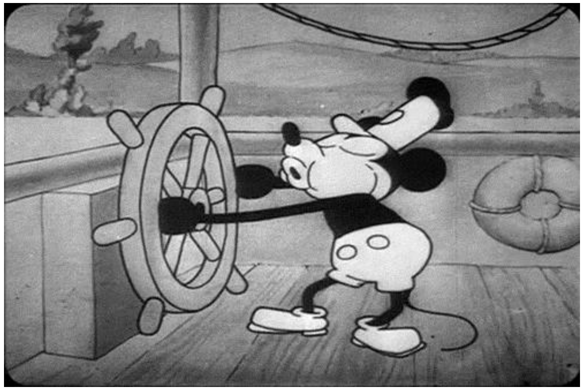
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, D'2020

We will get started soon...



06 April 2020

WORCESTER POLYTECHNIC INSTITUTE
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ES-2502, D'2020

We will get started soon...

Lecture 07:
Unit 5: Normal & Shear strain

06 April 2020

General information

Instructor: Cosme Furlong
HL-152
(508) 831-5126
Email: cfurlong @ wpi.edu
http://www.wpi.edu/~cfurlong/es2502.html

Teaching Assistant: Zachary Zolotarevsky
Email: zjzolotarevsky @ wpi.edu

Strain

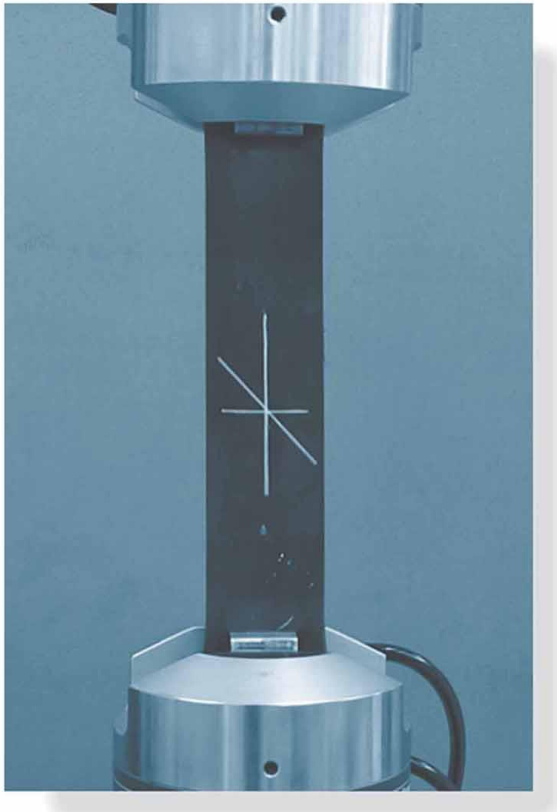


Figure: 02-01-A-UN

Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension.The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

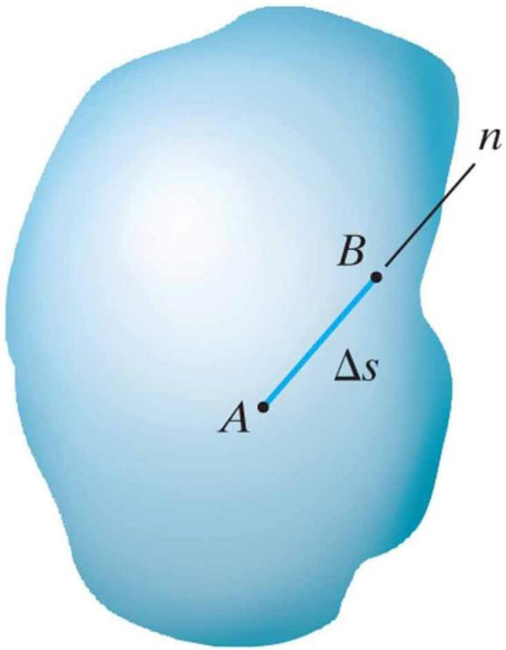
Observe what happened to the white line segments in this tensile test experiment

Figure: 02-01-B-UN

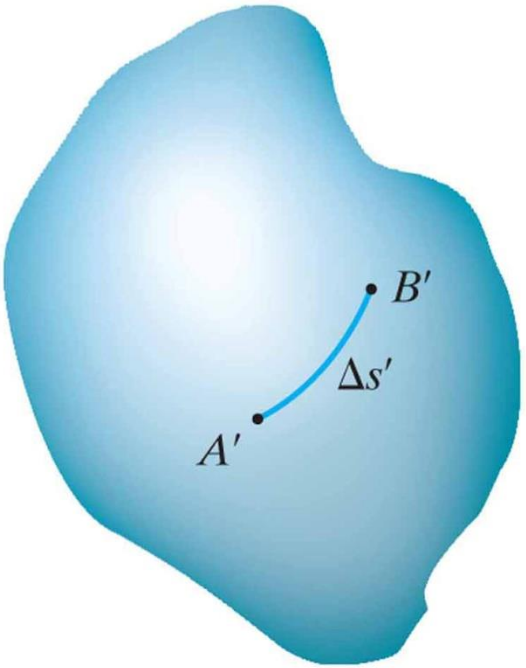
Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension.The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

Strain: definition: change in length per unit length

Normal strain



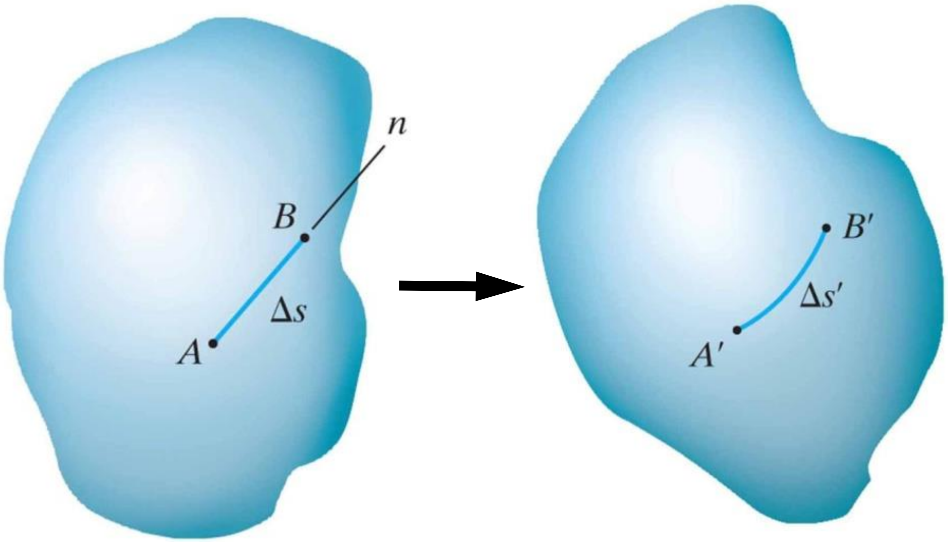
Undeformed body
(a)



Deformed body
(b)

<u>Strain</u>: definition: change in length per unit length

Normal strain



Undeformed body
(a)

Deformed body
(b)

<u>Average</u> normal strain:

$$\varepsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}$$

<u>Normal</u> strain:

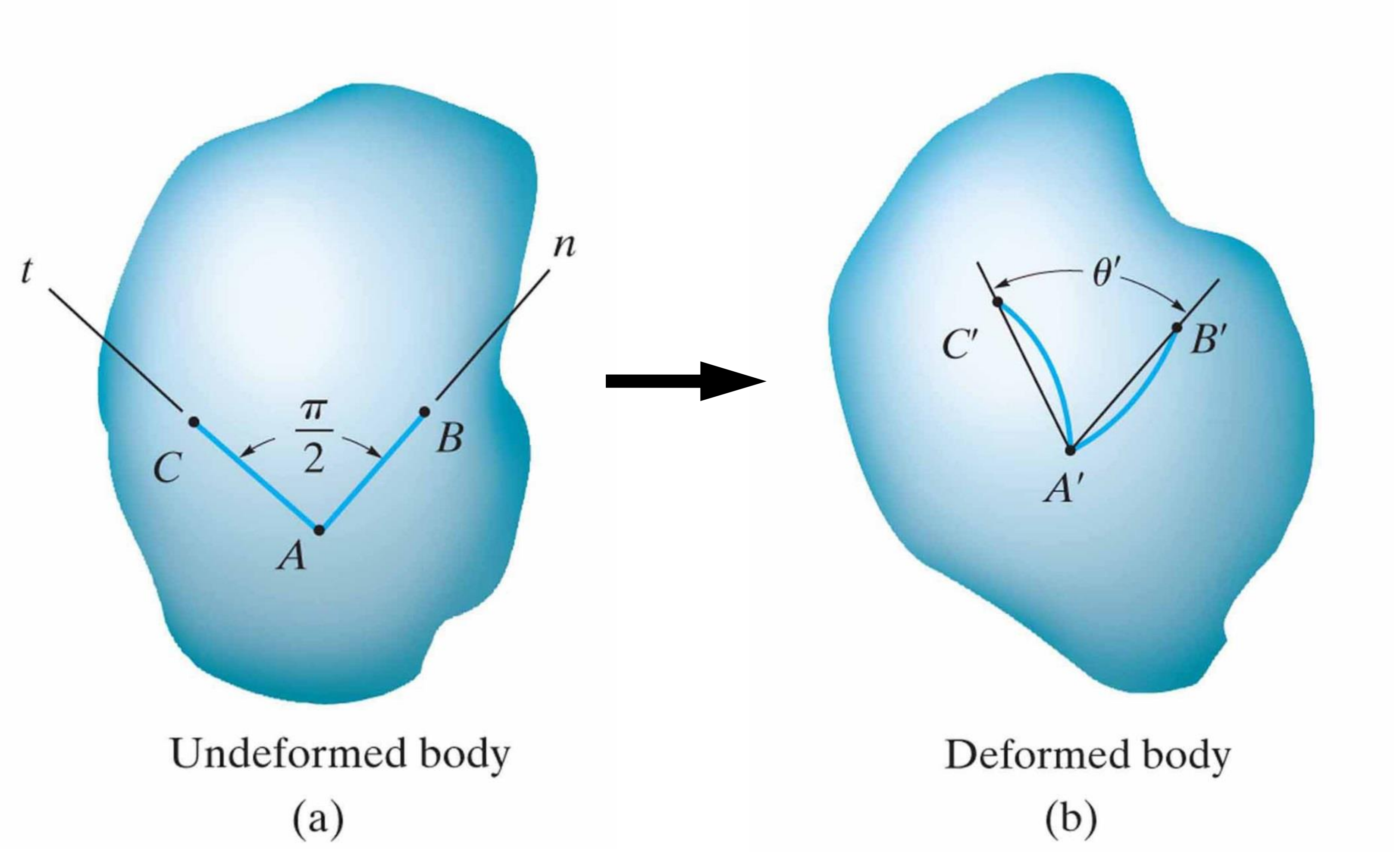
$$\varepsilon = \lim_{B \to A \ along \ n} \frac{\Delta s' - \Delta s}{\Delta s}$$

<u>Note that,</u>

$$\varepsilon = \frac{d(\Delta s)}{ds}$$

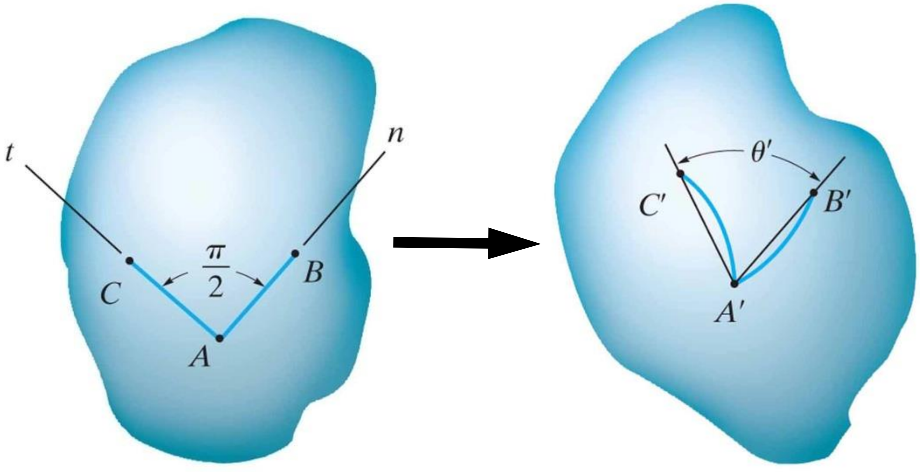
Strain: definition: change in length per unit length

Shear strain



<u>Strain</u>: definition: change in length per unit length

Shear strain



Undeformed body
(a)

Deformed body
(b)

<u>Shear </u>strain:

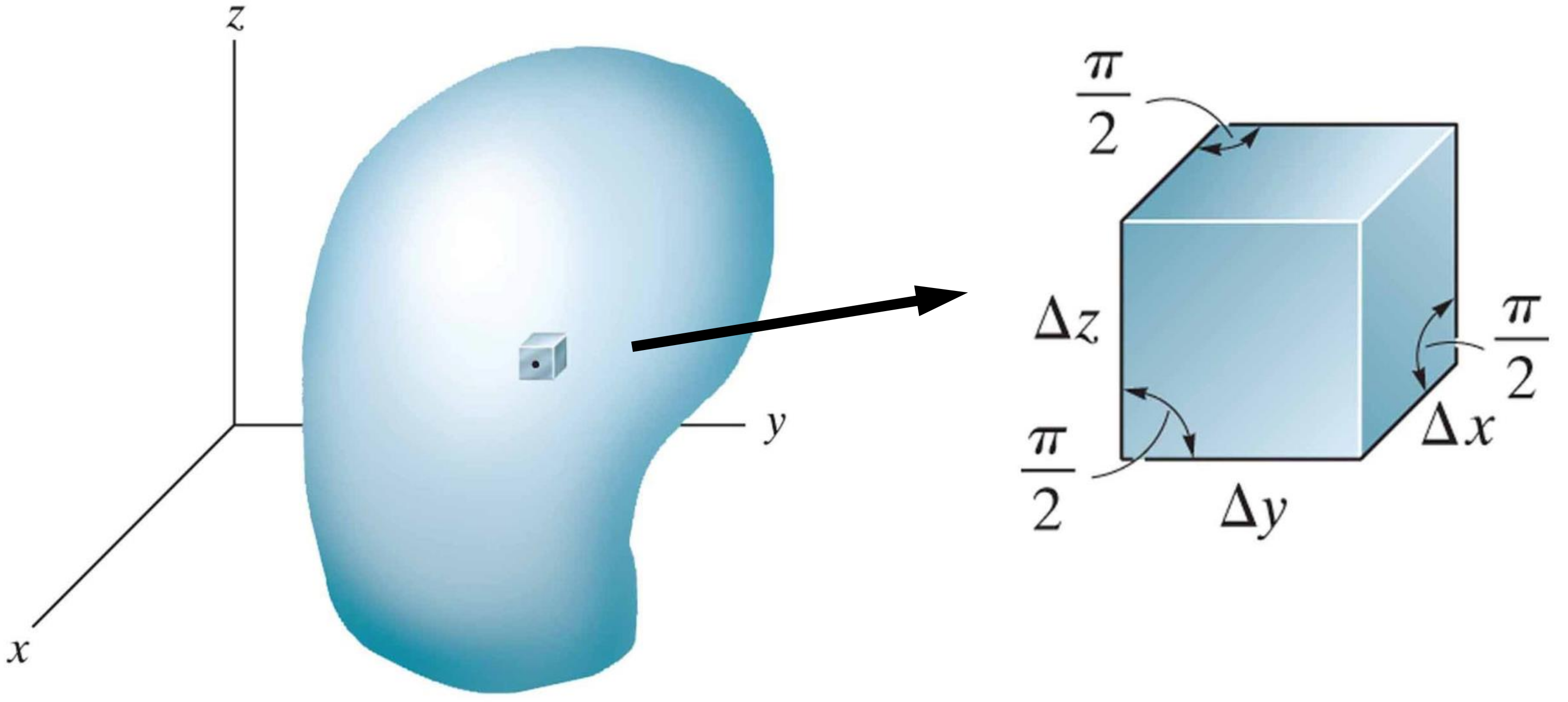
$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \ along \ n \\ C \to A \ along \ t}} \theta'$$

Cartesian strain components

Before deformations

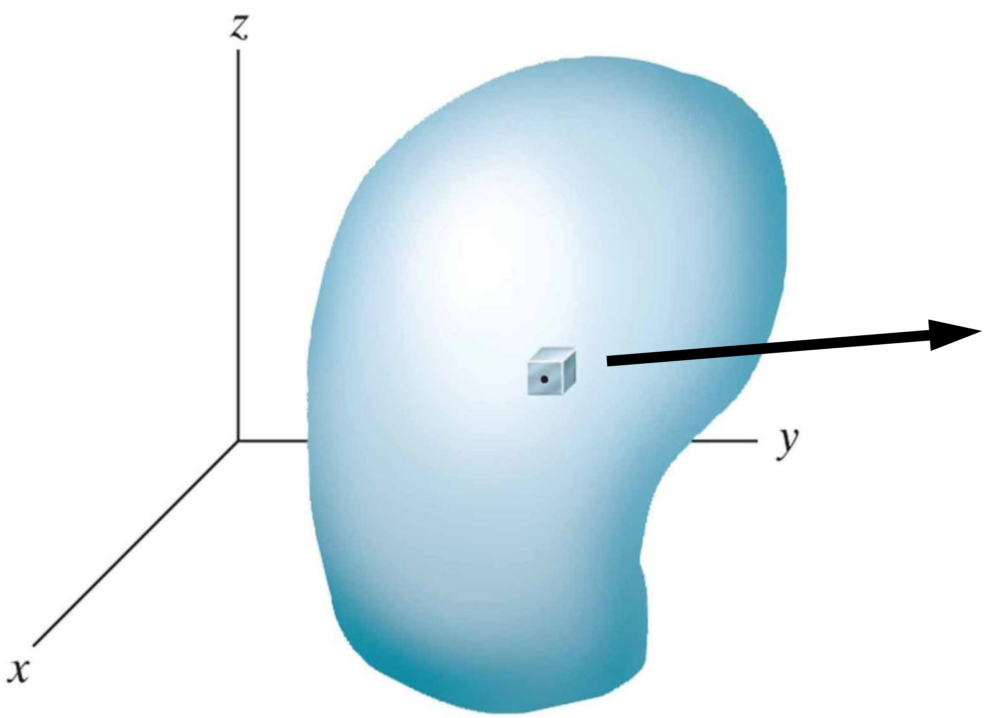
Differential element or cube (undeformed)

Dimensions: Δx, Δy, Δz



Cartesian strain components

After deformations

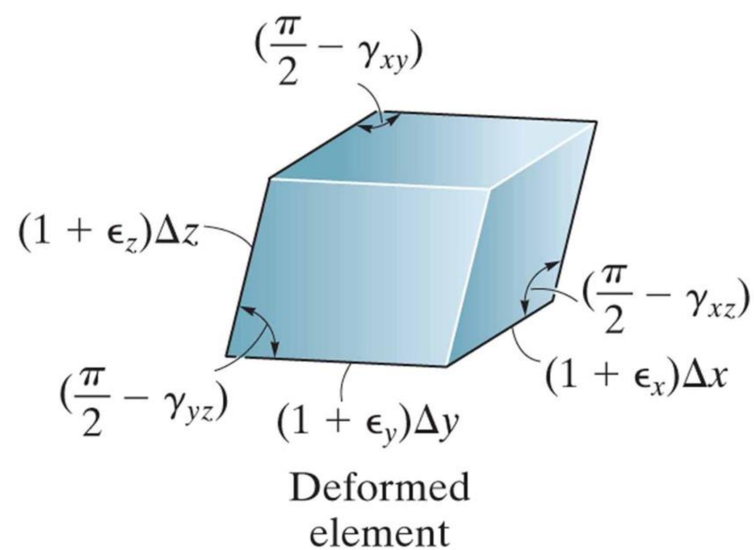


Differential element or cube (deformed)

New dimensions: $\Delta x + \varepsilon_x(\Delta x)$

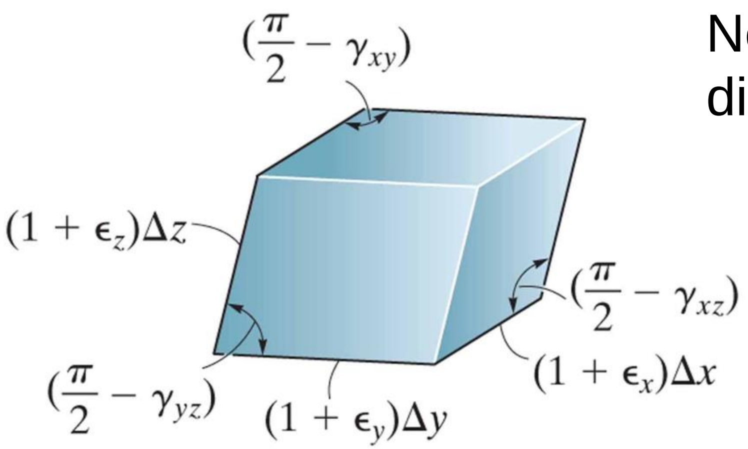
$\Delta y + \varepsilon_y(\Delta y)$

$\Delta z + \varepsilon_z(\Delta z)$



Cartesian strain components

Differential element or cube (deformed)



Deformed element

Original dimensions

Elongations

New dimensions:

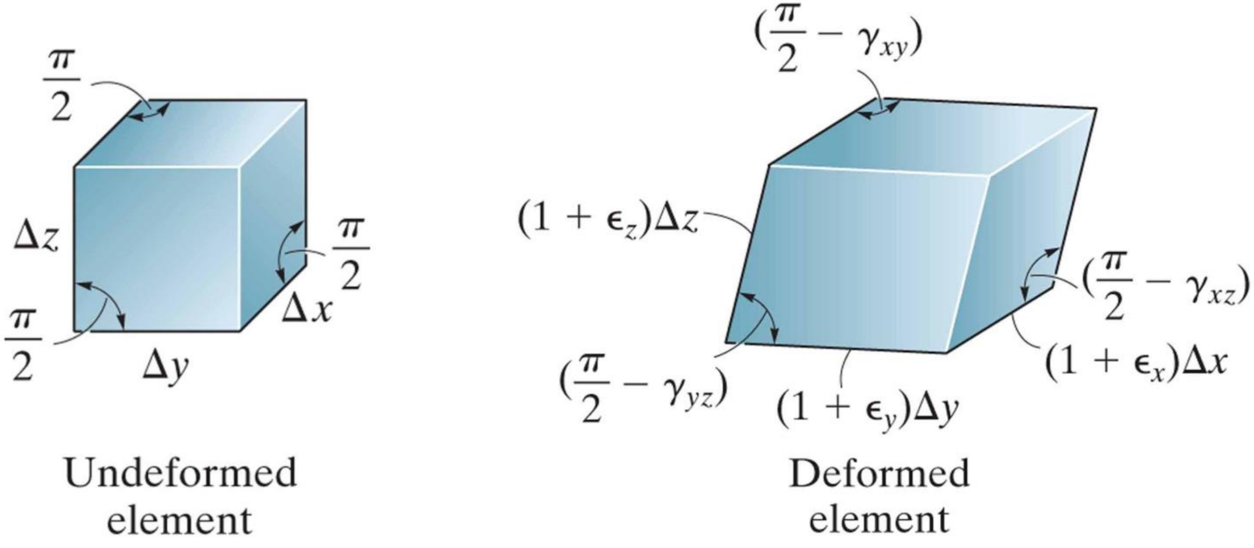
$$\Delta x + \varepsilon_x(\Delta x) = (1 + \varepsilon_x)\,\Delta x$$
$$\Delta y + \varepsilon_y(\Delta y) = (1 + \varepsilon_y)\,\Delta y$$
$$\Delta z + \varepsilon_z(\Delta z) = (1 + \varepsilon_z)\,\Delta z$$

Approximate angles between sides:

$$\frac{\pi}{2} - \gamma_{xy} \;,\quad \frac{\pi}{2} - \gamma_{yz} \;,\quad \frac{\pi}{2} - \gamma_{xz}$$

Cartesian strain components

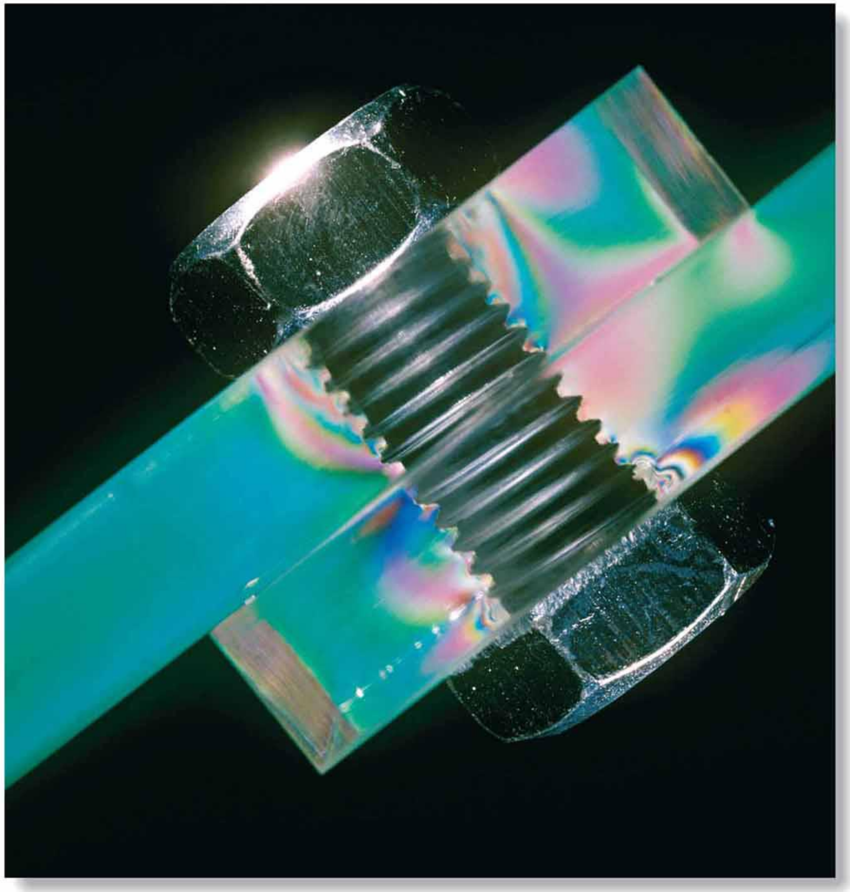
1) Normal strains cause a change in volume of the element
2) Shear strains cause a change in its shape
3) Normal and shear strains occurs simultaneously during deformation
4) State of strain at a point on a body requires specifying 6 strain components: ε_x, ε_y, ε_z, and γ_{xy}, γ_{yz}, γ_{xz}



Typical strain distributions generated inside a bolted assembly

Polarized light used in experiment shown: bolted assembly

Strains are related to stresses in the materials

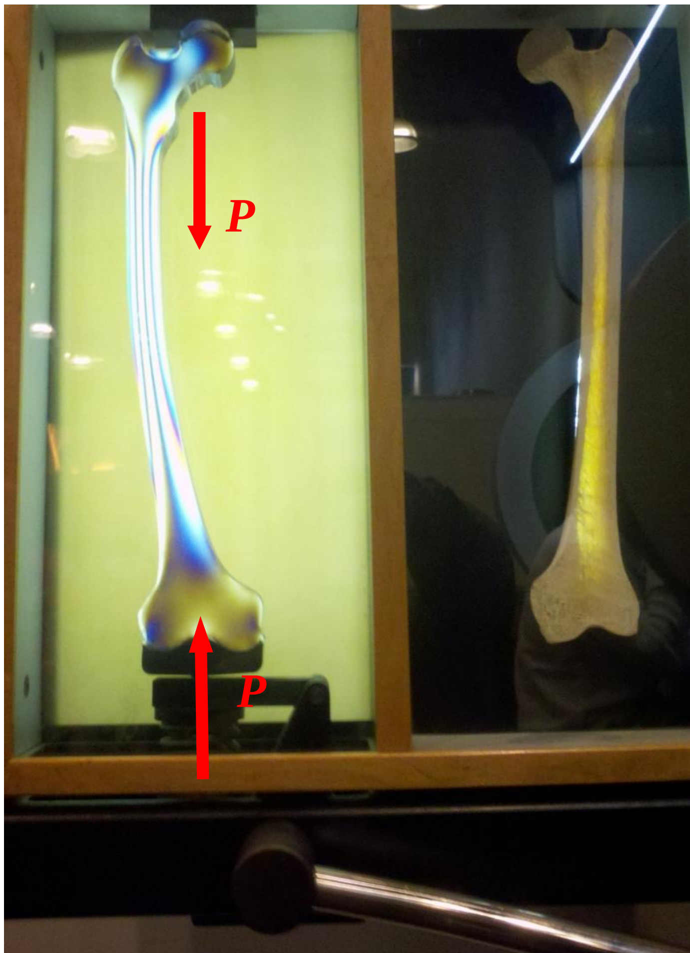


Strains can be measured and stresses estimated from strains

Typical strain distributions generated inside a bolted assembly

Polarized light used in experiment shown: component in compression

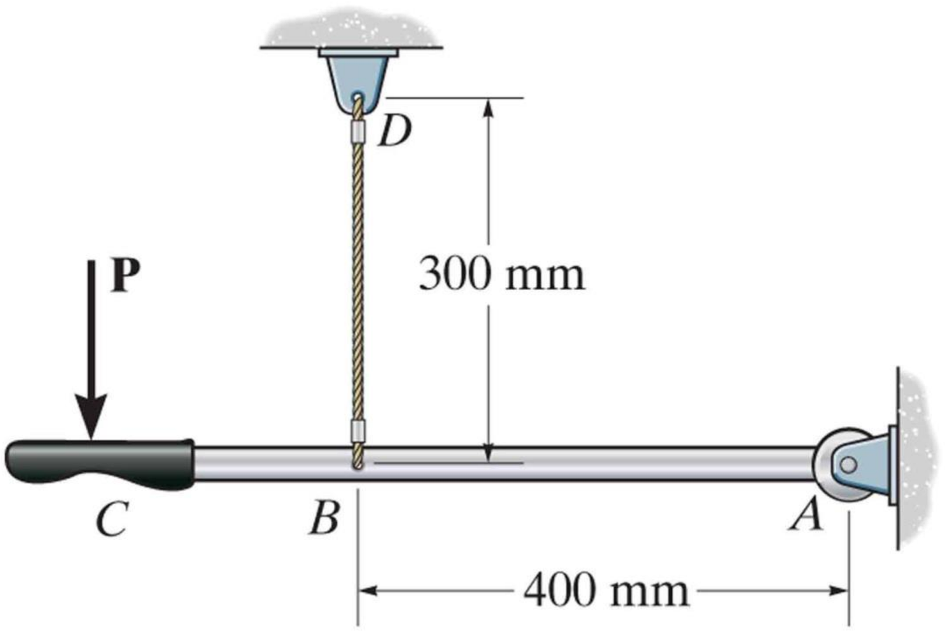
Strains are related to stresses in the materials



Strains can be measured and stresses estimated from strains

Strain: example A

When force **P** is applied to the rigid lever arm *ABC* shown, the arm rotates counterclockwise about pin *A* through and angle of 0.05^O. Determine the normal strain developed in wire *BD*.



Approach:

1) Define geometry
2) Determine change in geometry
3) Compute required strains

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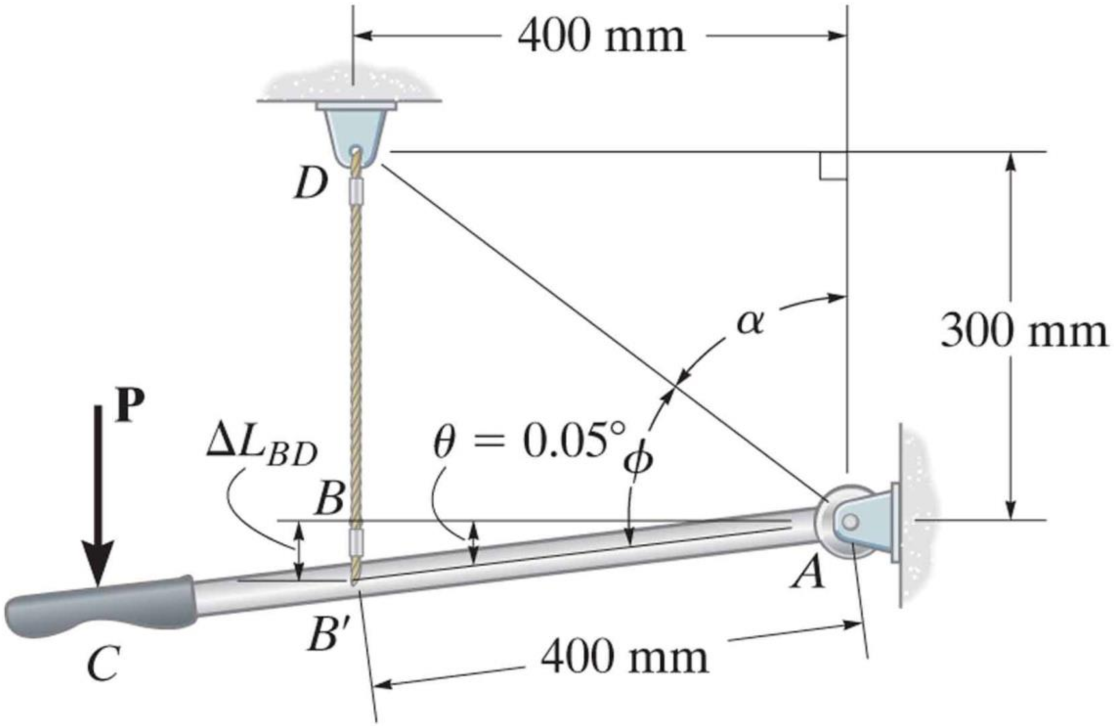
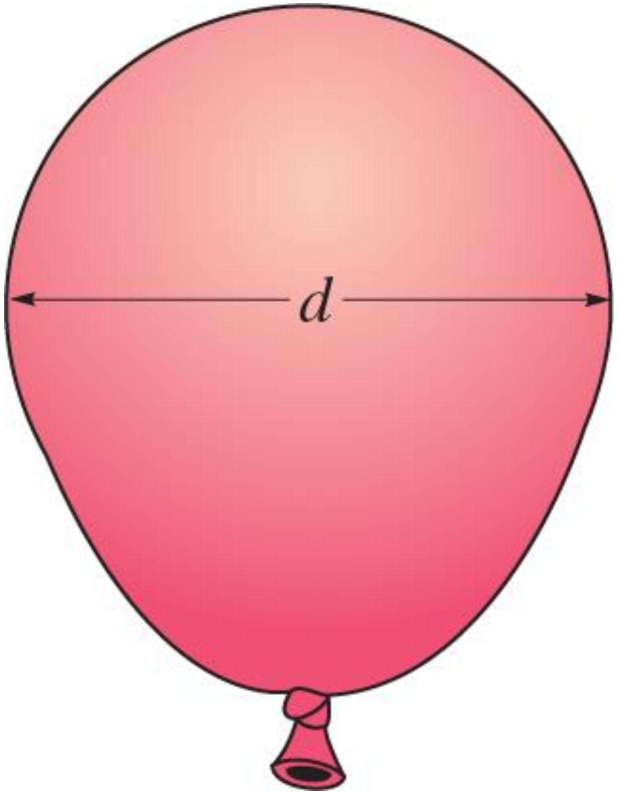


Change in geometry
&
Computation of
normal strain

Strain: example B

An air-filled rubber balloon has a diameter of 6 in. If the air pressure within it is increased until the ball's diameter becomes 7 in, determine the average normal strain in the rubber



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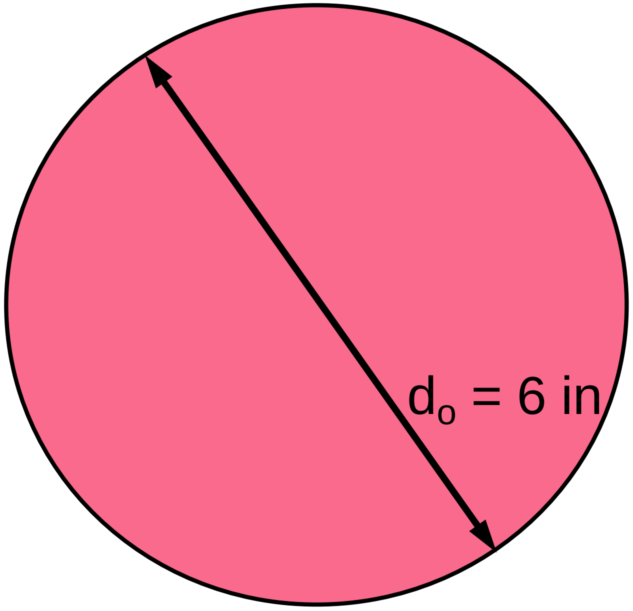

Approach:

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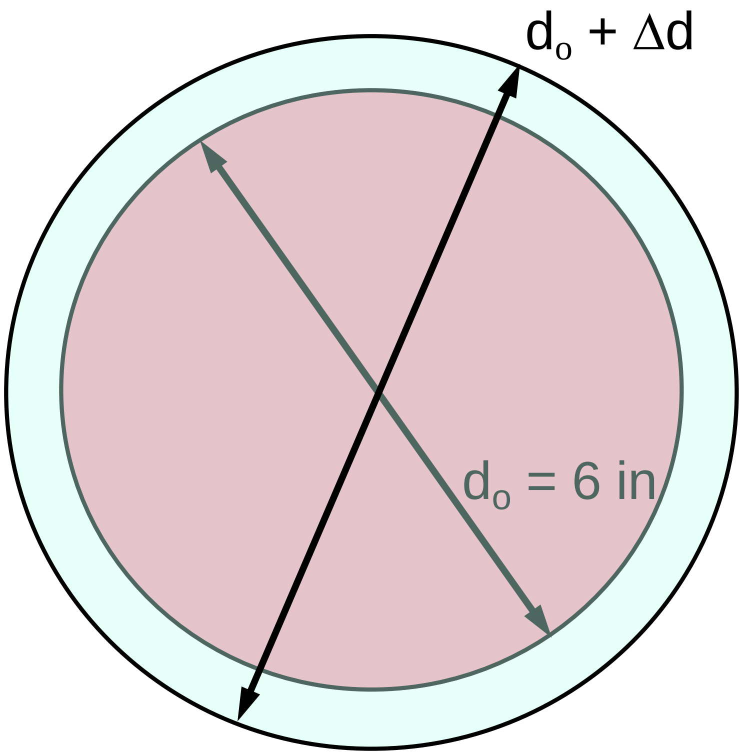
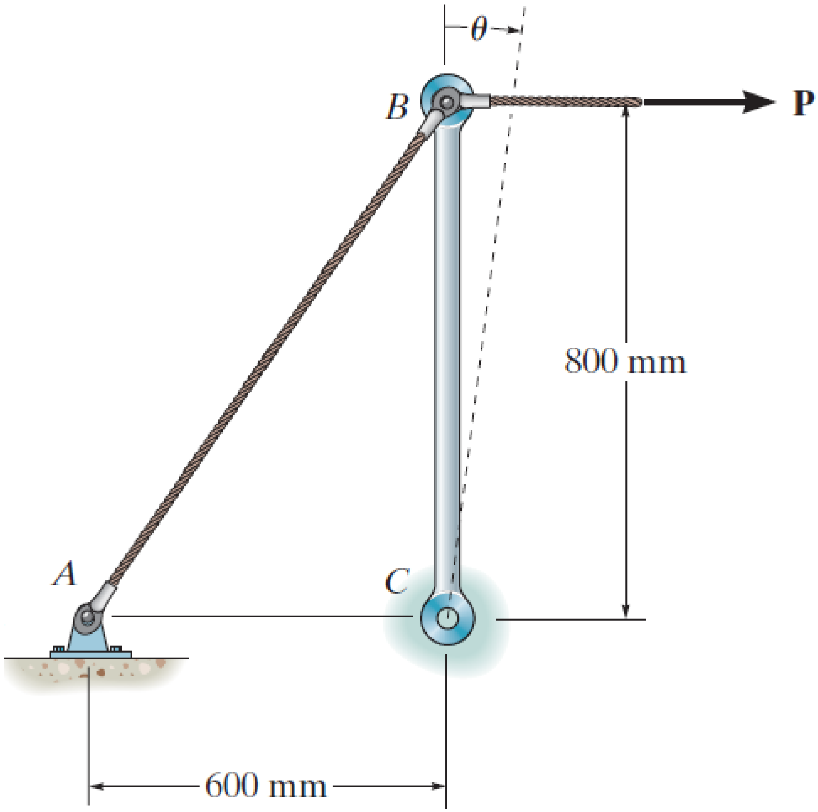


Change in geometry
&
Computation of normal strain

change in length of circumference $\rightarrow \varepsilon$

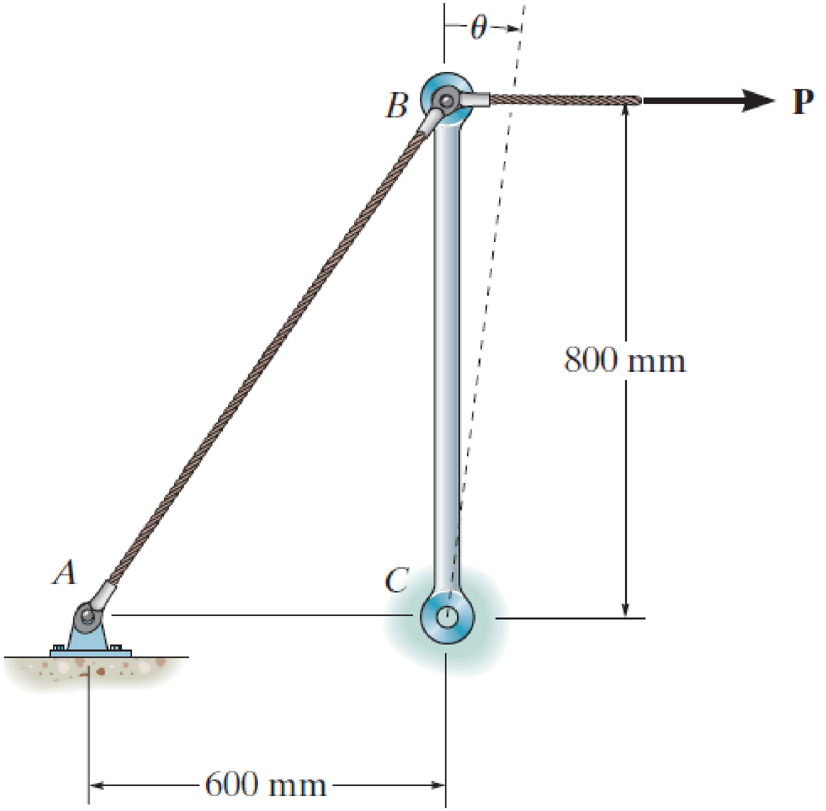
Strain: example C

Part of a control linkage for an airplane consists of a rigid member CB and a flexible cable AB. If a force is applied to the end B of the member and causes it to rotate by $\theta = 0.5^{o}$, determine the normal strain in the cable. Originally the cable is unstretched.



Strain: example D

Part of a control linkage for an airplane consists of a rigid member CB and a flexible cable AB. If a force is applied to the end B of the member and causes a normal strain in the cable of 0.004 mm/mm, determine the displacement of point B. Originally the cable is unstretched.



Reading assignment

- Chapter 2 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course