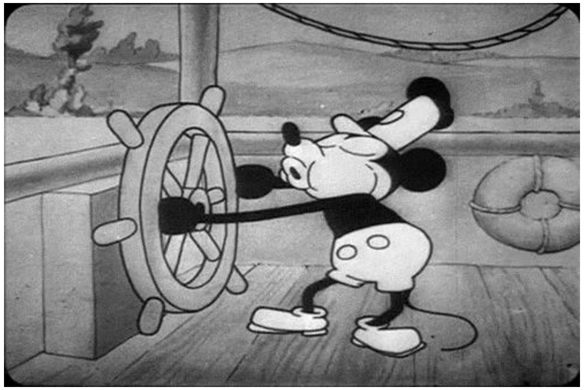
# WORCESTER POLYTECHNIC INSTITUTE
# MECHANICAL ENGINEERING DEPARTMENT

## STRESS ANALYSIS
## ES-2502, D'2020

**We will get started soon...**



**10 April 2020**

# WORCESTER POLYTECHNIC INSTITUTE
# MECHANICAL ENGINEERING DEPARTMENT

## STRESS ANALYSIS
## ES-2502, D'2020

**We will get started soon...**

**Lecture 10:**
**Unit 6: tension/compression of slender longitudinal bars: *general***

**10 April 2020**

# General information

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# Strain: example A

A concrete cylinder having a diameter of 6.0 in and a gauge length of 12 in is tested in compression. The results of the test are reported in the table as load versus contraction. Draw stress-strain diagram and estimate modulus of elasticity.

| Displacement, in | Load, kip |
|---|---|
| 0.0000 | 0.0 |
| 0.0006 | 5.0 |
| 0.0012 | 9.5 |
| 0.0020 | 16.5 |
| 0.0026 | 20.5 |
| 0.0036 | 25.5 |
| 0.0040 | 30.0 |
| 0.0045 | 34.5 |
| 0.0050 | 38.5 |
| 0.0062 | 46.5 |
| 0.0070 | 50.0 |
| 0.0075 | 53.0 |

# Strain: example A

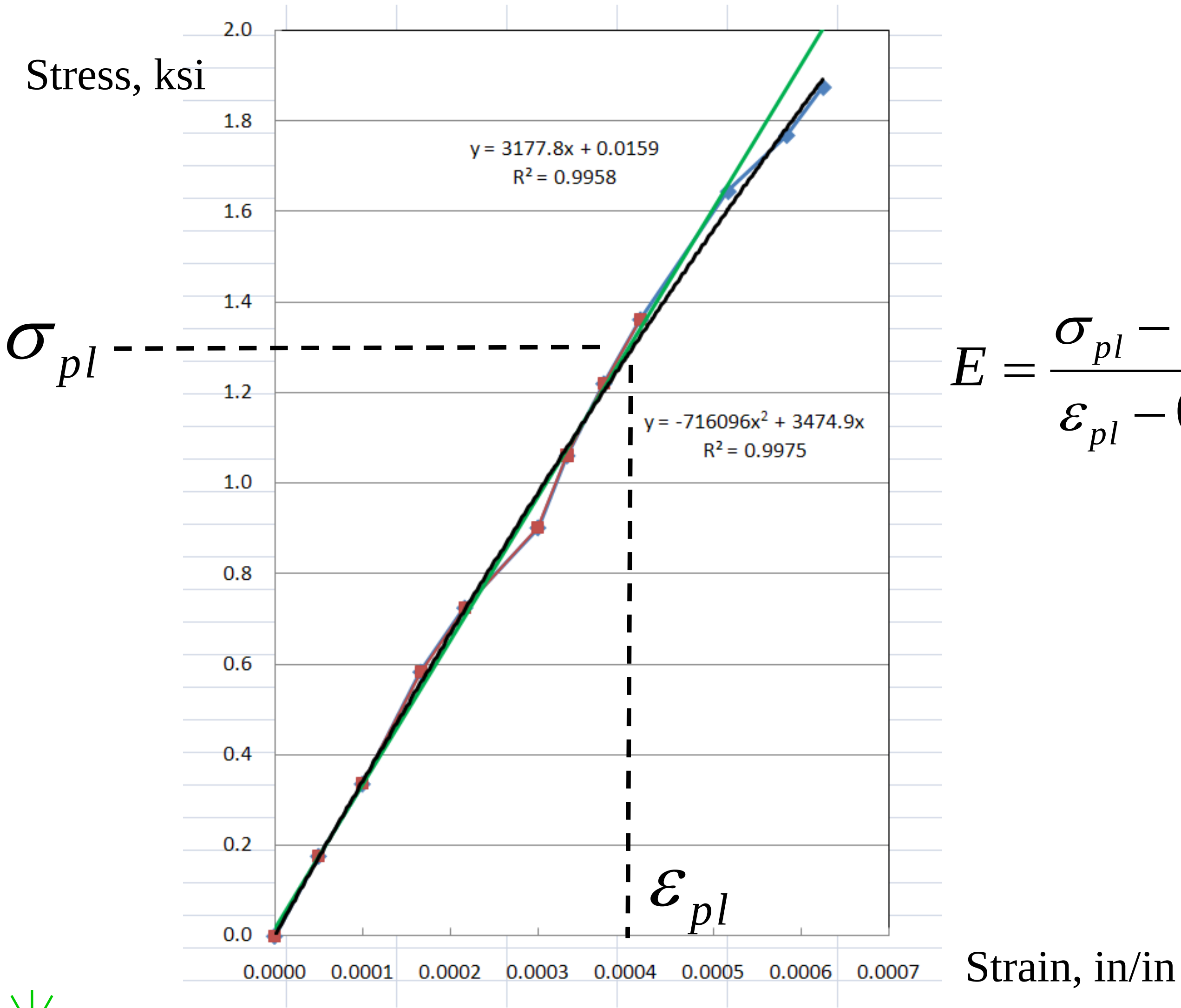
Compute stress and strain table:

| Displacement, in | Load, kip |
|---|---|
| 0.0000 | 0.0 |
| 0.0006 | 5.0 |
| 0.0012 | 9.5 |
| 0.0020 | 16.5 |
| 0.0026 | 20.5 |
| 0.0036 | 25.5 |
| 0.0040 | 30.0 |
| 0.0045 | 34.5 |
| 0.0050 | 38.5 |
| 0.0062 | 46.5 |
| 0.0070 | 50.0 |
| 0.0075 | 53.0 |

| Strain, in/in | Stress, kpsi |
|---|---|
| 0.000000 | 0.000 |
| 0.000050 | 0.177 |
| 0.000100 | 0.336 |
| 0.000167 | 0.584 |
| 0.000217 | 0.725 |
| 0.000300 | 0.902 |
| 0.000333 | 1.061 |
| 0.000375 | 1.220 |
| 0.000417 | 1.362 |
| 0.000517 | 1.645 |
| 0.000583 | 1.768 |
| 0.000625 | 1.874 |

# Strain: example A

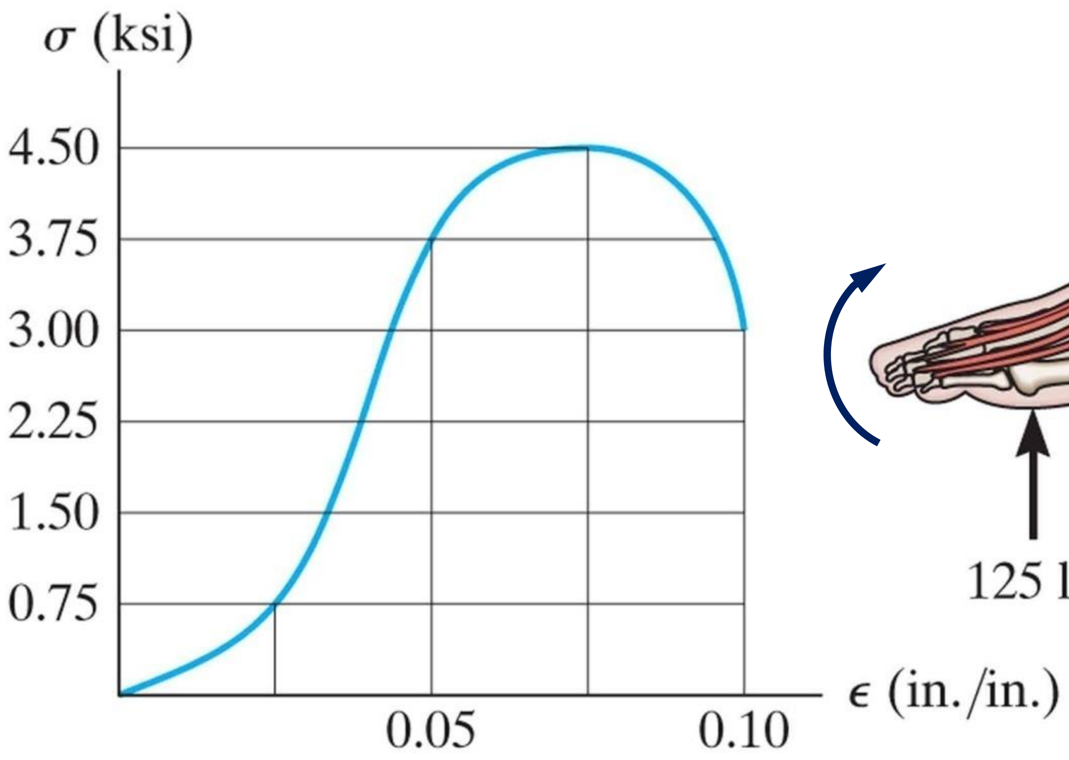
Plot data and estimate yield point:

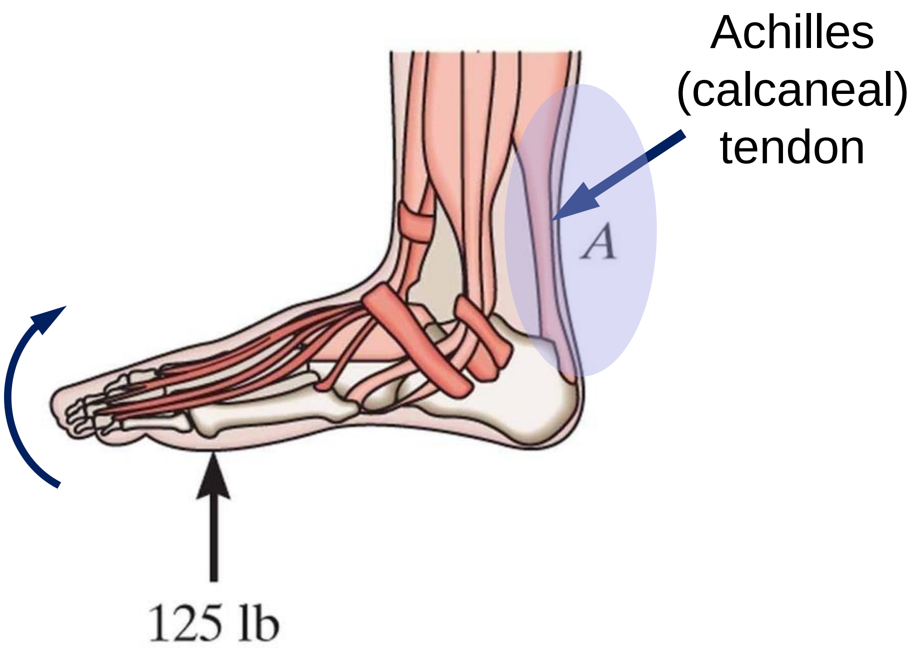



$$E = \frac{\sigma_{pl} - 0}{\varepsilon_{pl} - 0} \approx 3.27 \times 10^3 \ ksi$$

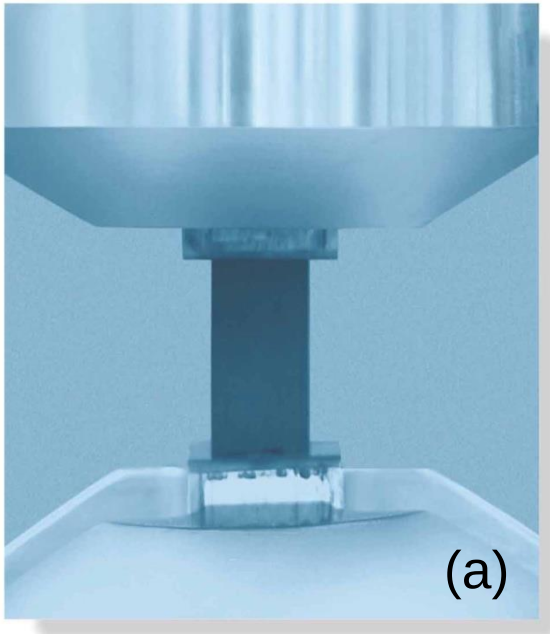
# Strain: example B

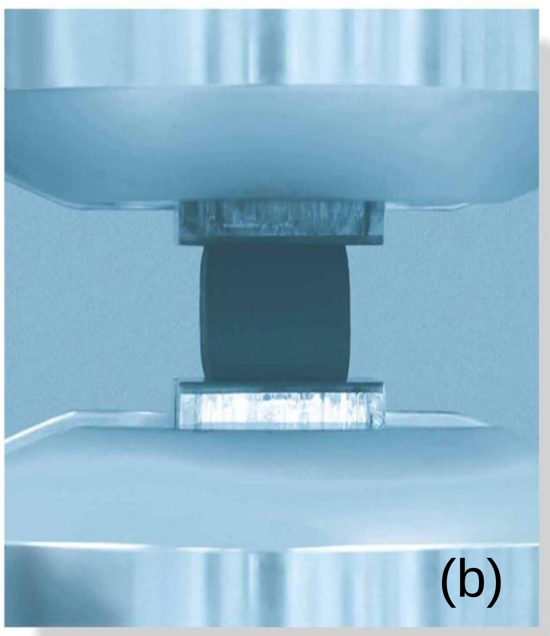
The $\sigma$–$\varepsilon$ diagram for a collagen fiber bundle from which a human tendon is composed is shown. If a segment of the Achilles tendon at $A$ has a length of $6.5$ in and an approximate cross-sectional area of $0.23\ \text{in}^2$ determine its elongation if the foot supports a load of $125$ lb, which causes a tension in the tendon of $343.7$ lb



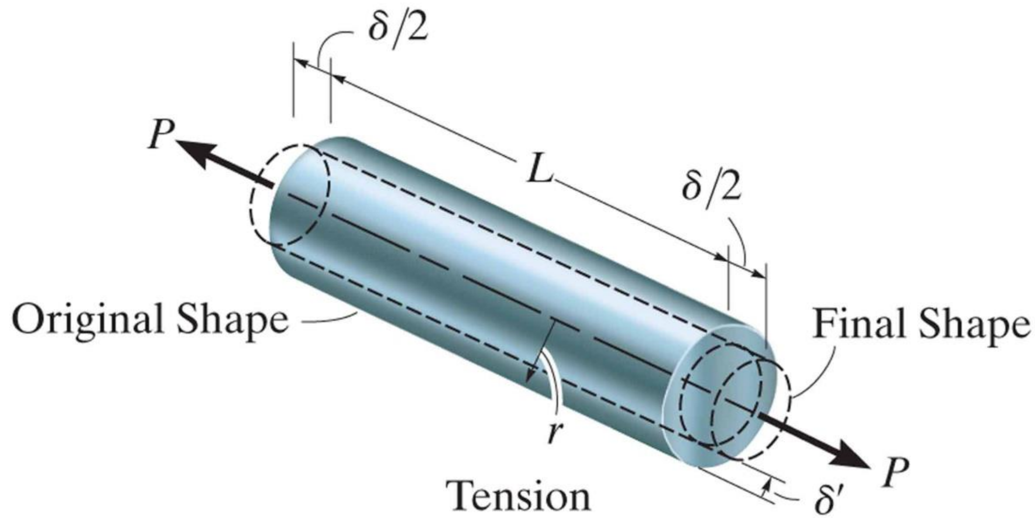



# Poisson's ratio:




Poisson's ratio: $$\nu = -\frac{\varepsilon_{lateral}}{\varepsilon_{longitudinal}}$$



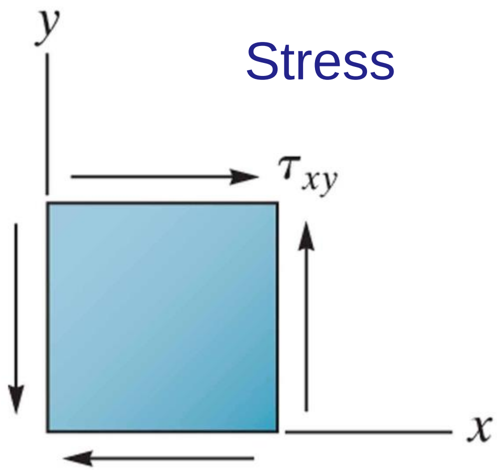
# Shear stress ↔ strain

Pure shear

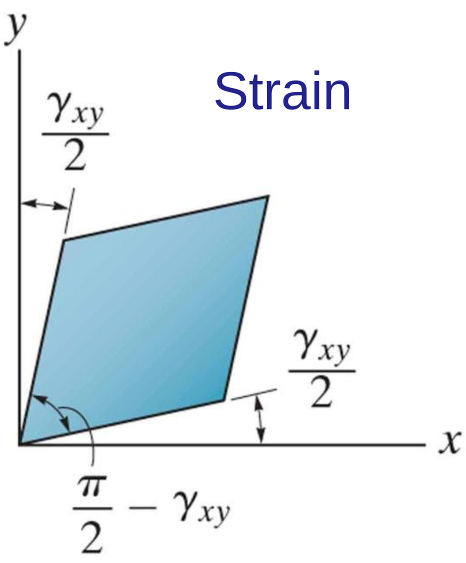
Hook's law for shear: $\tau = G\,\gamma$

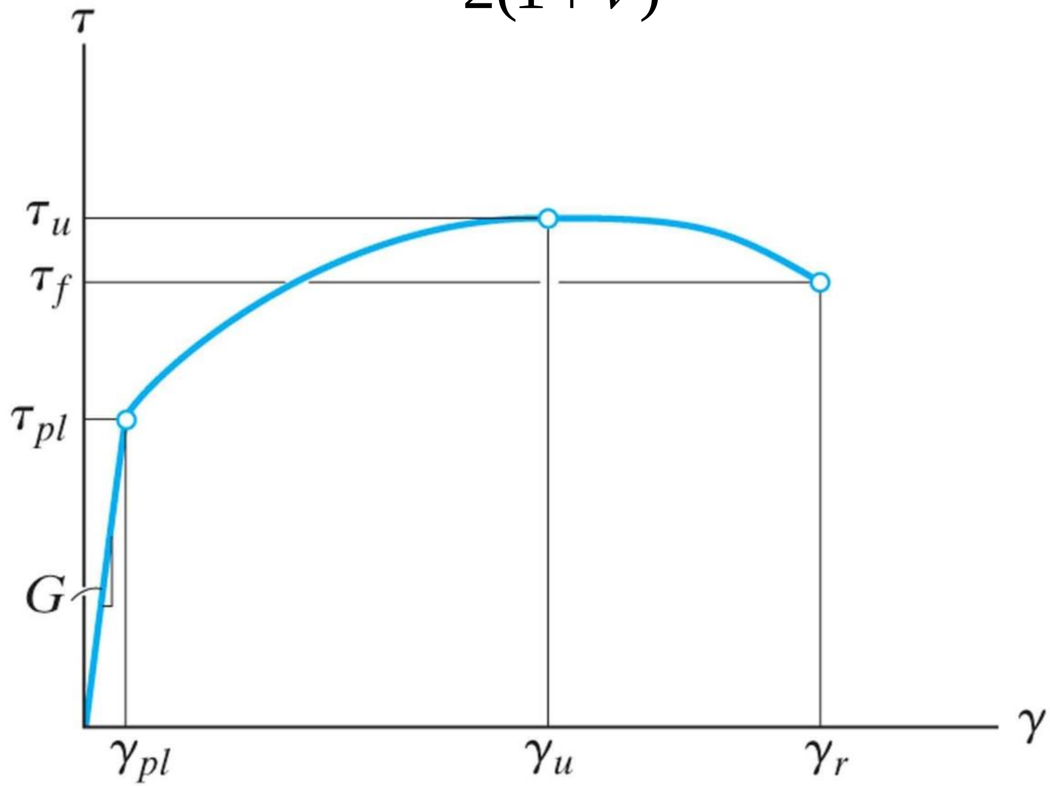
with $G = \dfrac{E}{2(1+\nu)}$ (shear modulus)

Stress



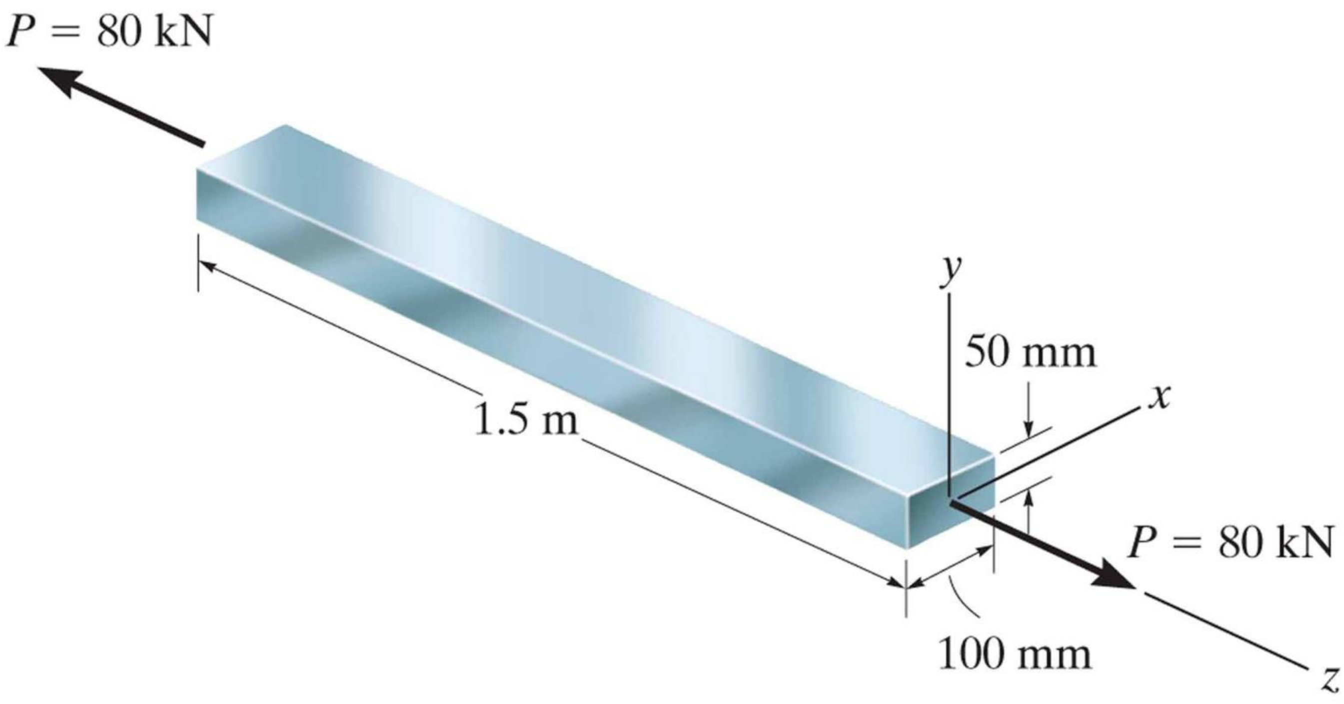

Strain



# Strain: example C

A bar made of ASTM A-36 steel has the dimensions shown. If the axial force of $P = 80$ kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section after applying the load. The material behaves elastically.



# Axial load

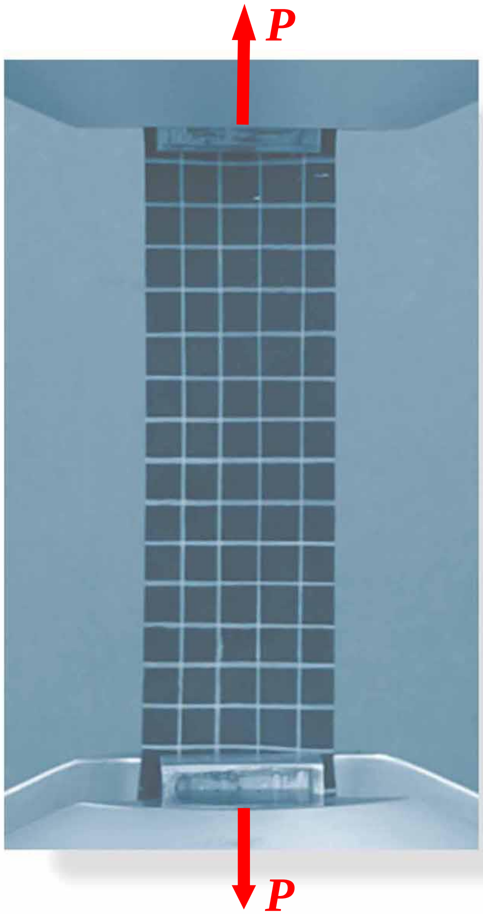



Figure: 04-01-UN-A
Notice how the lines on this rubber membrane distort after it is stretched. The localized distortions at the grips smooth out as stated by Saint-Venant's principle.

Note distortion lines: follow Saint-Venant's principle

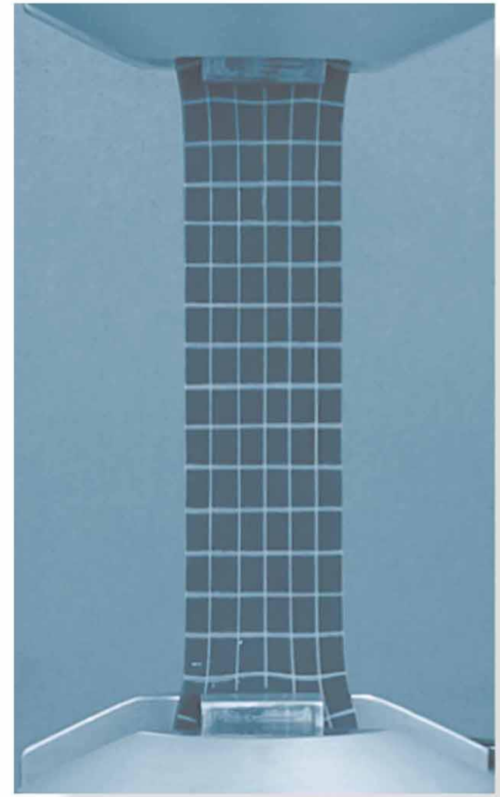
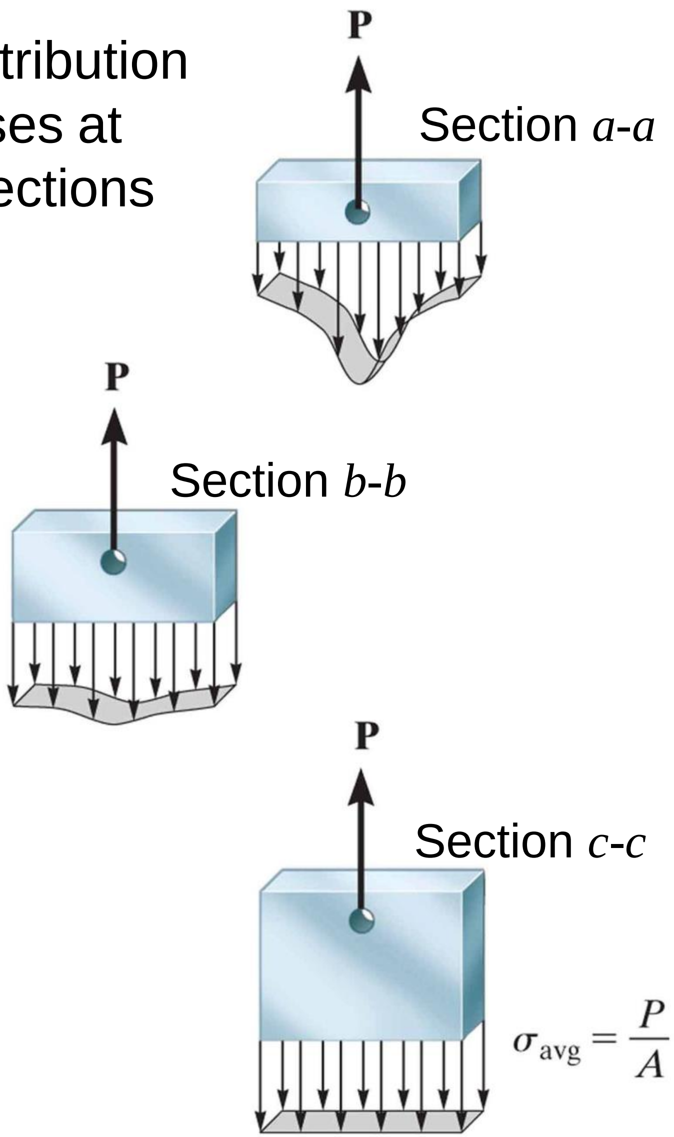
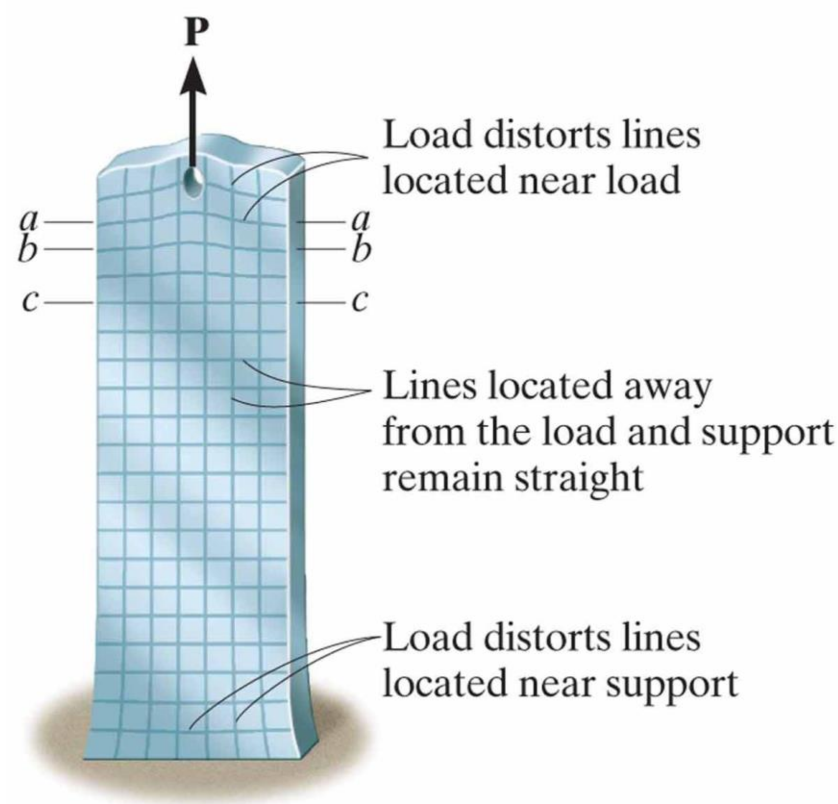


Figure: 04-01-UN-B
Notice how the lines on this rubber membrane distort after it is stretched. The localized distortions at the grips smooth out as stated by Saint-Venant's principle.

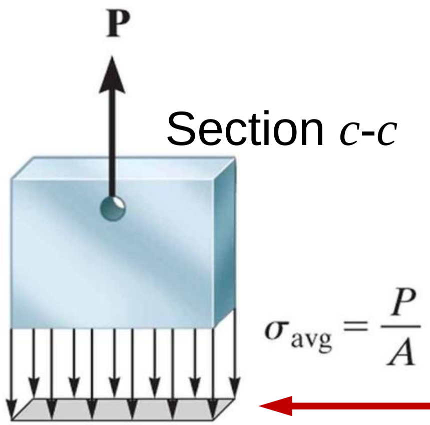
# Axial load: Saint-Venant's principle

Internal distribution of stresses at various sections



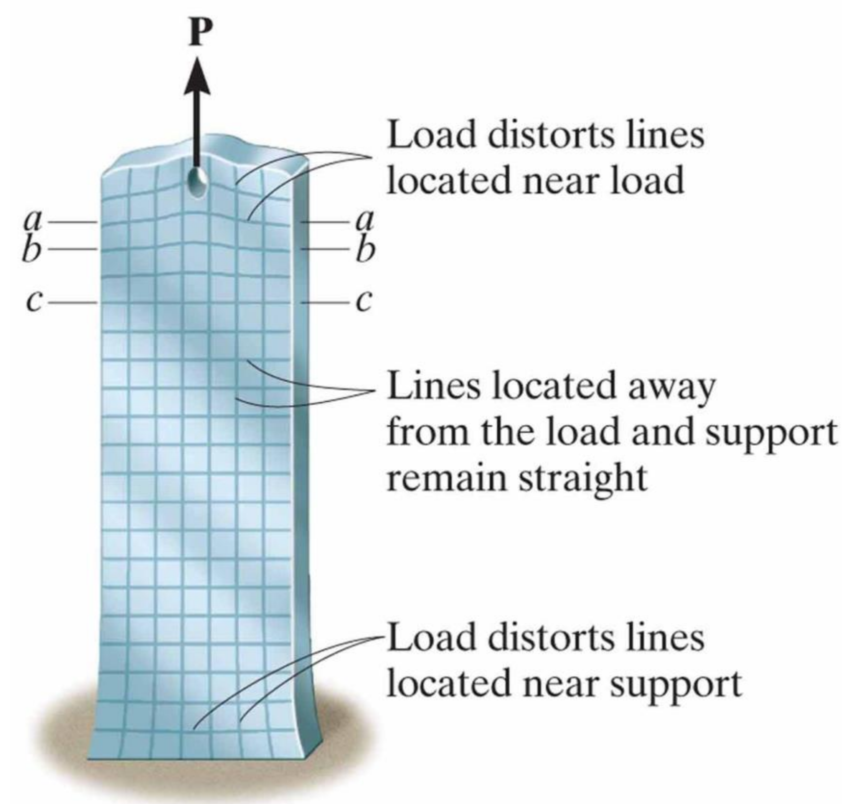
# Axial load: Saint-Venant's principle




Section *c*-*c*

**Nominal stress**

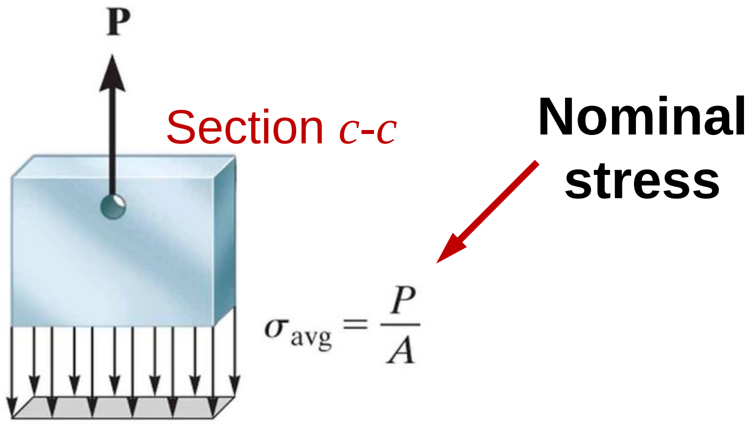
Uniform stress distribution



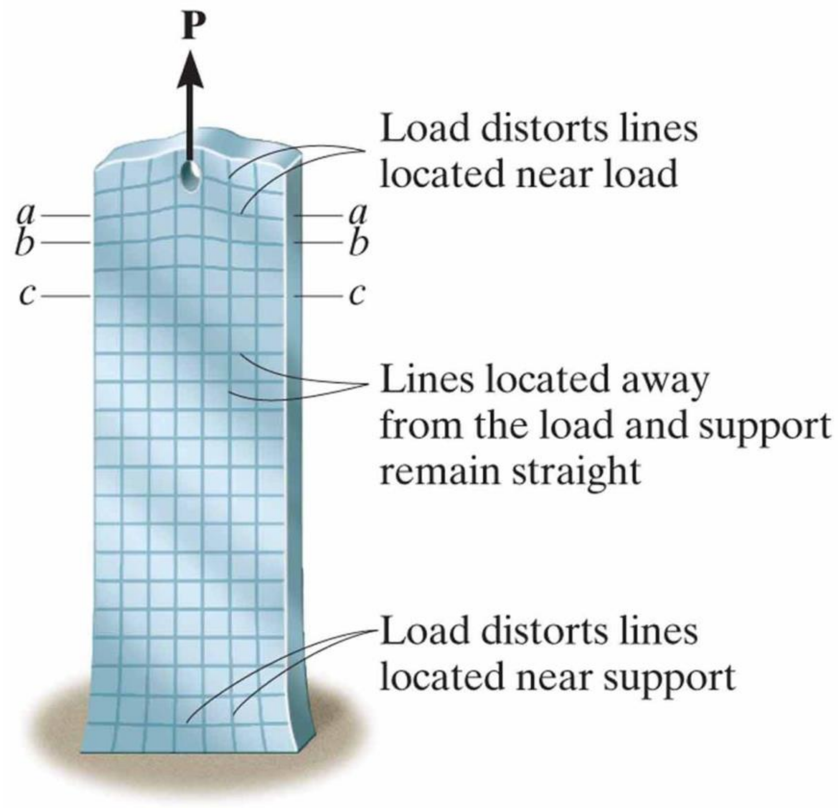
# Axial load: Saint-Venant's principle

In your analyses, select locations (sections / points) located away from regions that are subjected to load application (to eliminate "end" effects)

**Saint-Venant's principle:** stresses and strains within a section will approach their nominal values as the section locates away from regions of load application

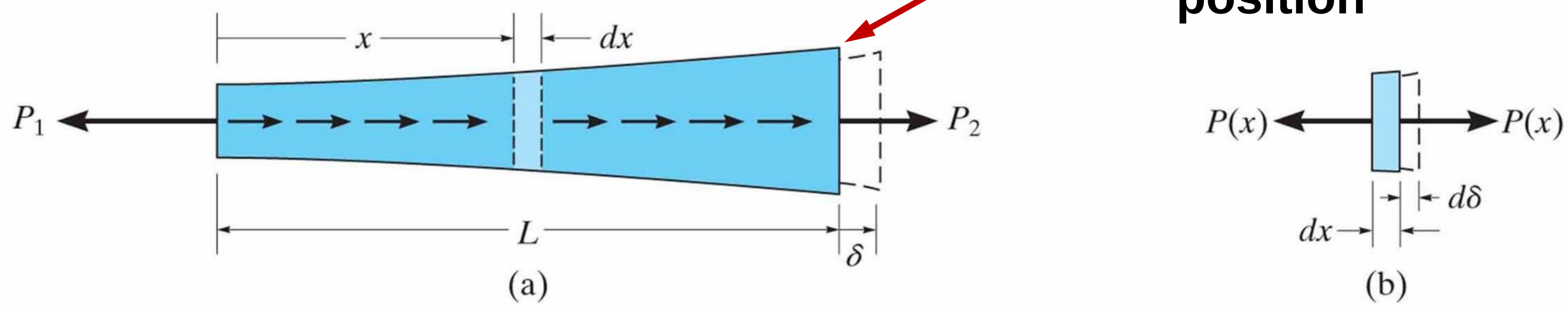



$$\sigma_{avg} = \frac{P}{A}$$



# Elastic deformation of an axially loaded member

Section is a function of position

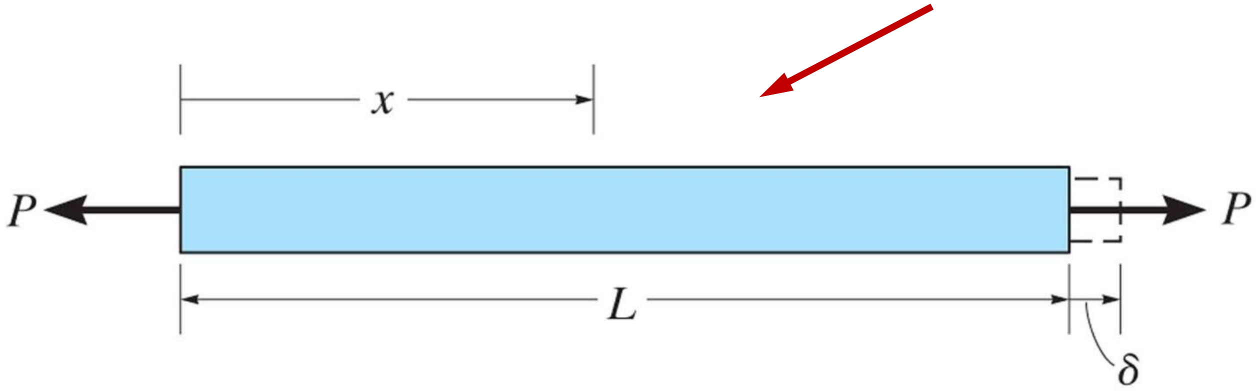



$$\sigma = \frac{P(x)}{A(x)} \qquad and \qquad \varepsilon = \frac{d\delta}{dx}$$

Therefore, $d\delta = \dfrac{P(x)\,dx}{A(x)\,E}$ $\longrightarrow$ $\delta = \displaystyle\int_0^L \frac{P(x)}{A(x)\,E}\,dx$

# Elastic deformation of an axially loaded member

**Constant load and cross-sectional area**

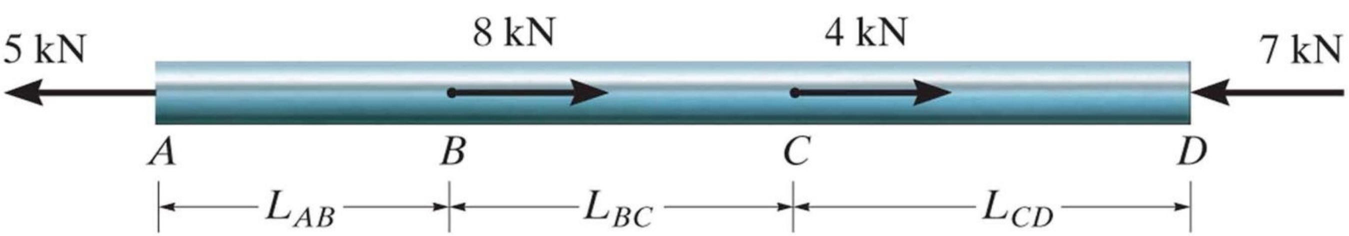



Elastic deformation:

$$\delta = \int_0^L \frac{P(x)}{A(x)\,E}\,dx \quad = \quad \frac{P}{A\,E}\int_0^L dx \quad = \quad \frac{P\,L}{A\,E}$$

# Elastic deformation of an axially loaded member

**Bar subjected to multiple axial loads**

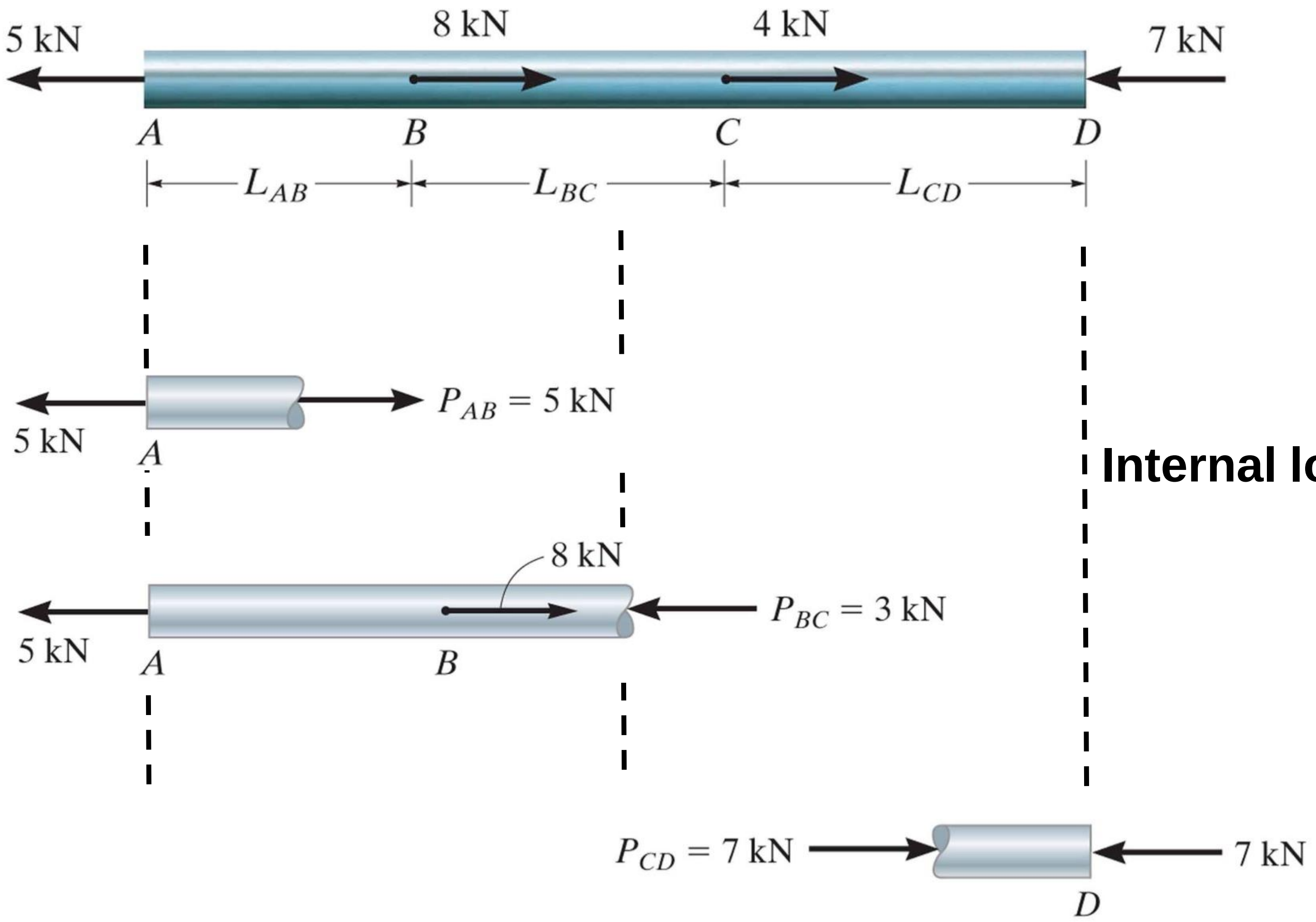



Elastic deformation:

$$\delta = \sum_i \left( \frac{P\,L}{A\,E} \right)_i$$

# Elastic deformation of an axially loaded member

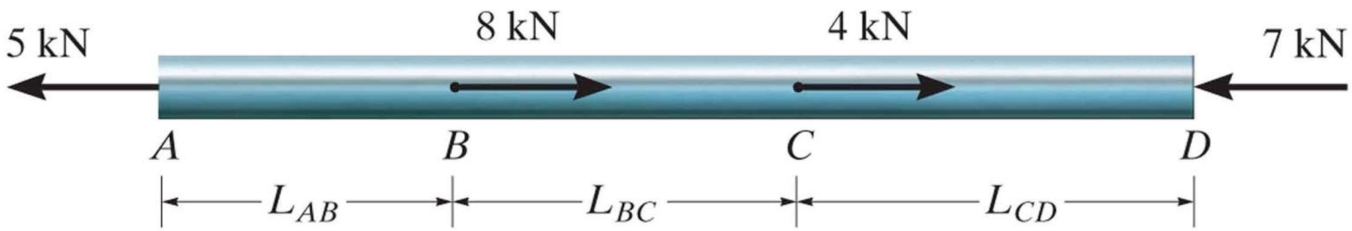
## Procedure for analysis




**Internal loads**

# Elastic deformation of an axially loaded member

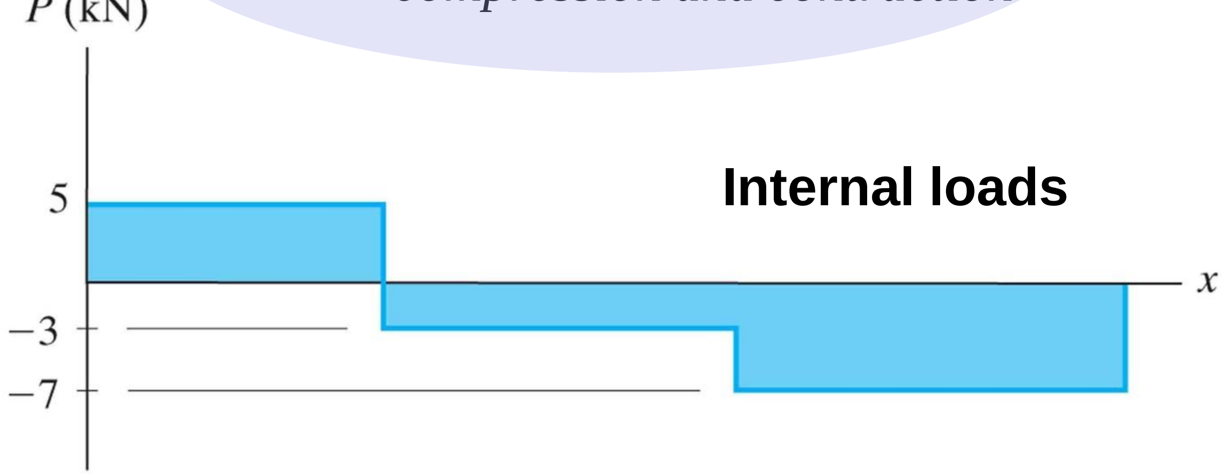
## Procedure for analysis




Sign convention:

+ *tension and elongation*

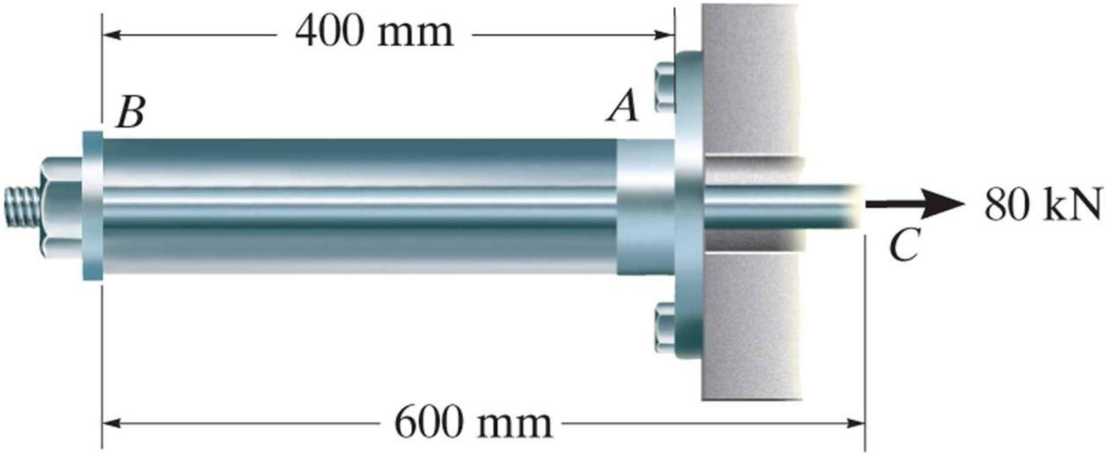
– *compression and contraction*




**Internal loads**

# Axial load: example D

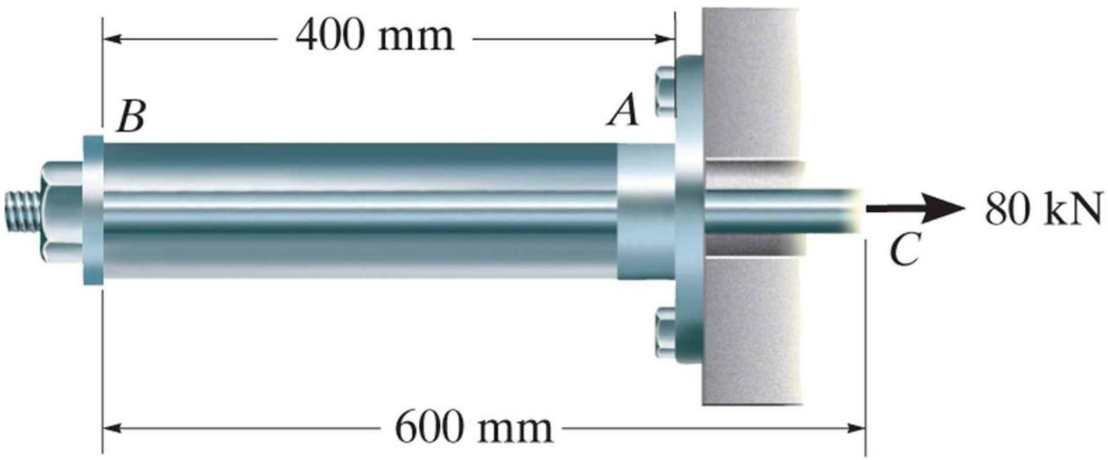
The assembly shown consists of an aluminum tube $AB$ having a cross sectional area of $400\ \text{mm}^2$. A steel rod having a diameter of $10\ \text{mm}$ is attached to a rigid collar and passes through the tube. If a tensile load of $80\ \text{kN}$ is applied to the rod, determine the displacement of the end $C$ of the rod. Elastic modules: $E_{\text{steel}} = 200\ \text{GPa}$ and $E_{\text{alum}} = 200\ \text{GPa}$




**Approach:**

1) Determine internal loading
2) Compute displacement

# Axial load: example D

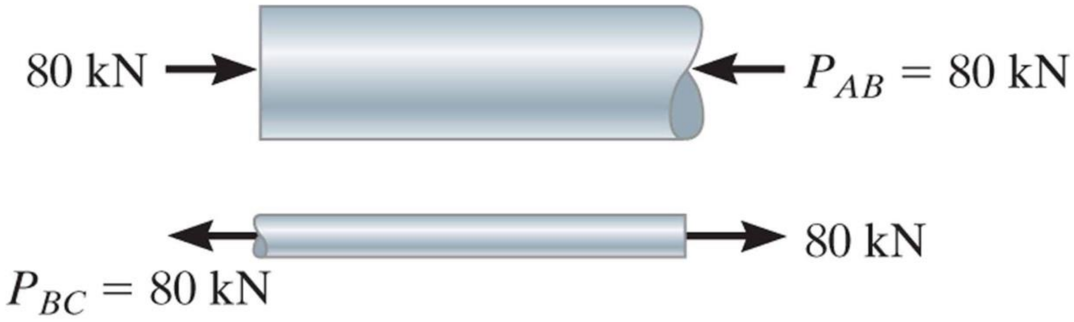



**Displacement of $C$:**

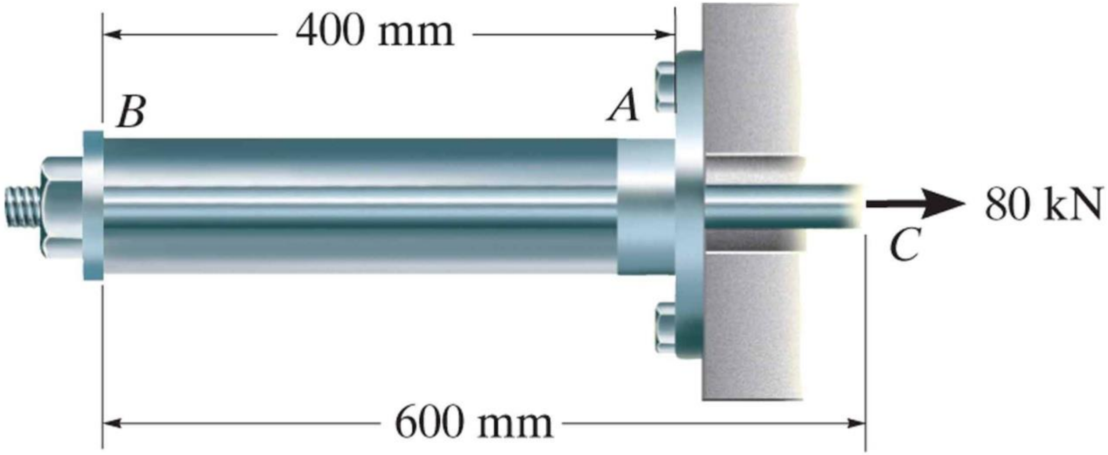
$$\delta_C = \delta_B + \delta_{C/B}$$

# Axial load: example D

**(1) Internal loading**




**(2) → find displacement at $C$**



# Reading assignment

- Chapters 3 and 4 of textbook
- Review notes and text: ES2001, ES2501

# Homework assignment

- **As indicated on webpage of our course**