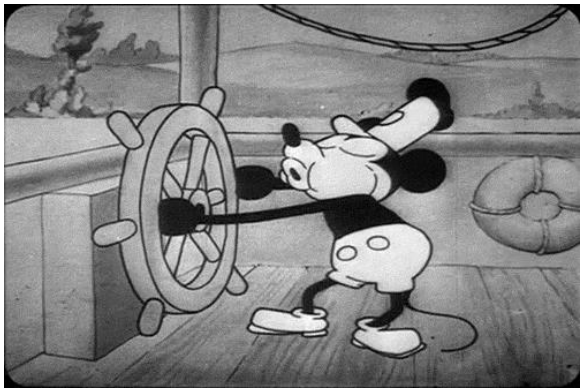
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, D'2020

We will get started soon...



09 April 2020

WORCESTER POLYTECHNIC INSTITUTE
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STRESS ANALYSIS
ES-2502, D'2020

We will get started soon...

Lecture 09:
Unit 6: Stress & Strain/ Mechanical Properties

09 April 2020

General information

Instructor: Cosme Furlong
HL-152
(508) 831-5126
Email: cfurlong @ wpi.edu
http://www.wpi.edu/~cfurlong/es2502.html

Teaching Assistant: Zachary Zolotarevsky
Email: zjzolotarevsky @ wpi.edu

Stress ↔ Strain

Tensile test

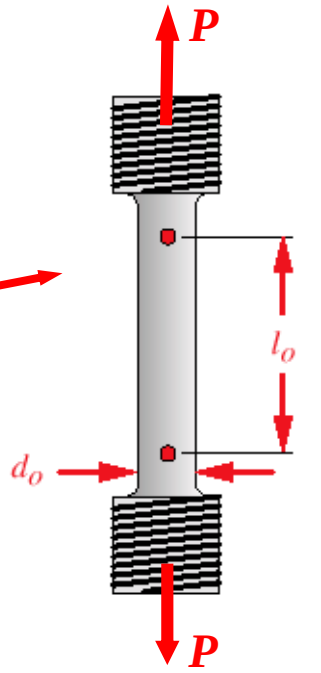


FIGURE 2-1
A Tensile Test Specimen

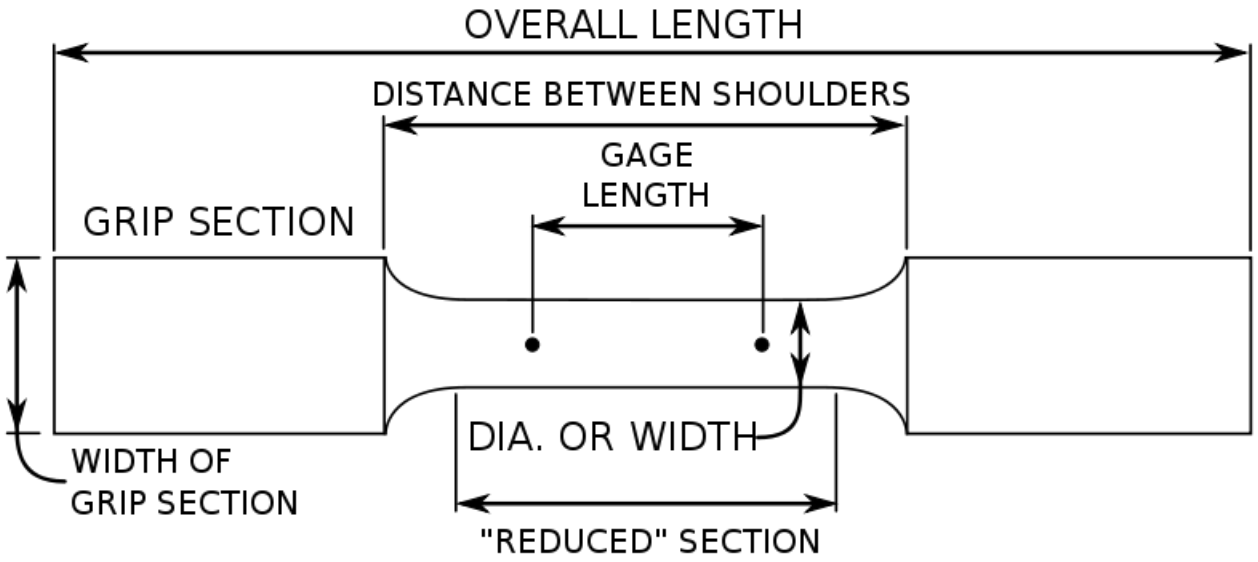
ASTM standards

Average Stress: (normal)
$$\sigma = \frac{P}{A}$$

Average Strain: (normal)
$$\varepsilon = \frac{l - l_o}{l_o}$$

Modulus of elasticity:
$$E = \frac{\sigma}{\varepsilon}$$

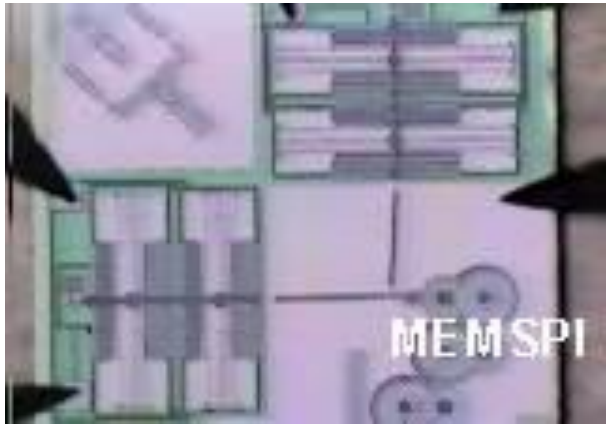
Stress ↔ Strain: tensile test



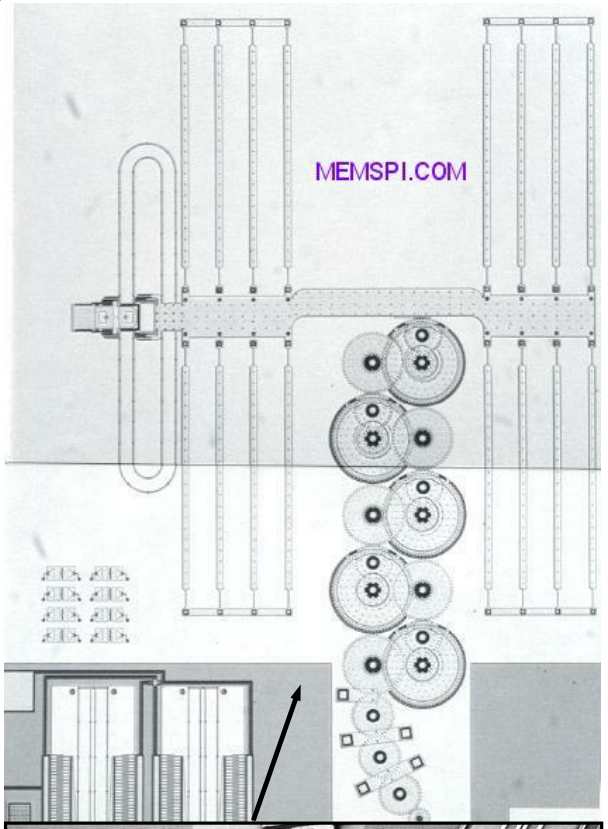
All values in inches	Standard specimen at nominal diameter:		Small specimen at nominal diameter:		
	0.500	0.350	0.25	0.160	0.113
Gauge length	2.00±0.005	1.400±0.005	1.000±0.005	0.640±0.005	0.450±0.005
Diameter tolerance	±0.010	±0.007	±0.005	±0.003	±0.002
Fillet radius (min.)	3/8	0.25	5/16	5/32	3/32
Length of reduced section (min.)	2.5	1.75	1.25	0.75	5/8

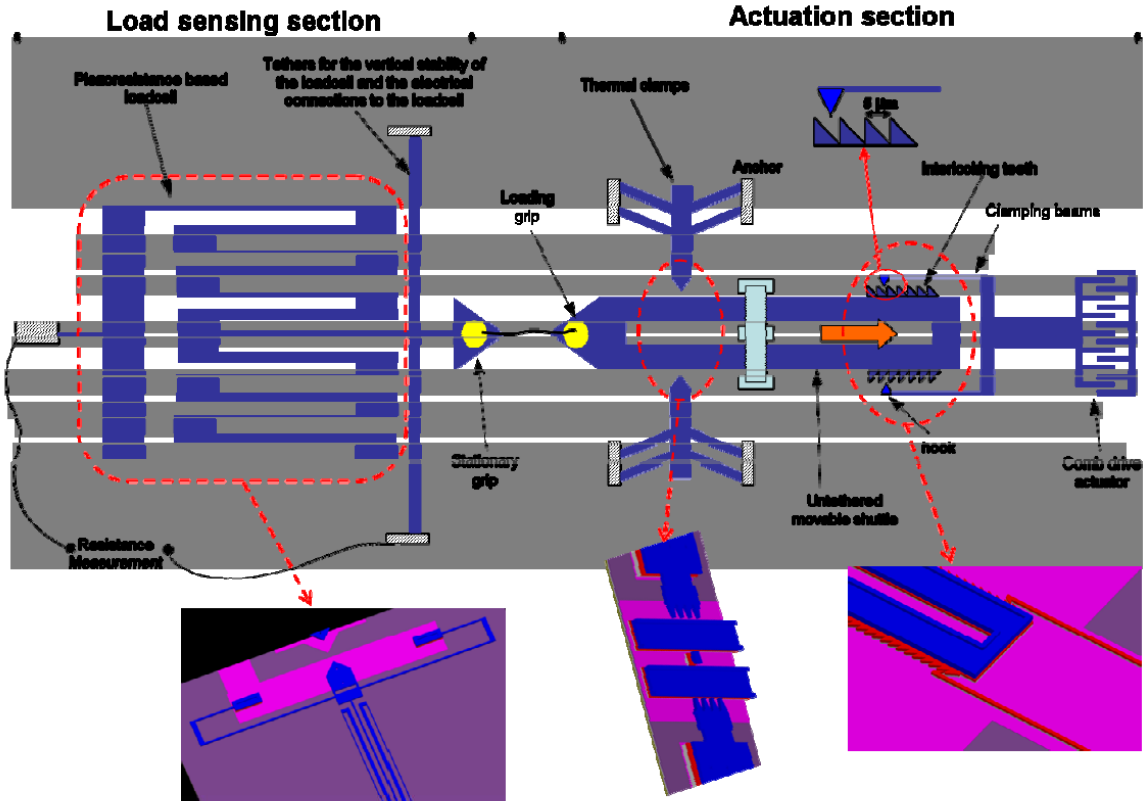
Material properties

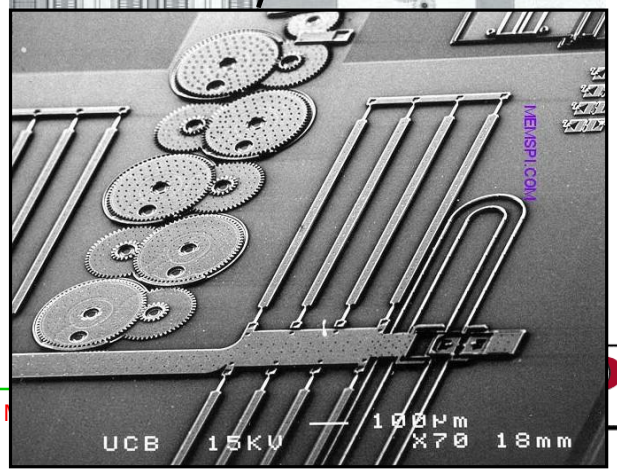
Microscale tensile test



Machine is
about 1.5 mm
in height !!



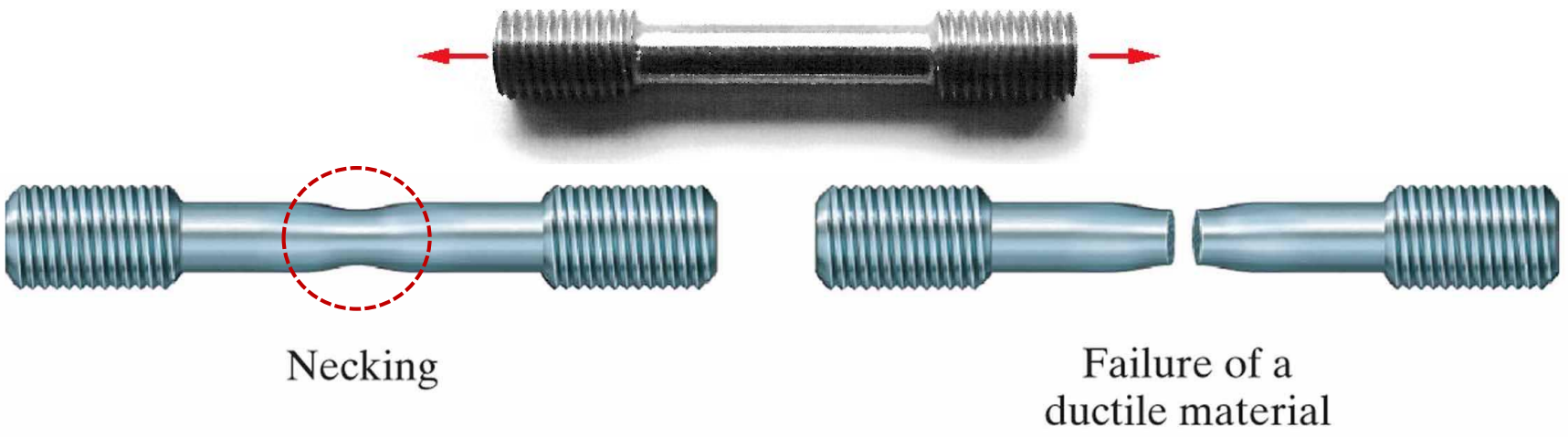




UIUC/Sandia Labs

Material properties: Stress - Strain

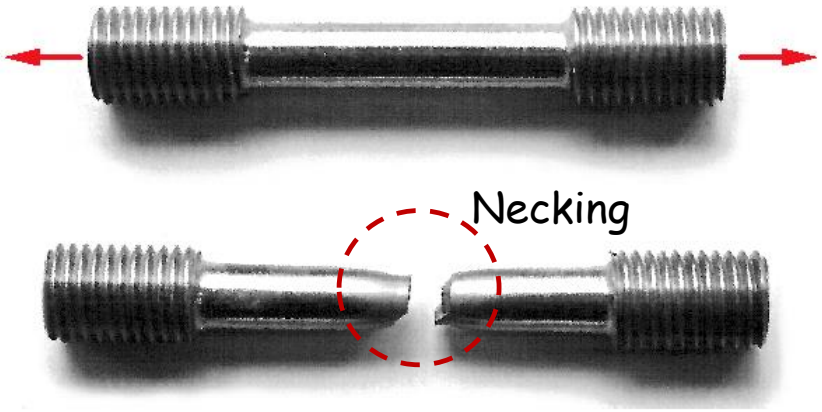
Tensile test: ductile material

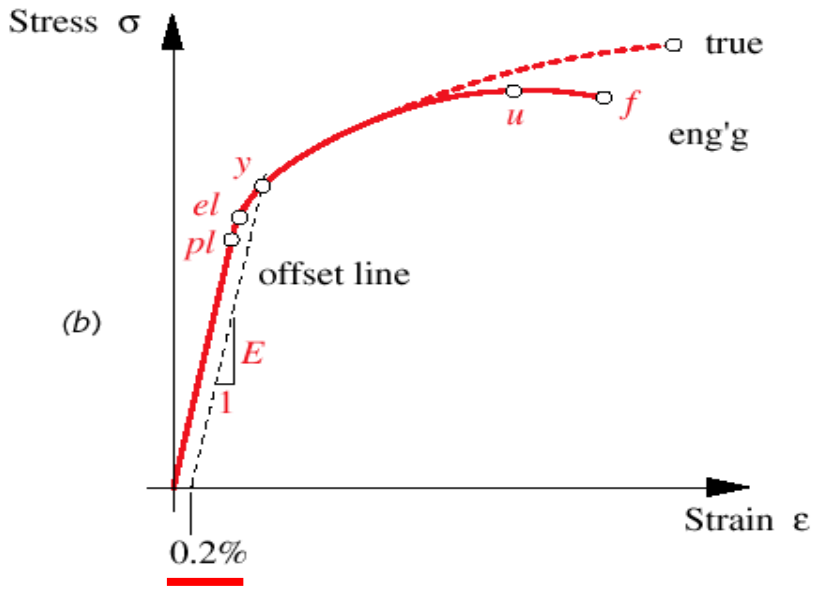


Failure surface

Material properties: Stress - Strain

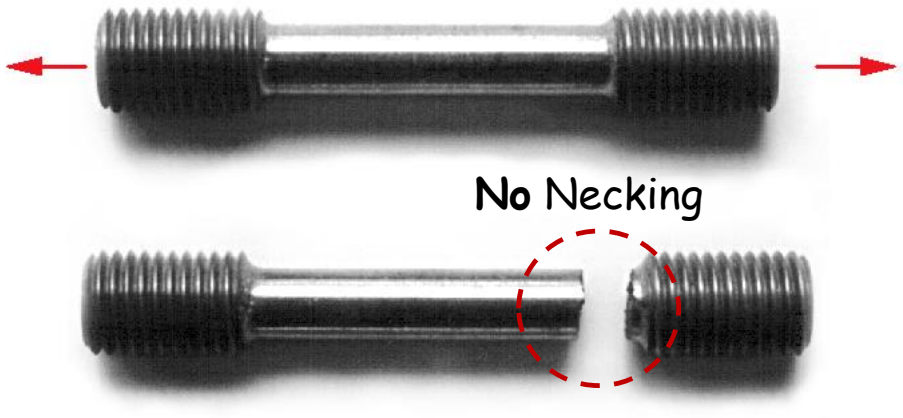
Tensile test: ductile material

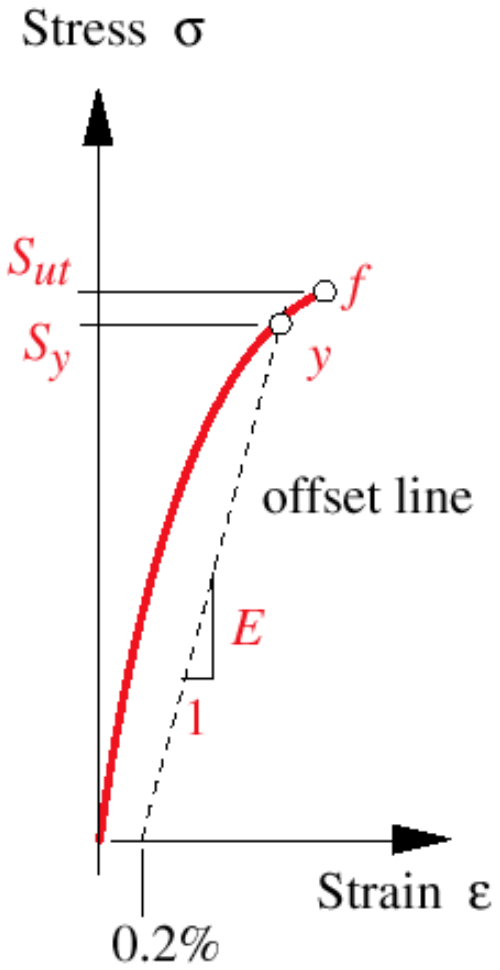




Material properties: Stress - Strain

Tensile test: brittle material



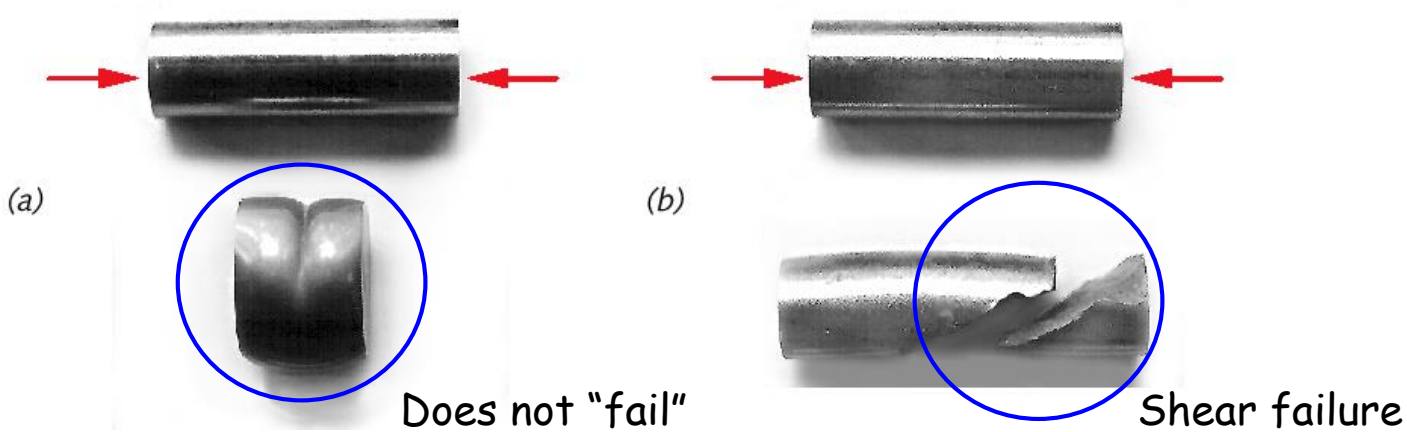


Material properties: Stress - Strain

Compression test: ductile & brittle materials

Ductile

Brittle



Does not "fail"

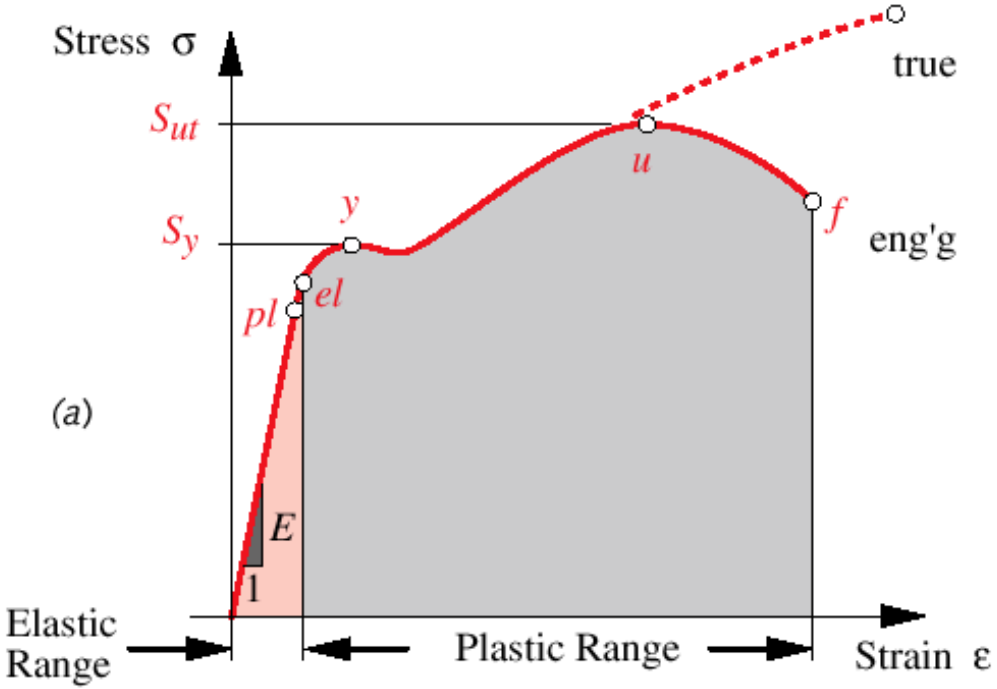
Shear failure

Even materials: same behavior in tension as in compression.

Stress - Strain

Stress-strain diagrams: ductile materials

Ductile material



Engineering stress: $\sigma = \frac{P}{A_{initial}}$

True stress: $\sigma = \frac{P}{A_{true}}$

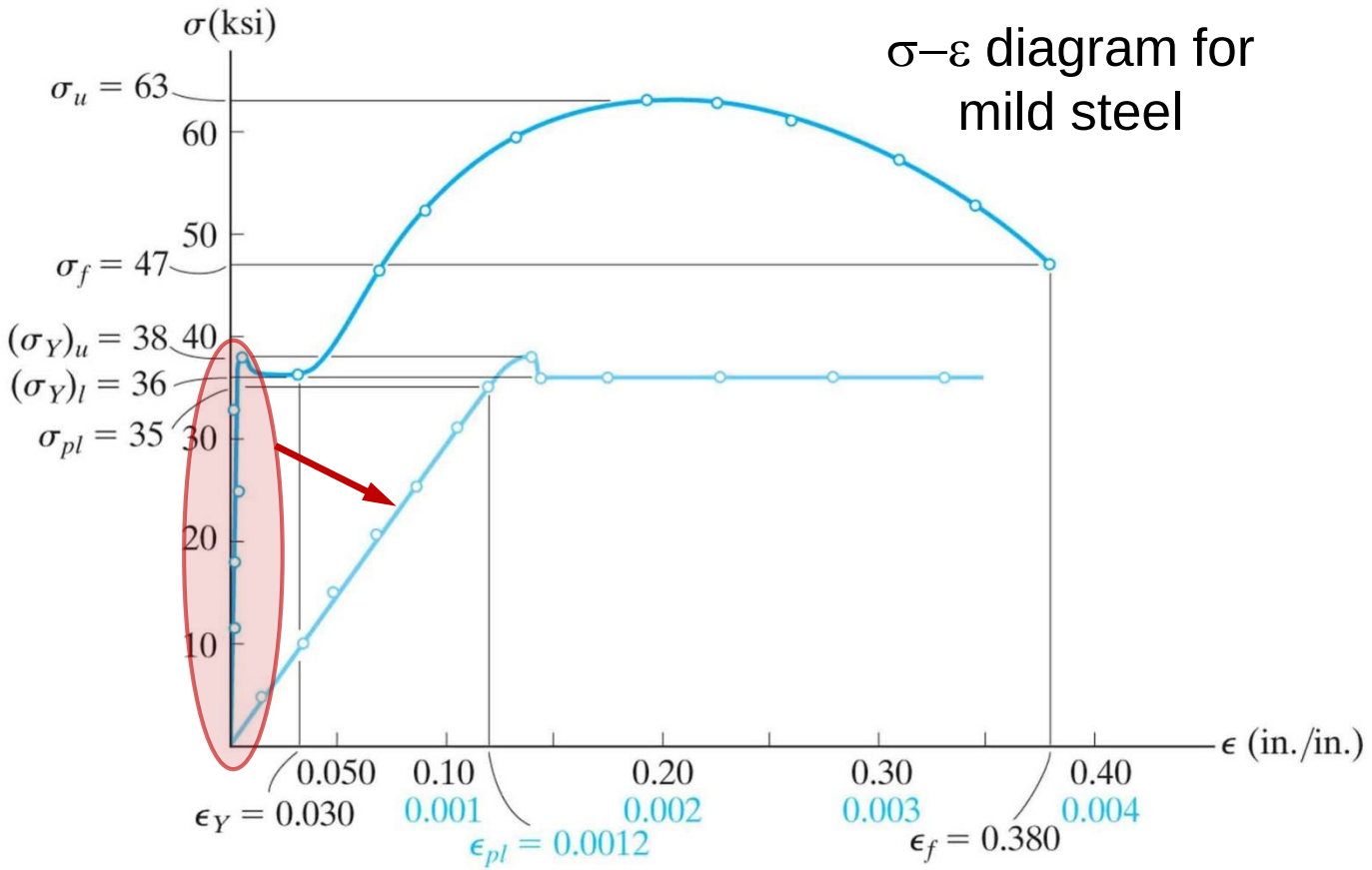
Stress ↔ Strain: Hook's Law

$$\sigma = E \cdot \varepsilon$$

← Elastic range

E = proportionality constant ← E (aka Young's modulus)

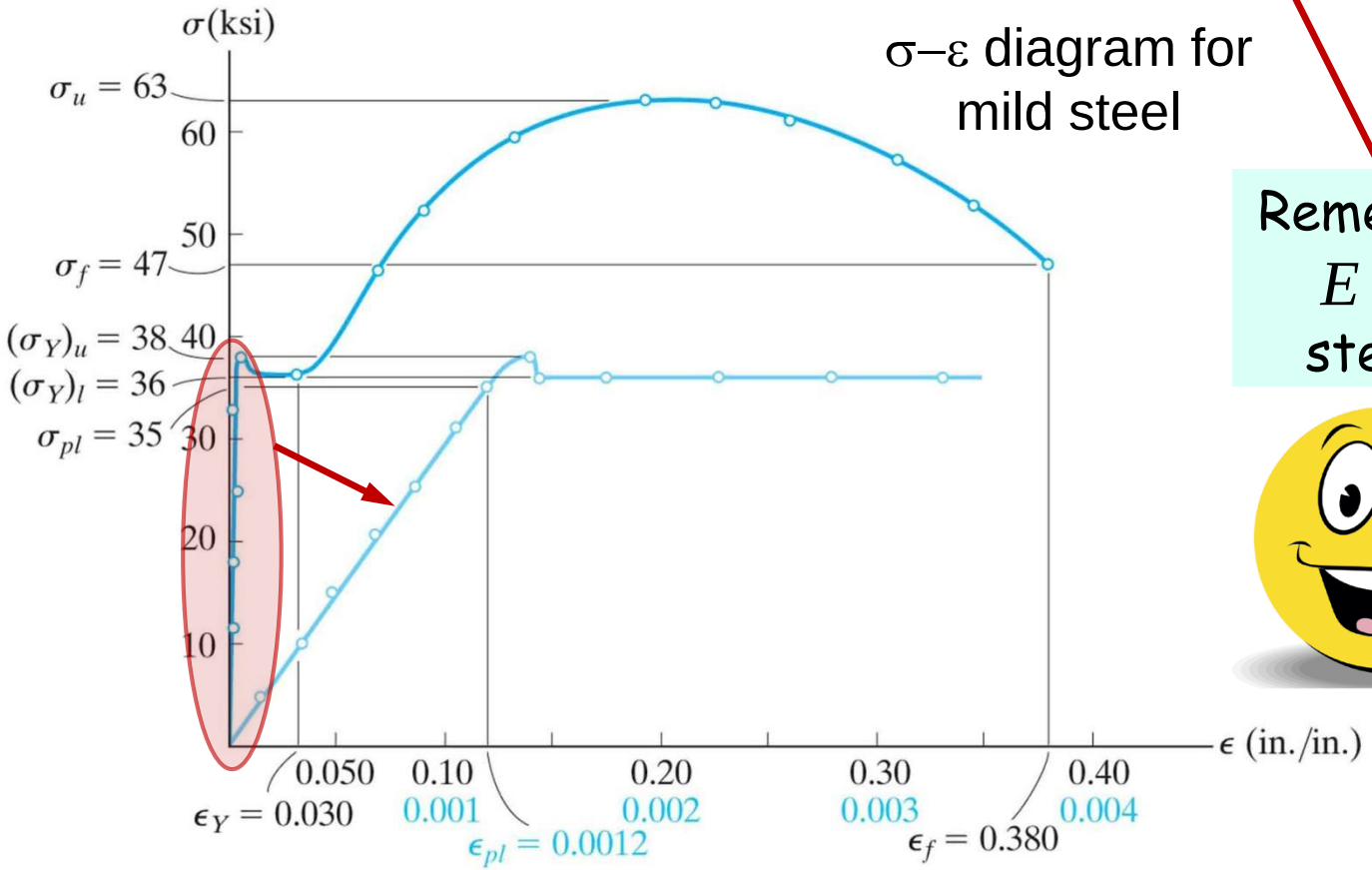
Engineering stress



Stress ↔ Strain: elastic properties

For diagram shown:
(E for steel)

$$E = \frac{35 \times 10^3 \ psi}{0.0012 \ in/in} \approx 30 \times 10^6 \ psi$$



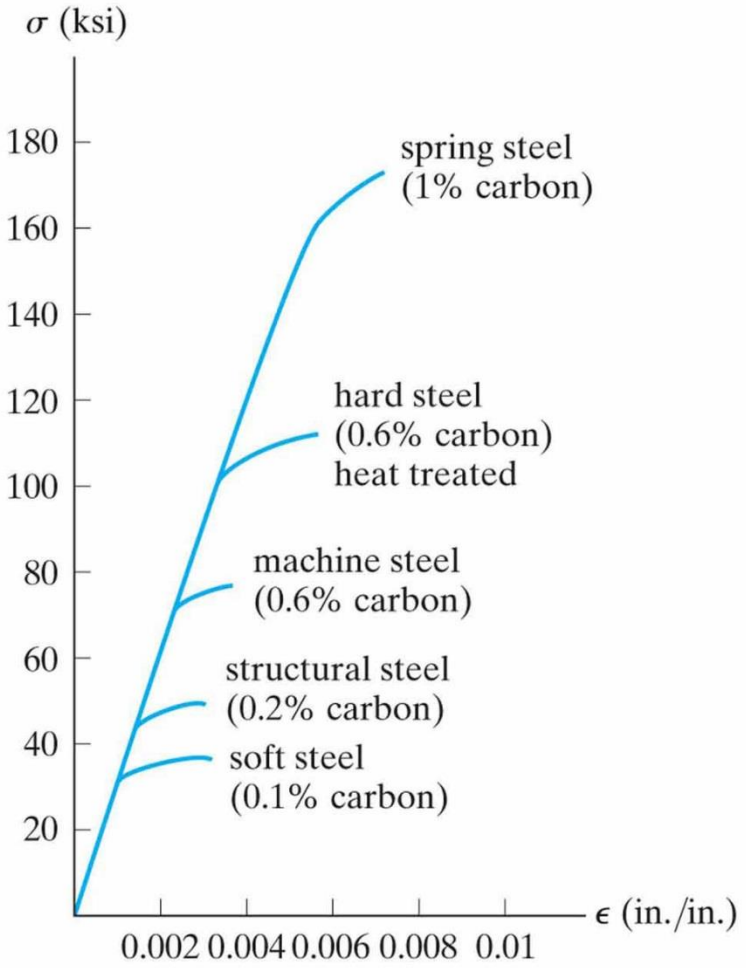
σ–ε diagram for mild steel

Remember E for steel !!

Stress ↔ Strain: Hook's Law

$$\sigma = E \cdot \varepsilon$$

E = Elastic modulus (aka)

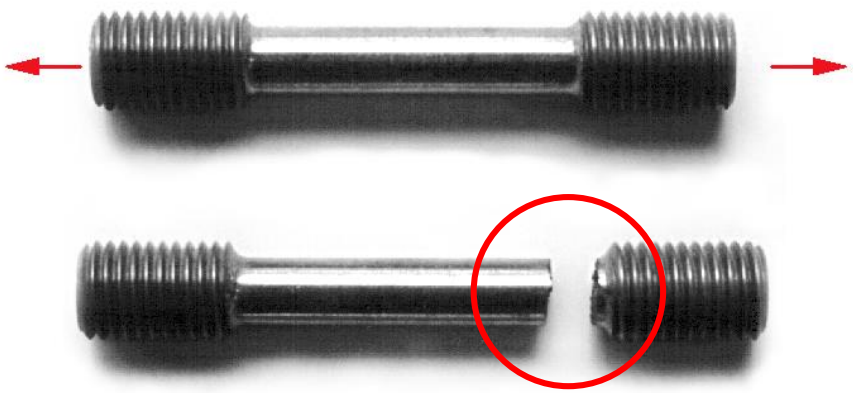


Remember: E is practically the same for different classes of **steels** !!

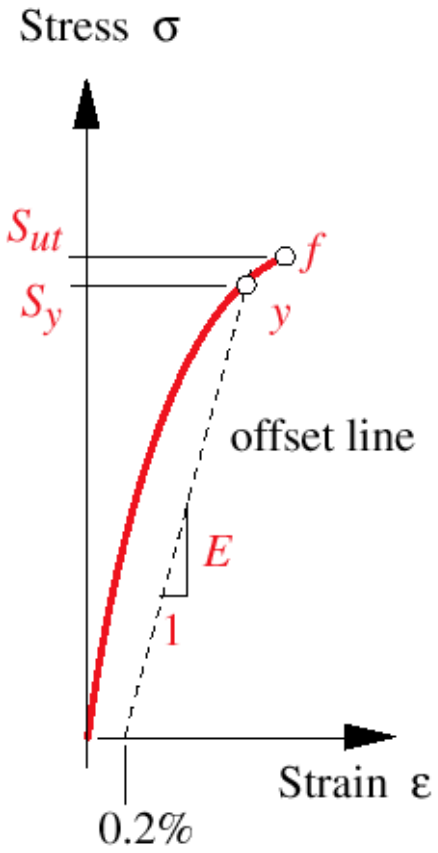
Stress - Strain

Stress-strain diagrams: brittle materials

Brittle material



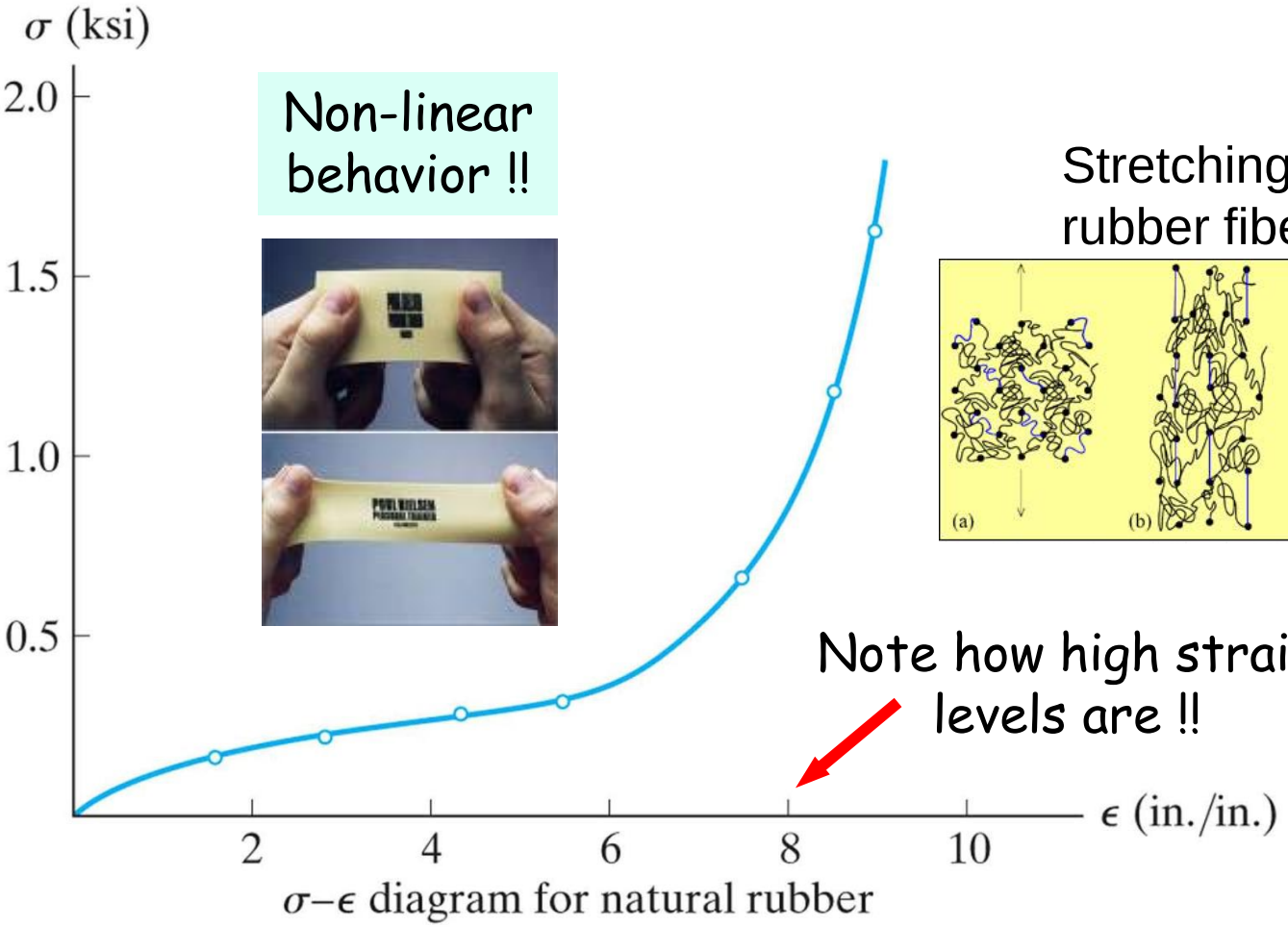
minimum necking



Offset method

Stress - Strain

Stress-strain diagrams: natural rubber



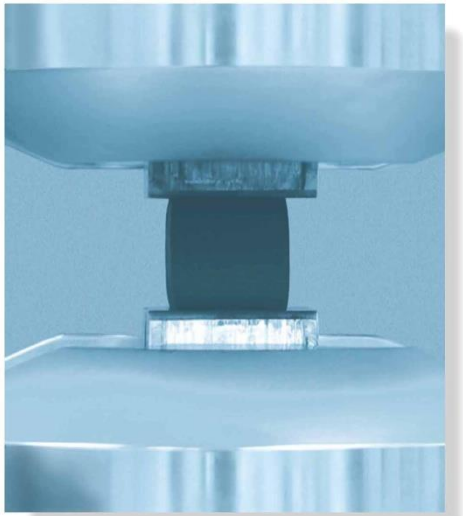
Stress - Strain

Stress-strain diagrams: gray cast iron

Why is gray cast iron tested in compression?

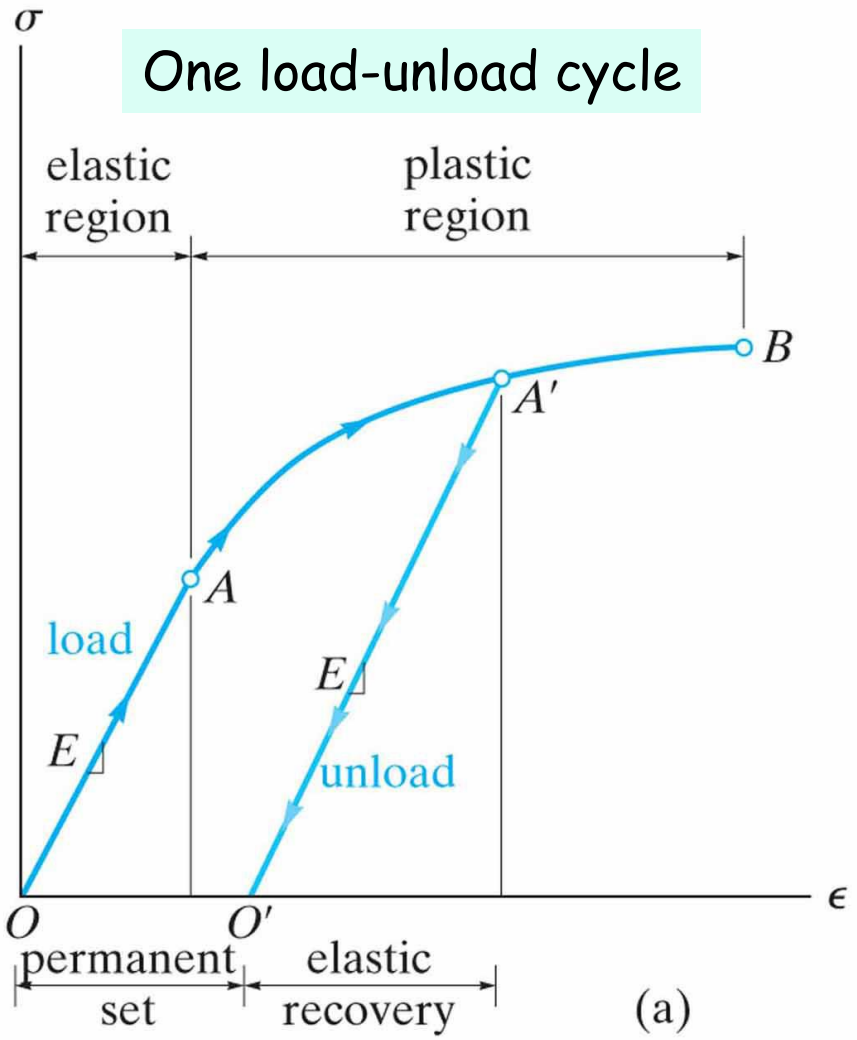
σ–ϵ diagram for gray cast iron

Non-linear behavior !!

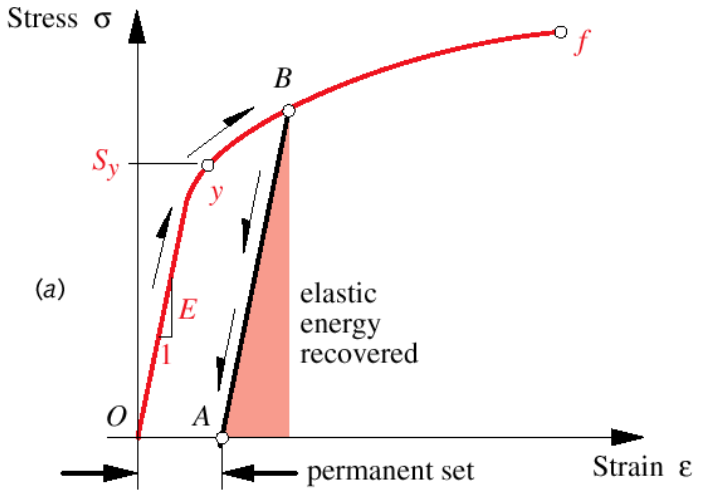


Strain hardening

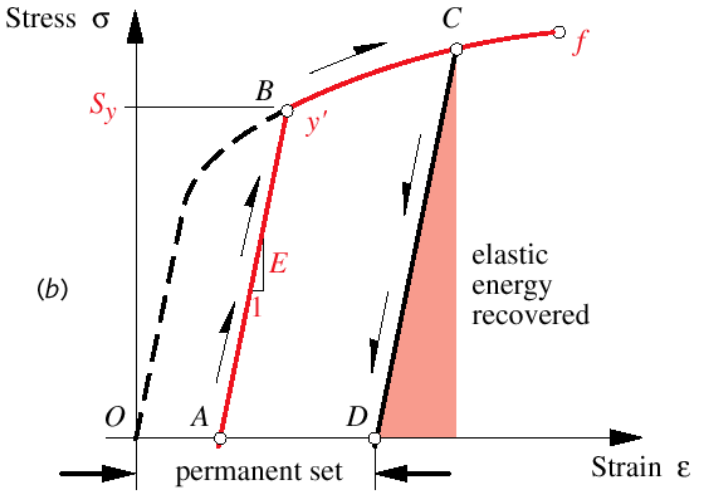
One load-unload cycle



One load-unload cycle



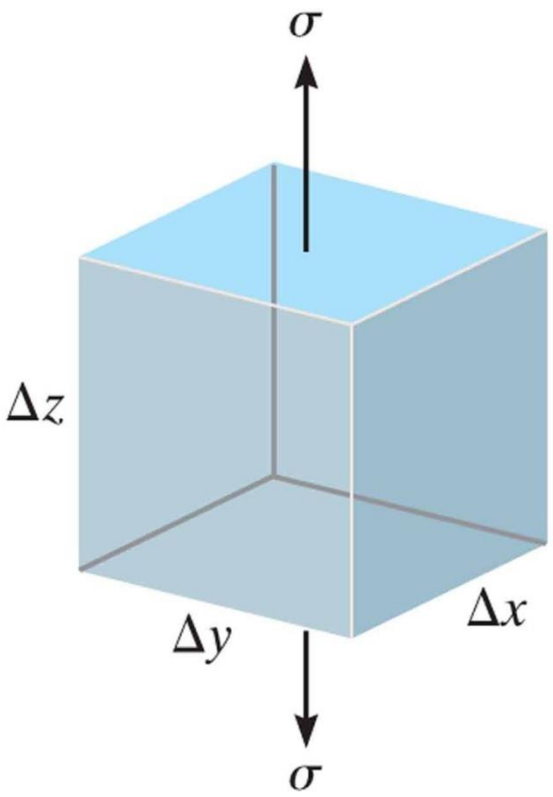
Two load-unload cycles



WPI

Strain energy:

$$\Delta V = volume = \Delta x\ \Delta y\ \Delta z$$



$$\Delta U = \left(\frac{\Delta F}{2}\right) \cdot \varepsilon\, \Delta z$$

Strain energy

Average force

Displacement

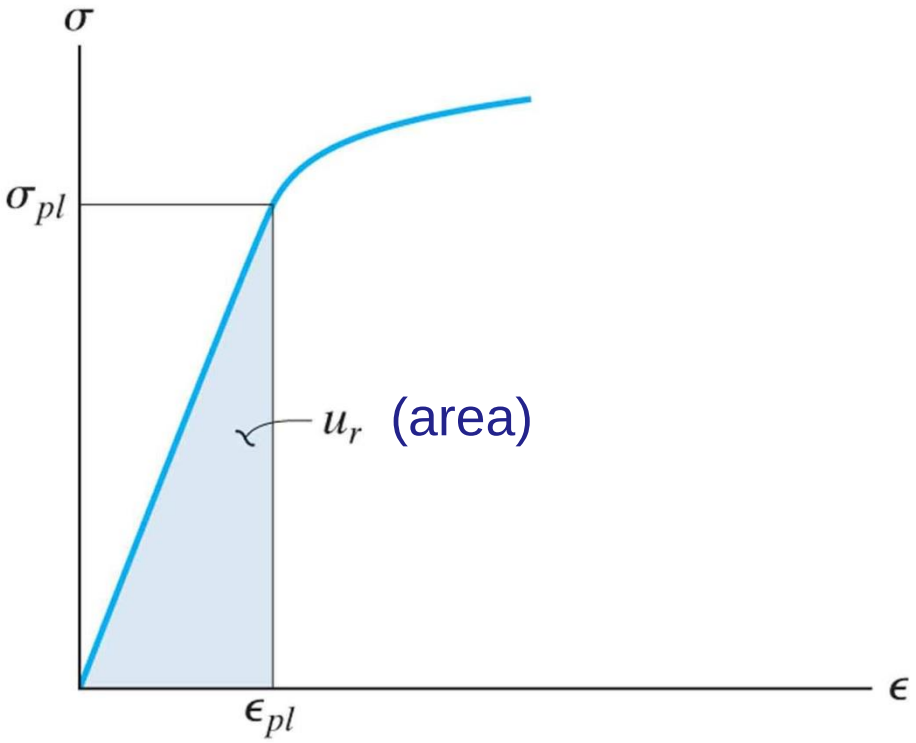
$$\Delta U = \left(\frac{1}{2}\sigma\ \Delta x \cdot \Delta y\right) \cdot \varepsilon\, \Delta z$$

$$= \frac{1}{2}\sigma\ \varepsilon\ (\Delta x \cdot \Delta y \cdot \Delta z)$$

$$\boxed{\Delta U = \frac{1}{2}\sigma \cdot \varepsilon \cdot \Delta V}$$

Strain energy ***density***:

$$u = \frac{\Delta U}{\Delta V} = \frac{1}{2}\sigma \cdot \varepsilon$$



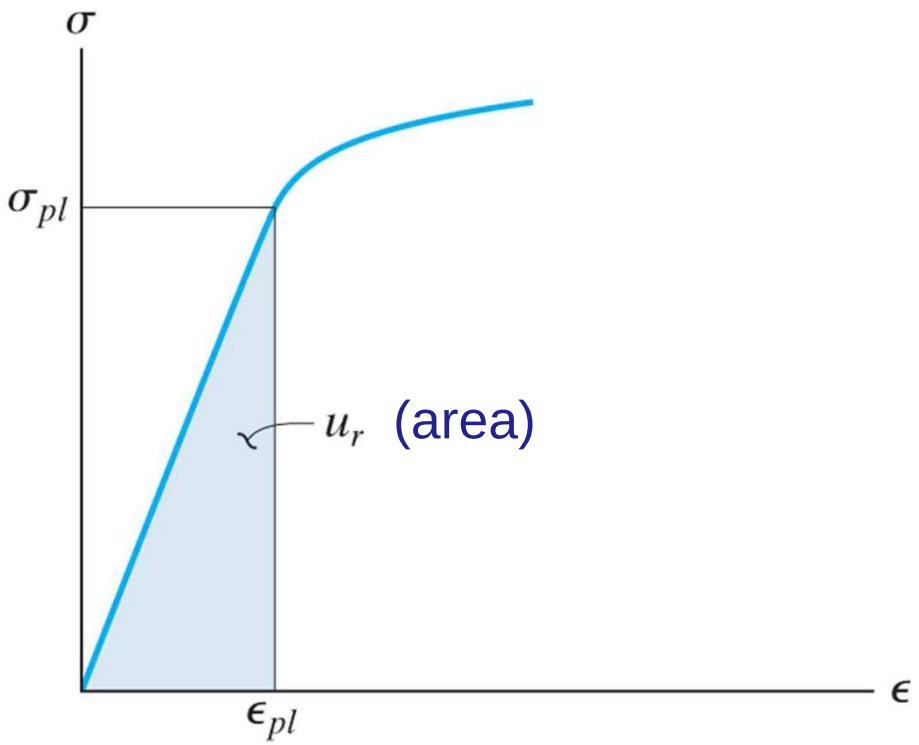
Strain energy: resilience

$$u = \frac{\Delta U}{\Delta V} = \frac{1}{2}\sigma \cdot \varepsilon$$

Modulus of resilience:

$$u_r = \int_0^{\varepsilon_{pl}} \sigma \, d\varepsilon$$

$$u_r = \frac{1}{2}\sigma_{pl}\,\varepsilon_{pl} = \frac{1}{2}\frac{\sigma_{pl}^2}{E}$$



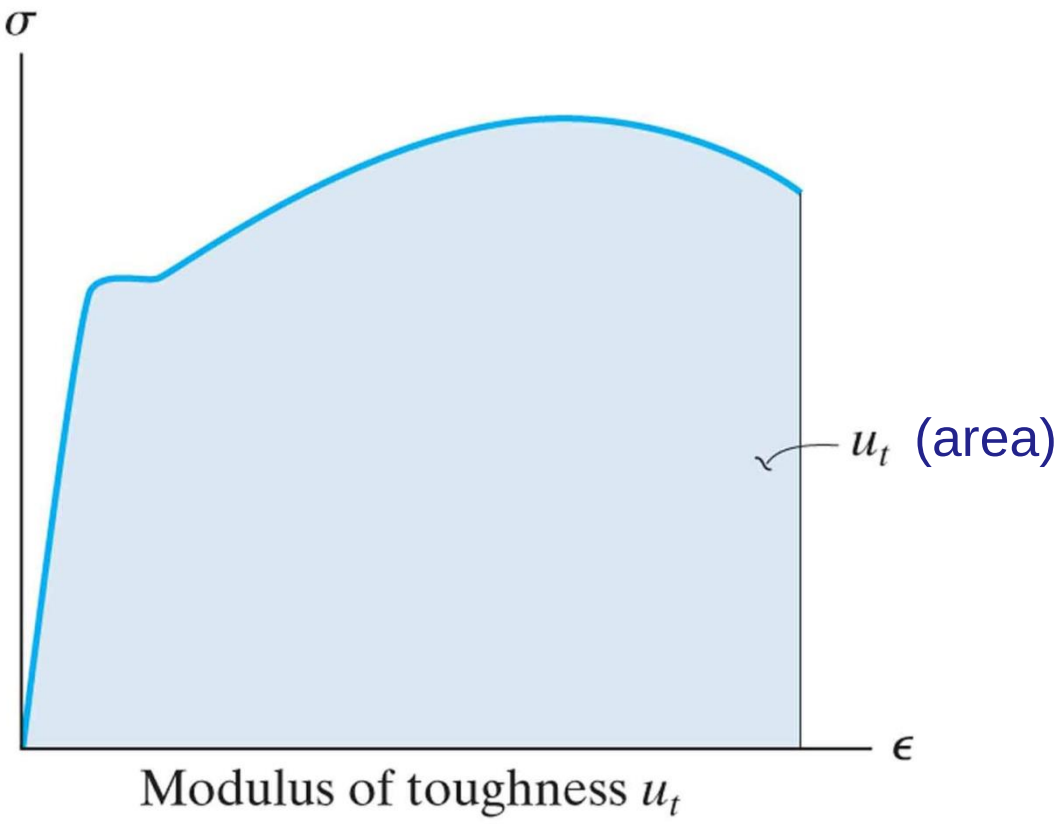
Modulus of resilience u_r

Strain energy: toughness

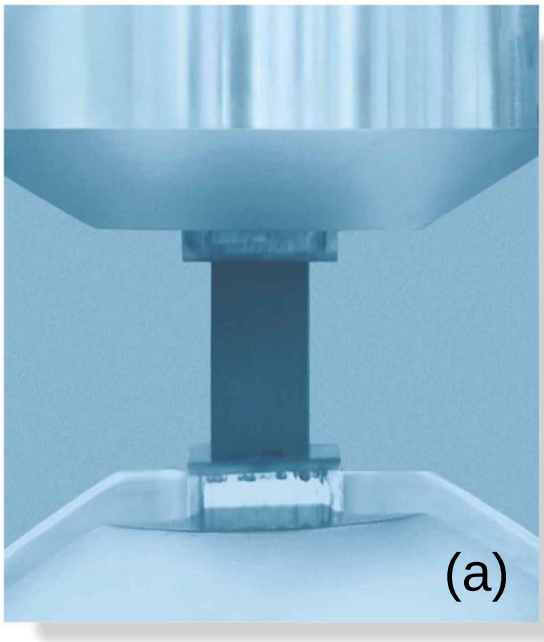
$$u = \frac{\Delta U}{\Delta V}$$

Modulus of toughness:

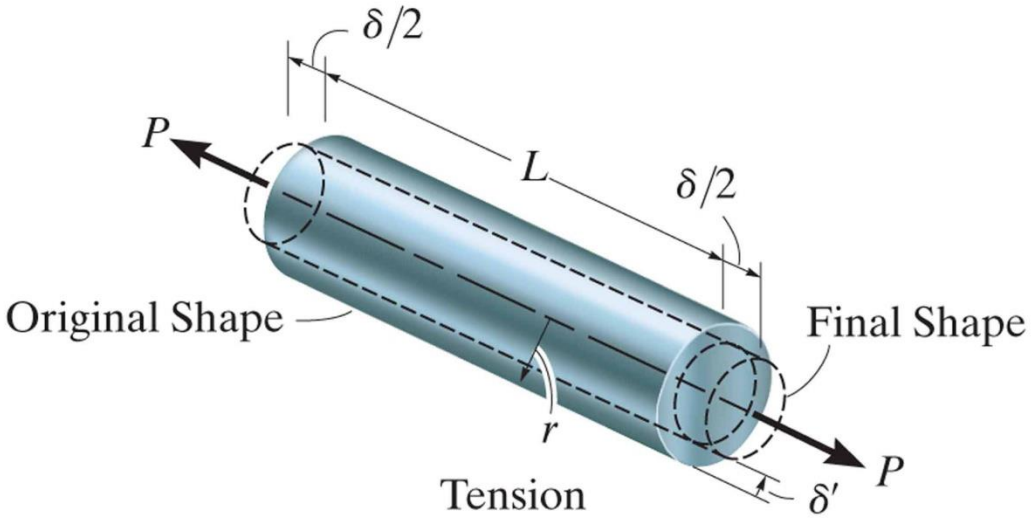
$$u_t = \int_0^{\varepsilon_f} \sigma \, d\varepsilon$$



Poisson's ratio:



Poisson's ratio: $\nu = -\dfrac{\varepsilon_{lateral}}{\varepsilon_{longitudinal}}$

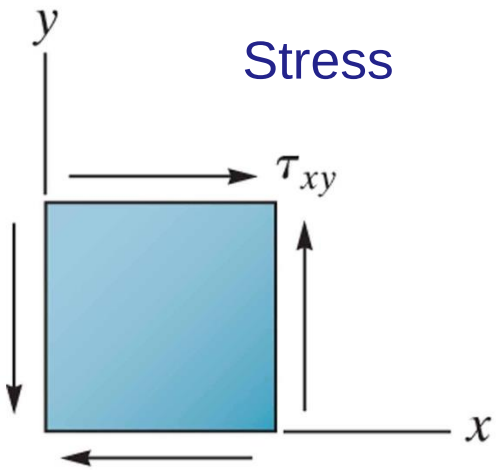


Shear stress ↔ strain

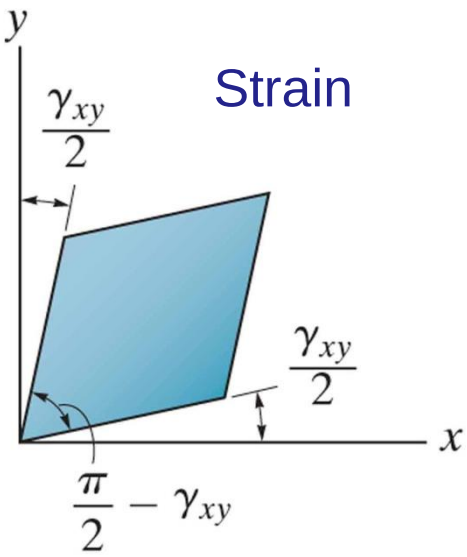
Pure shear

Hook's law for shear: $\tau = G\,\gamma$

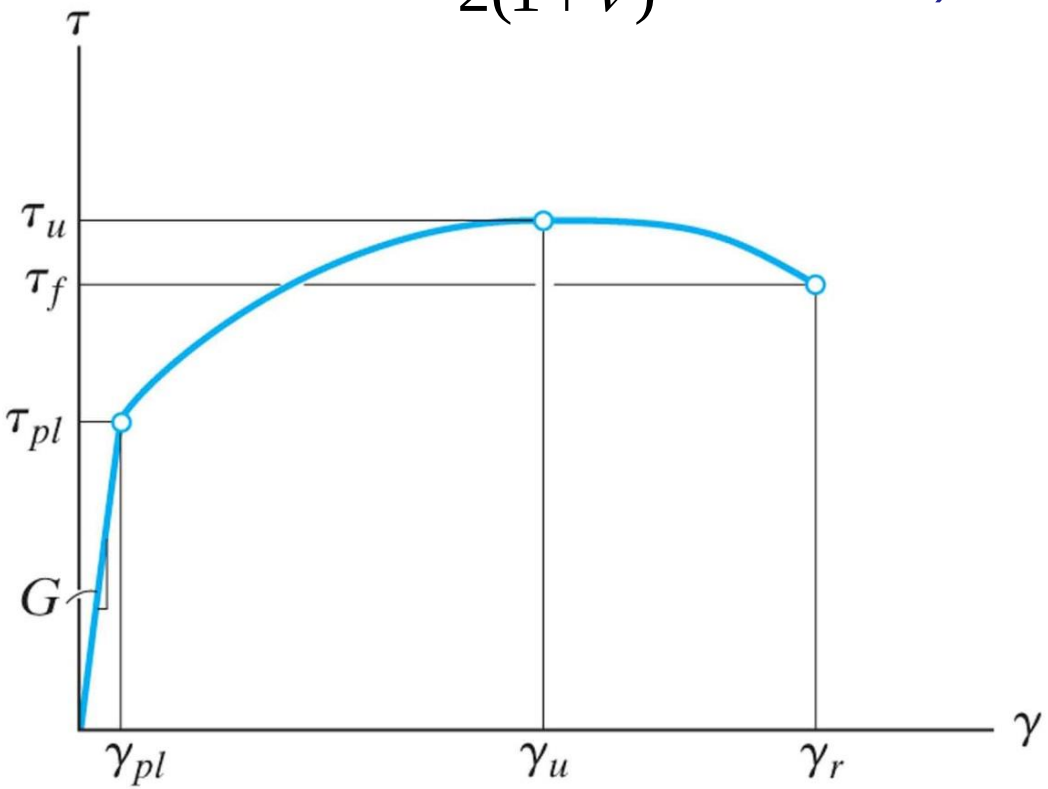
with $G = \dfrac{E}{2(1+\nu)}$ (shear modulus)



Stress



Strain



Reading assignment

- Chapter 3 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course