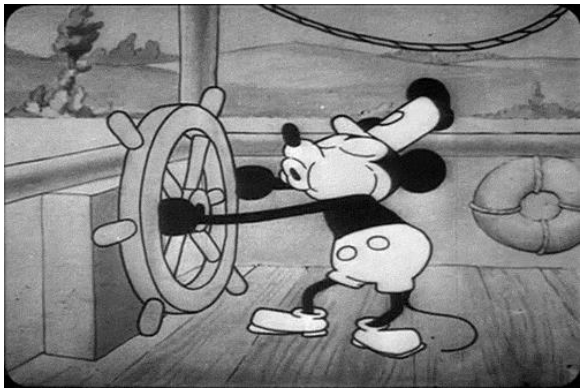
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, D'2020

We will get started soon...



07 May 2020

WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, D'2020

We will get started soon...

Lecture 23:
Unit 18, 19: Bending of beams::
Bending of beams: transverse shear; section properties

07 May 2020

General information

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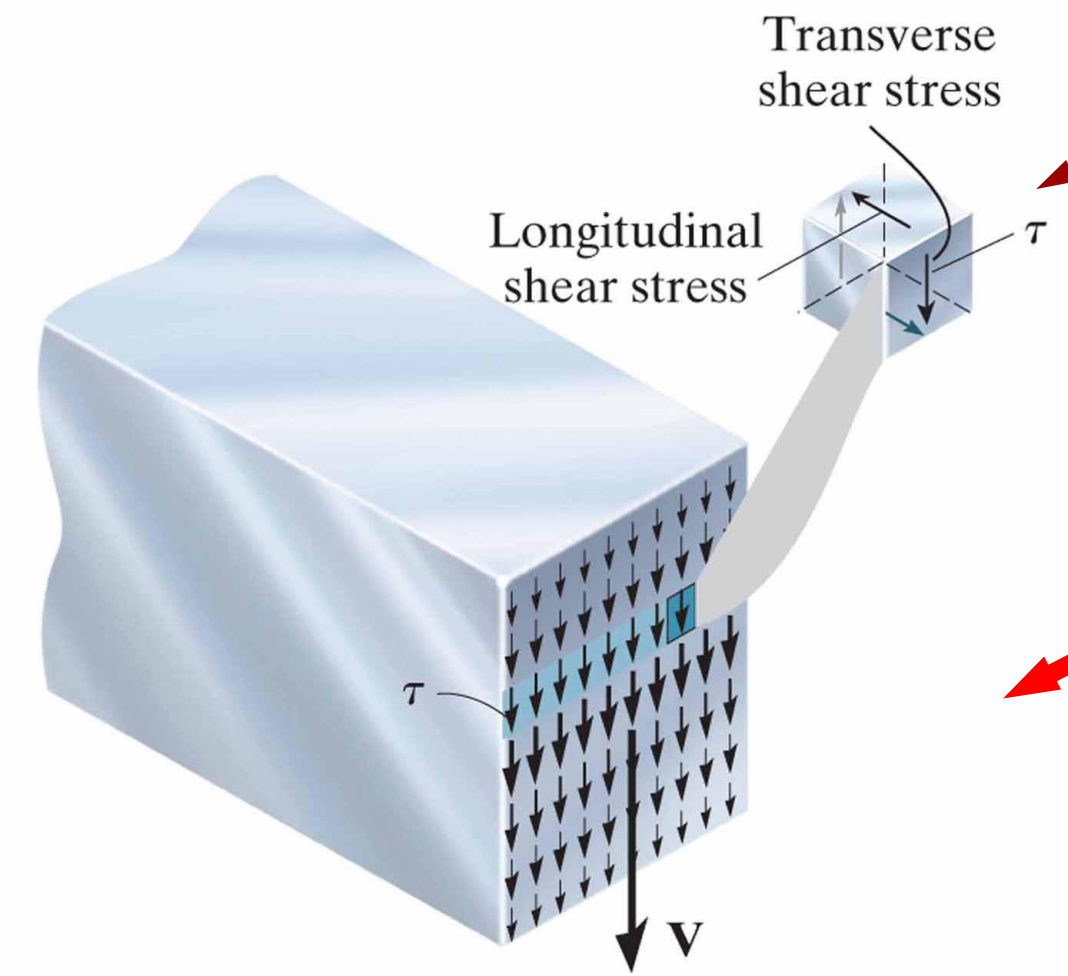
Email: zjzolotarevsky @ wpi.edu

Transverse shear: *produced by bending*

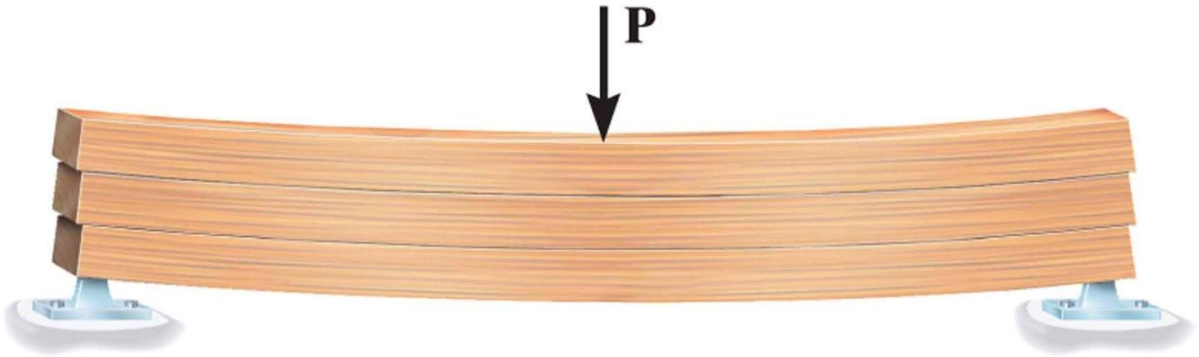
Observed in components subjected to bending loads



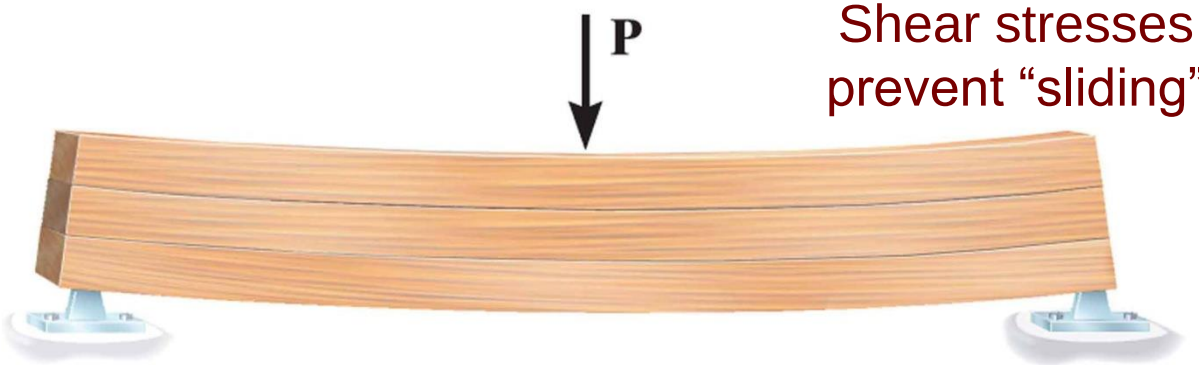
Differential stress element in equilibrium

Transverse shear: *produced by bending*

Observed in components subjected to bending loads



Boards not bonded together

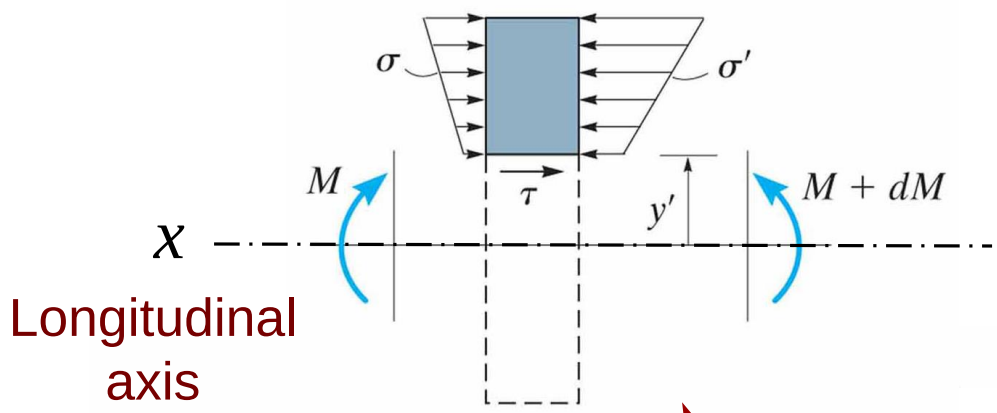


Shear stresses prevent "sliding"

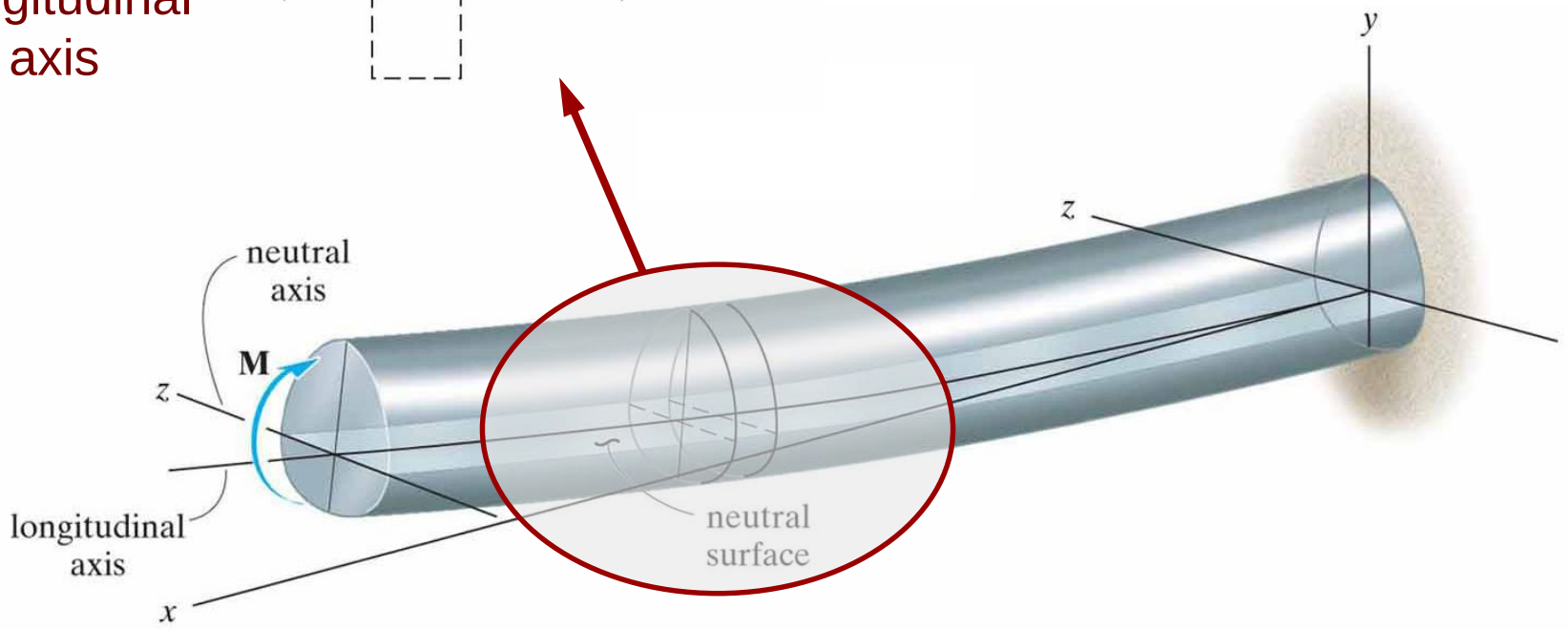
Boards bonded together

Transverse shear: *produced by bending*

Observed in components subjected to bending loads

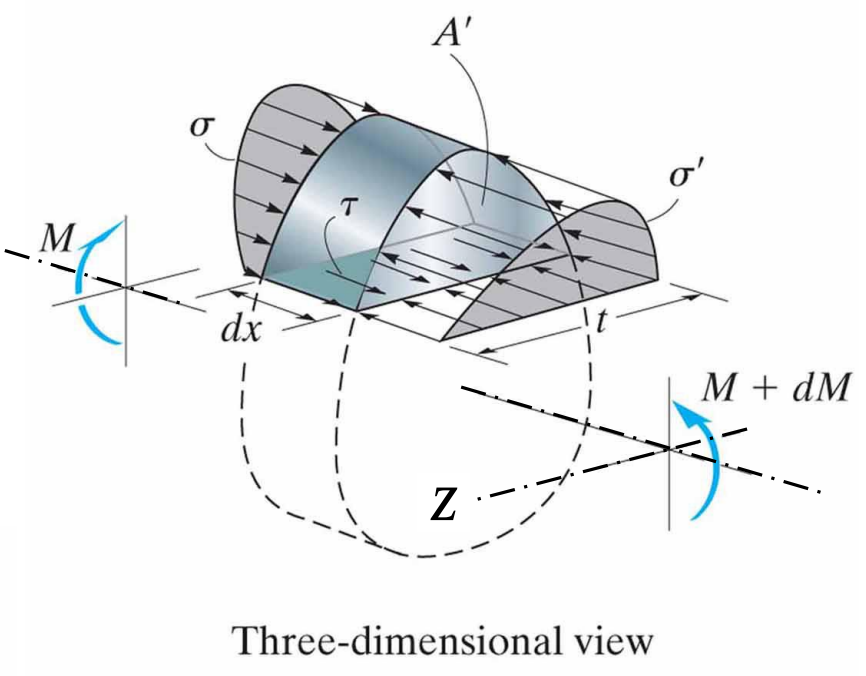


Longitudinal axis

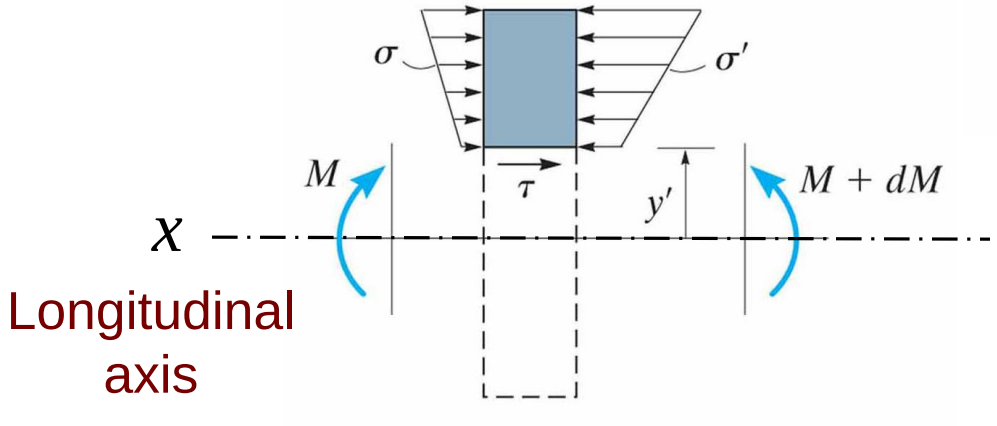


Transverse shear: *produced by bending*

Observed in components subjected to bending loads

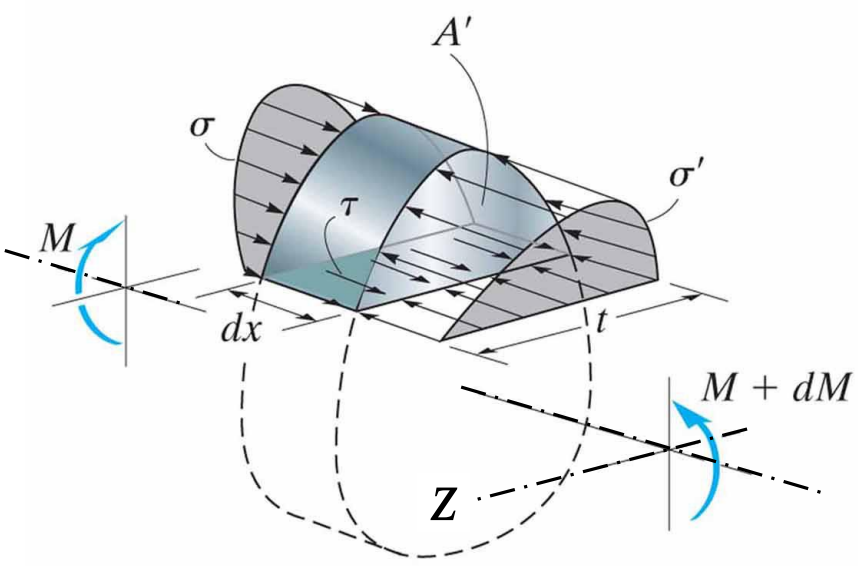


Three-dimensional view



Transverse shear: *produced by bending*

Observed in components subjected to bending loads

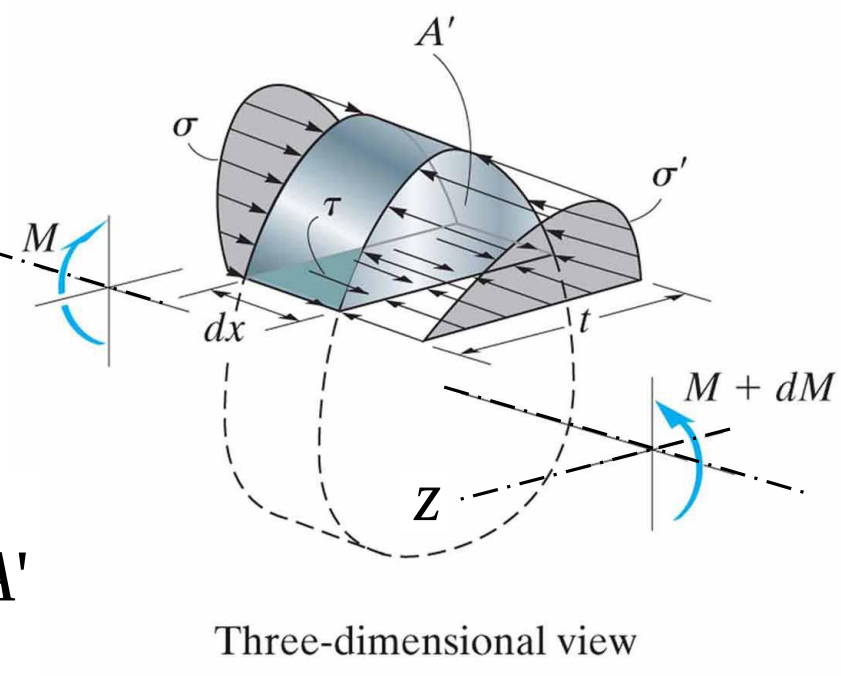


$$\sum F_x = 0; \quad +\leftarrow$$

$$\int_{A'} \sigma' \, dA' - \int_{A'} \sigma \, dA' - \tau (t \cdot dx) = 0$$

Transverse shear: *produced by bending*

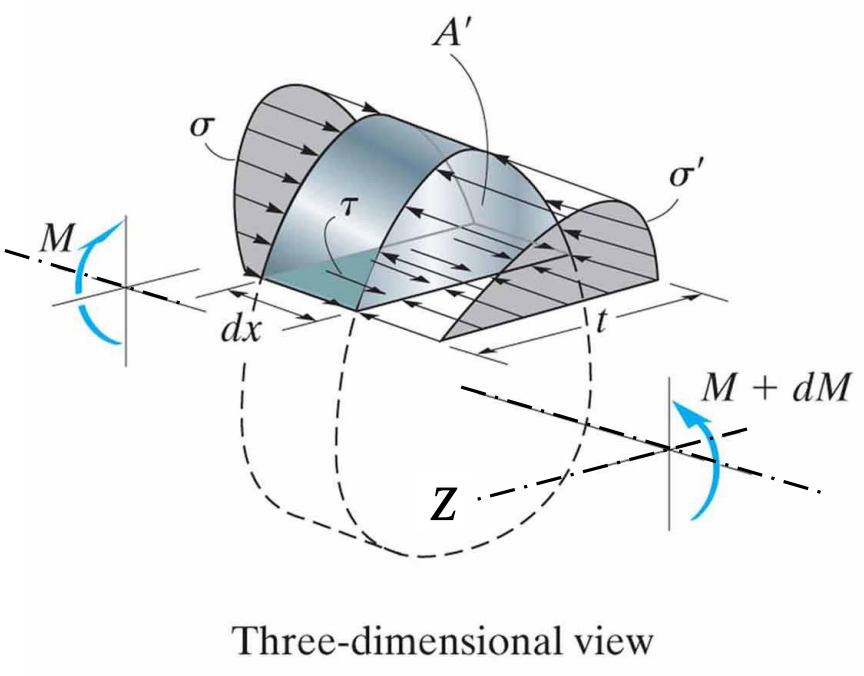
Observed in components subjected to bending loads



$$\int_{A'} \sigma' \, dA' = \int_{A'} \left(\frac{M + dM}{I_{zz}} \right) y \, dA'$$

Transverse shear: *produced by bending*

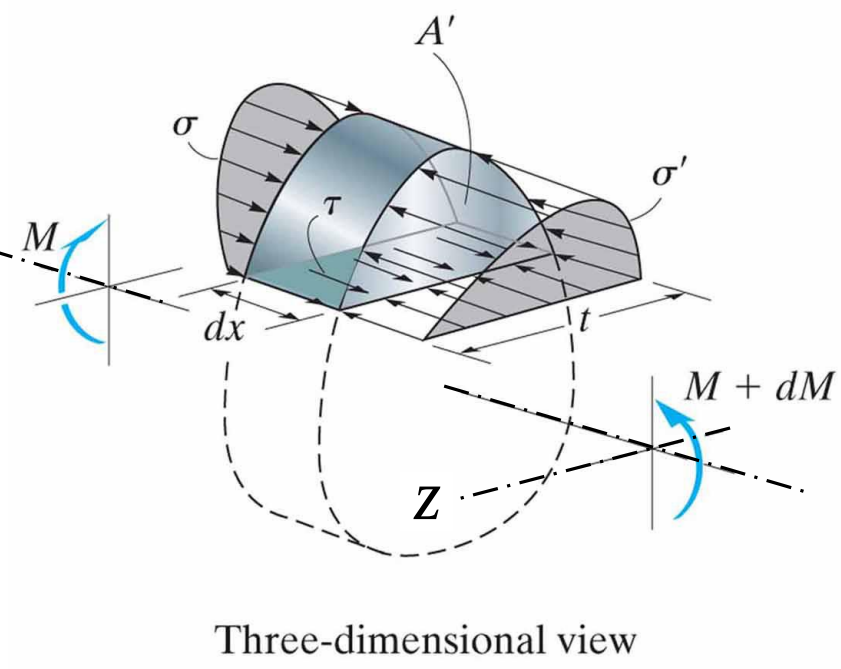
Observed in components subjected to bending loads



$$\int_{A'} \sigma \, dA' = \int_{A'} \left(\frac{M}{I_{zz}} \right) y \, dA'$$

Transverse shear: *produced by bending*

Observed in components subjected to bending loads

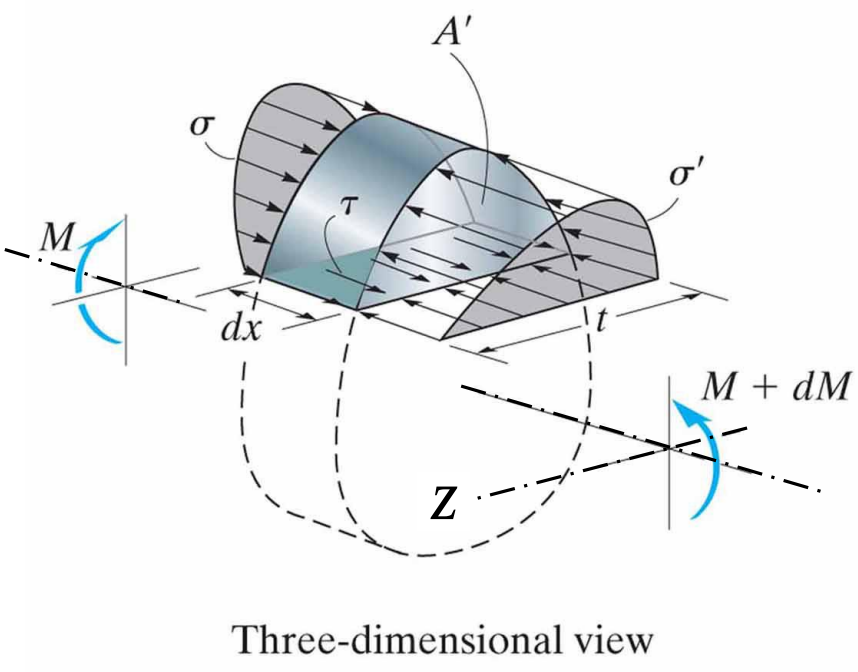


$$\sum F_x = 0; \quad + \leftarrow$$

$$\int_{A'} \left(\frac{M + dM}{I_{zz}} \right) y \, dA' - \int_{A'} \left(\frac{M}{I_{zz}} \right) y \, dA' - \tau (t \cdot dx) = 0$$

Transverse shear: *produced by bending*

Observed in components subjected to bending loads

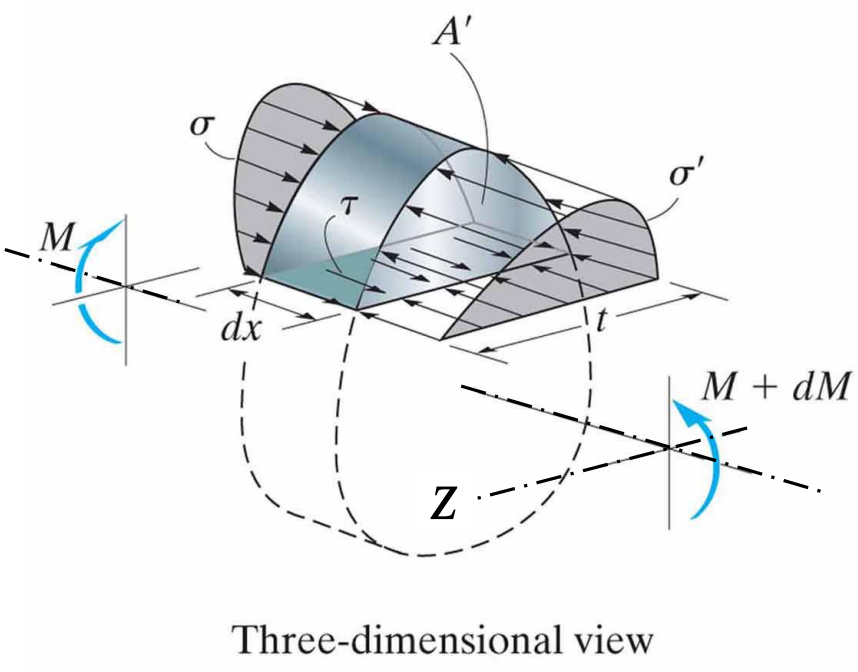


$$\sum F_x = 0; \quad + \leftarrow$$

$$\int_{A'} \left(\frac{dM}{I_{zz}} \right) y \, dA' - \tau (t \cdot dx) = 0$$

Transverse shear: *produced by bending*

Observed in components subjected to bending loads



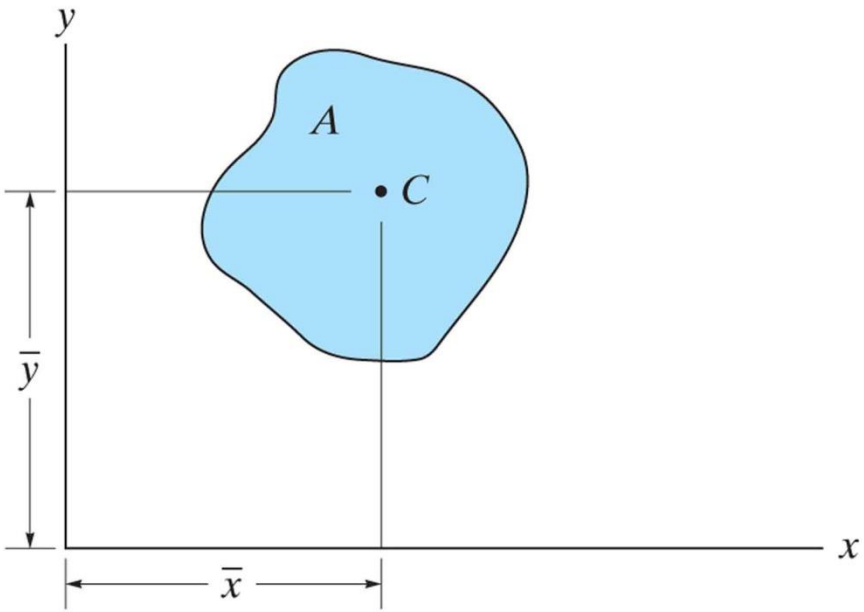
$$\sum F_x = 0; \quad +\leftarrow$$

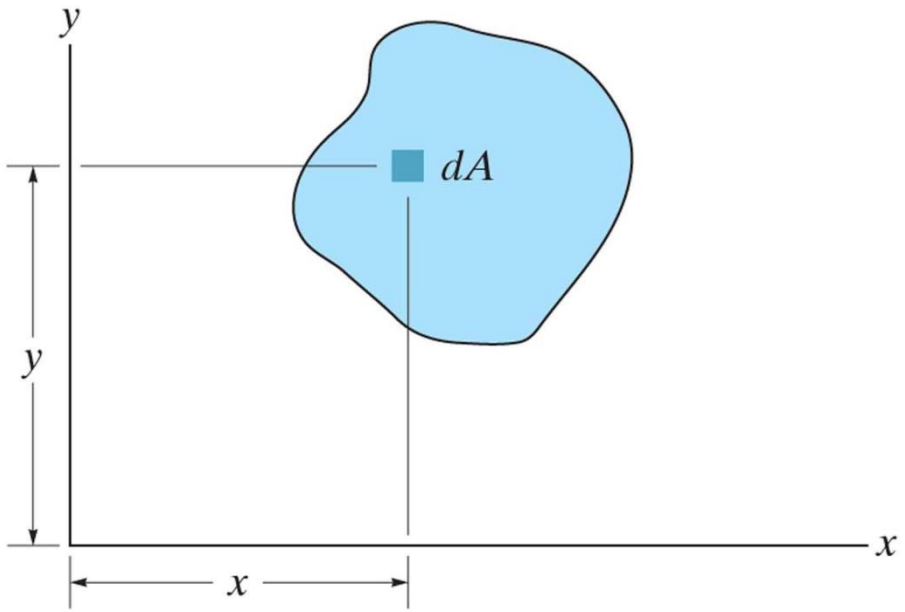
$$\tau = \frac{1}{I_{zz} \cdot t} \frac{dM}{dx} \int_{A'} y \, dA'$$

$$\frac{dM}{dx} = V$$

Centroid of an area

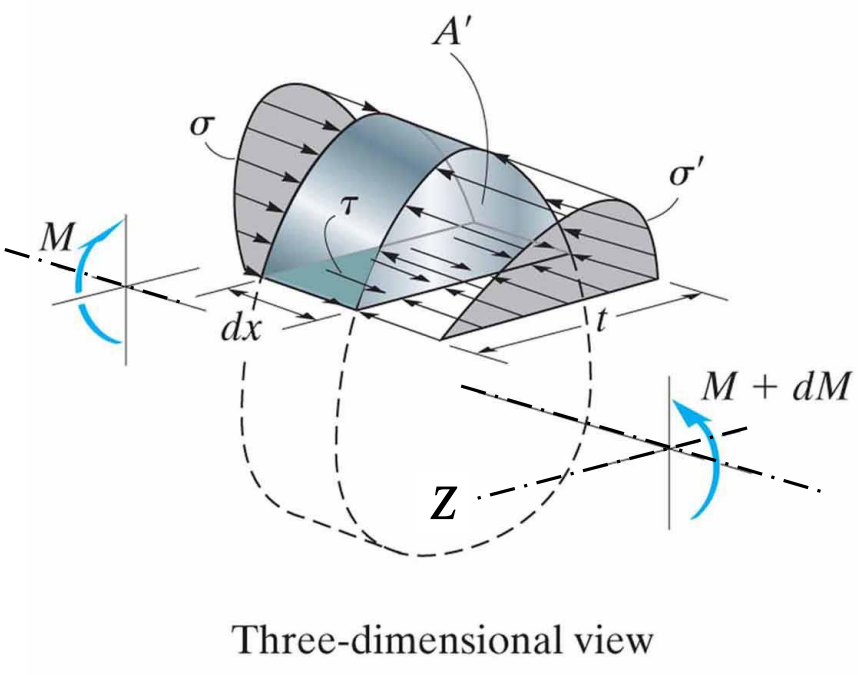
$$\bar{x} = \frac{\int_A x\, dA}{\int_A dA}; \qquad \bar{y} = \frac{\int_A y\, dA}{\int_A dA}$$





Transverse shear: *produced by bending*

Observed in components subjected to bending loads



$$\sum F_x = 0; \quad + \leftarrow$$

$$\tau = \frac{1}{I_{zz} \cdot t} \frac{dM}{dx} \int_{A'} y \, dA'$$

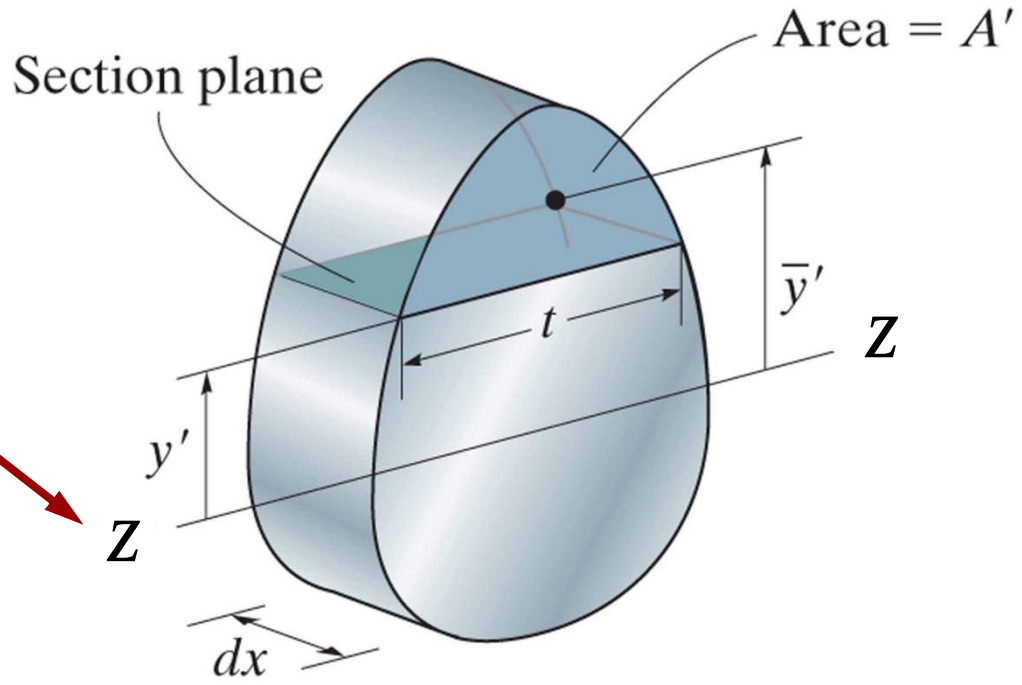
$$\int_{A'} y \, dA' = \bar{y}' A'$$

(related to centroid of A')

Transverse shear: *produced by bending*

Observed in components subjected to bending loads

$$\int_{A'} y\, dA' = \bar{y}'\, A' = Q$$

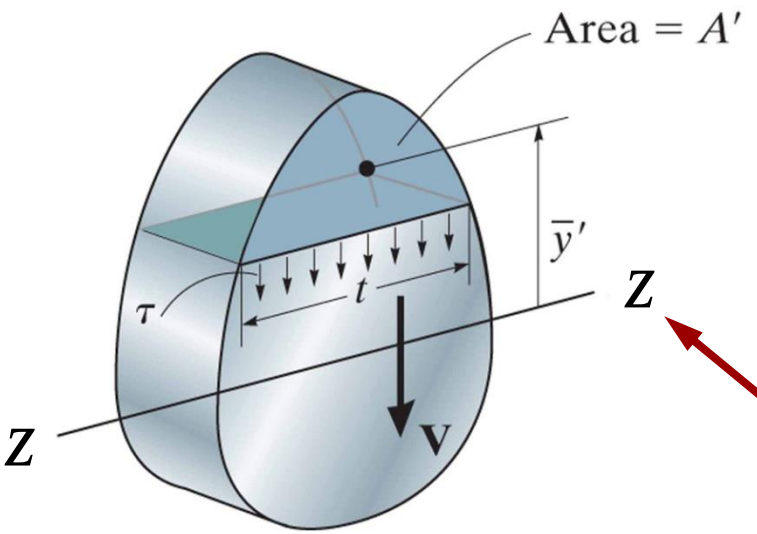


Passes through centroid of section

Transversal Shear formula: ***produced by bending***

Observed in components subjected to bending loads

$$\sum F_x = 0; \quad + \leftarrow \qquad \tau = \frac{1}{I_{zz} \cdot t} \frac{dM}{dx} \int_{A'} y \, dA'$$



$$\tau = \frac{V \cdot Q}{I_{zz} \cdot t}$$

$$Q = \bar{y}' A'$$

Important to remember!!

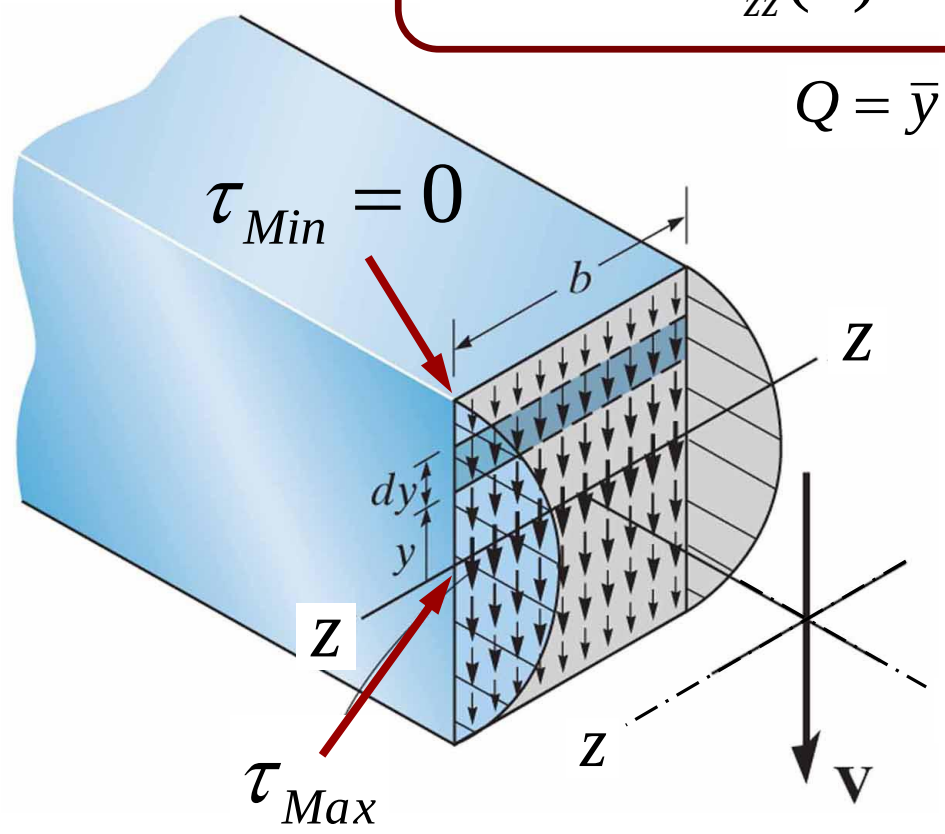
Passes through centroid of section

Transversal Shear formula: ***produced by bending***

Observed in components subjected to bending loads

$$\tau(x,y) = \frac{V(x)\cdot Q(y)}{I_{zz}(x)\cdot t(y)}$$

Important to remember!!



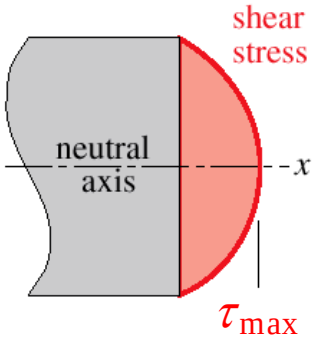
$$Q = \bar{y}'A'$$

Internal distribution of shear stresses:

$$\tau_{xy} = \tau_{yx}$$

Transversal Shear formula: ***produced by bending***

Observed in components subjected to bending loads

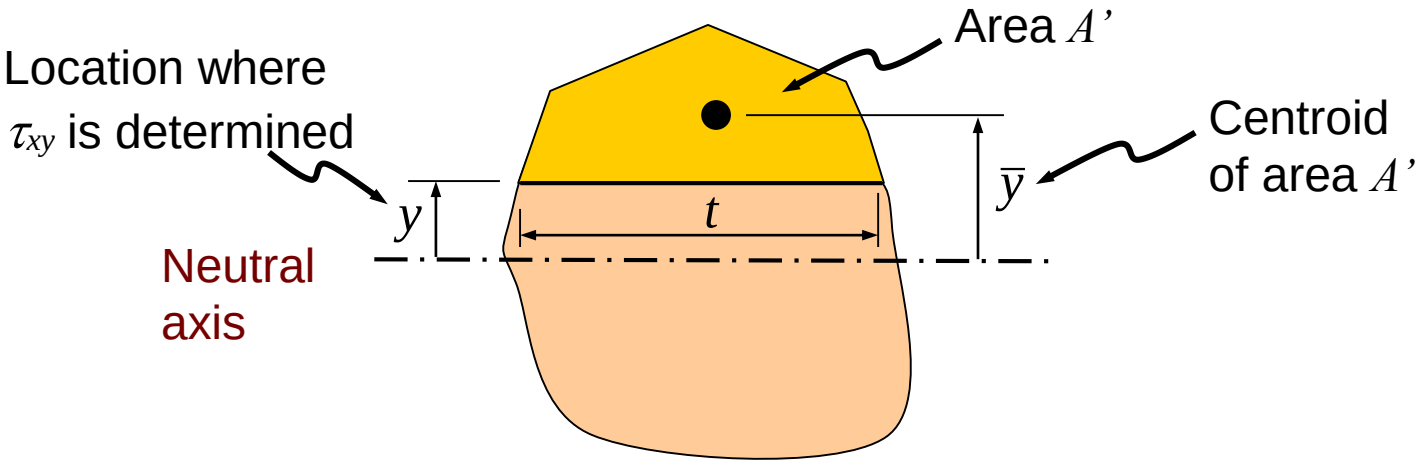


Transverse shear stress:

$$\tau_{xy} = \frac{V \cdot Q}{I \cdot t} \quad \text{with} \quad Q = \bar{y}\, A'$$

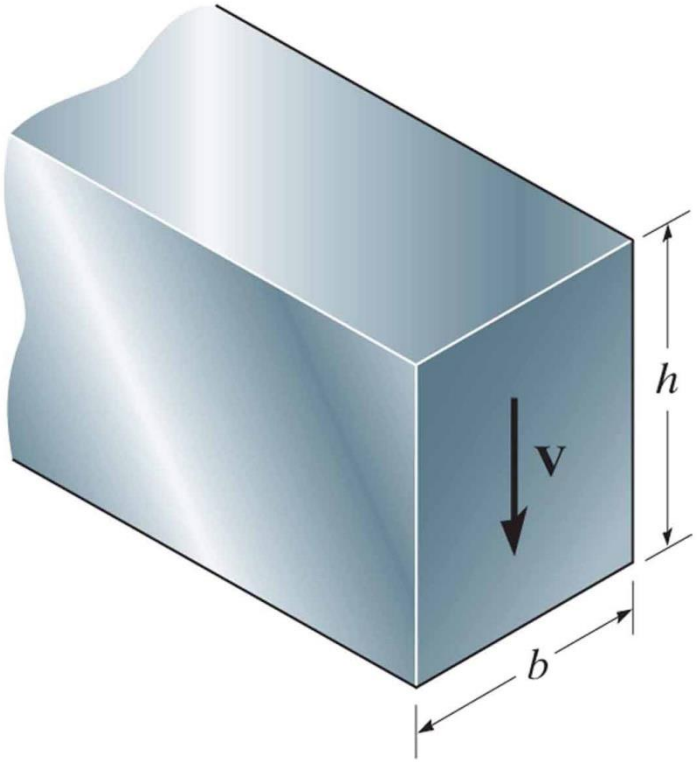
Recall that $V = V(x)$

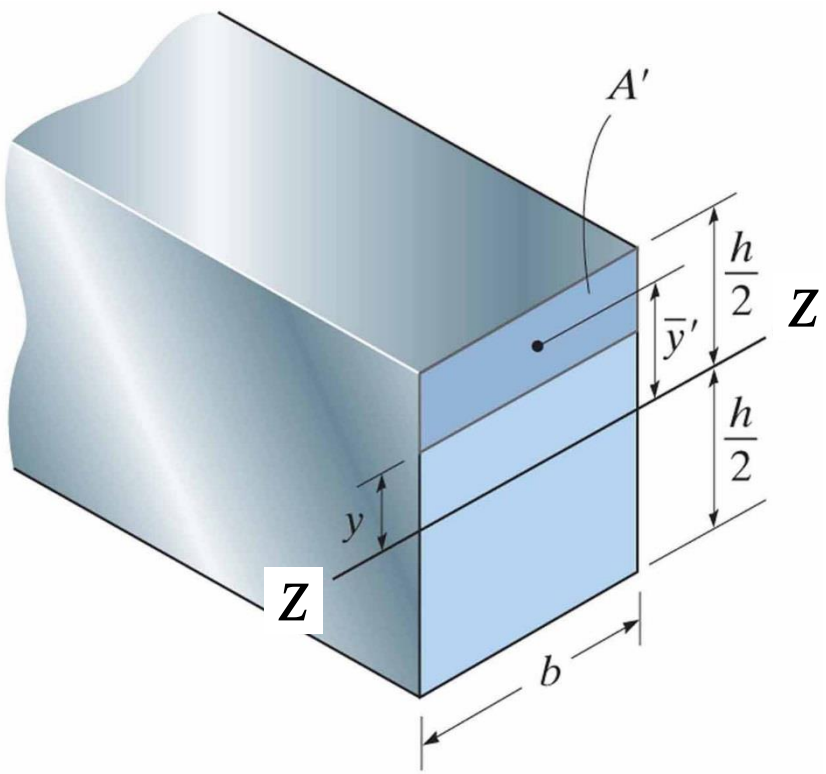
Generic cross-section:



Transversal shear formula**: example A**

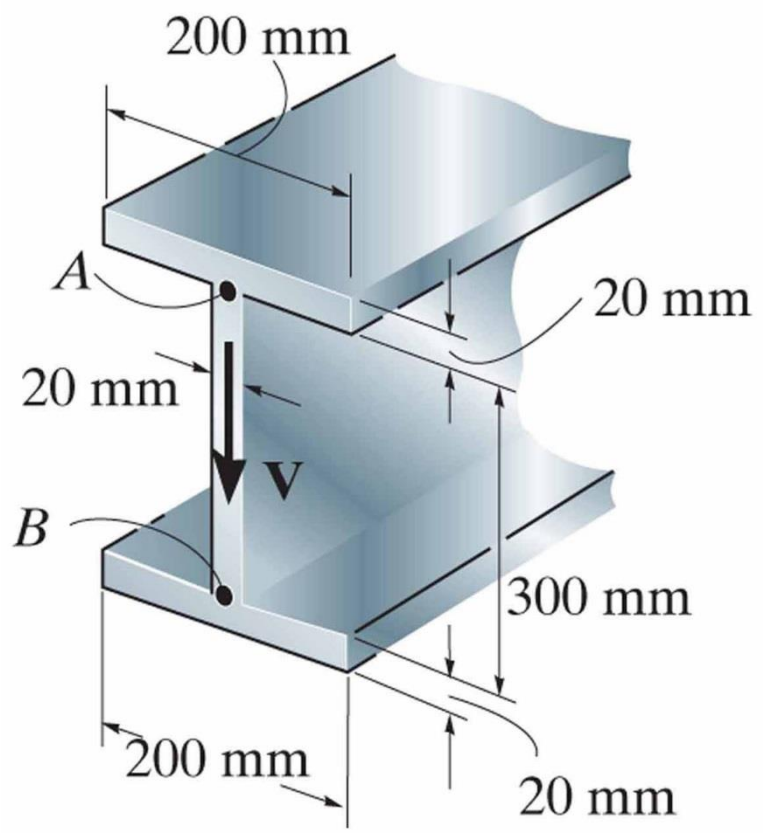
Determine the distribution of the transversal shear stresses over the cross section of the beam shown





Transversal shear formula**: example B**

If the wide-flange beam is subjected to a shear of $V = 20\ \text{kN}$, determine the shear stress on the web at A. Indicate the shear-stress components on a volume element located at this point. Determine the distribution of the transversal shear stresses over the cross section of the beam shown



Note that V is given.

It is obtained from $V(x)$ function evaluated at location x of interest.

Reading assignment

- Chapter 7 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course