\rho} = -\frac{\varepsilon_x}{y}
\]
Bending deformation of straight beams

From before:

\[
\frac{1}{\rho} = -\frac{\varepsilon_x}{y}
\]

Hook’s law:

\[
\varepsilon_x = \frac{\sigma_x}{E}
\]

Flexure formula:

\[
\sigma_x = -\frac{M \cdot y}{I_{zz}}
\]
Bending deformation of straight beams

The elastic curve

Radius of curvature is computed by:

\[
\frac{1}{\rho} = \frac{\frac{d^2 y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{2/3}}
\]

Therefore,

\[
\frac{\frac{d^2 y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{2/3}} = \frac{M}{E \cdot I_{zz}}
\]
Bending deformation of straight beams

The elastic curve

For small deformations:

\[ \frac{d^2 y}{dx^2} = \frac{M}{E \cdot I_{zz}} \]

Elastica equation

Important to remember!!
Bending deformation of straight beams
The elastic curve

For small deformations:

\[ \frac{d^2 y}{dx^2} = \frac{M}{E \cdot I_{zz}} \]

\[ \frac{d}{dx} \left( E \cdot I_{zz} \frac{d^2 y}{dx^2} \right) = V(x) \]

\[ \frac{d^2}{dx^2} \left( E \cdot I_{zz} \frac{d^2 y}{dx^2} \right) = w(x) \]
Bending deformation of straight beams

The elastic curve

\[ E \cdot I_{zz} \frac{d^4 y}{dx^4} = w(x) \]  \hspace{1cm} \text{Applied load}

\[ E \cdot I_{zz} \frac{d^3 y}{dx^3} = V(x) \]  \hspace{1cm} \text{Shear force}

\[ E \cdot I_{zz} \frac{d^2 y}{dx^2} = M \]  \hspace{1cm} \text{Elastica equation}
Bending deformation of straight beams

The elastic curve

\[
\frac{w}{EI} = \frac{d^4 y}{dx^4}
\]
Load function – deflection

\[
\frac{V}{EI} = \frac{d^3 y}{dx^3}
\]
Shear function – deflection

\[
\frac{M}{EI} = \frac{d^2 y}{dx^2}
\]
Moment function – \textit{elastica}

\[
\theta = \frac{dy}{dx}
\]
Slope – deflection

\[
y = f(x)
\]
Deflection
Bending deformation of straight beams

**Boundary and continuity conditions**

\[ \Delta = \text{displacement} \]

\[ \theta = \text{slope of displacement} \]
Bending deformation of straight beams: example A

The cantilever shown is subjected to a vertical load $P$ at its end. Determine the equation of the deformation (elastic) curve. $E \cdot I$ is constant.
Bending deformation of straight beams: example B

The beam is made of two rods and is subjected to the concentrated load $P$. Determine the maximum deflection of the beam if the moments of inertia of the rods are $I_{AB}$ and $I_{BC}$, and the modulus of elasticity is $E$.

Apply boundary and continuity conditions
Reading assignment

- Chapter 12 of textbook
- Review notes and text: ES2001, ES2501
Homework assignment

• As indicated on webpage of our course