

# WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

## STRESS ANALYSIS ES-2502, D'2020

We will get started soon...



03 April 2020



# WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

## STRESS ANALYSIS ES-2502, D'2020

We will get started soon...

Lecture 06:  
Unit 5: Strain: definition of normal  
strain and shear strain

03 April 2020



# General information

Instructor: Cosme Furlong

HL-152

(508) 831-5126

Email: cfurlong @ wpi.edu

<http://www.wpi.edu/~cfurlong/es2502.html>

Teaching Assistant: Zachary Zolotarevsky

Email: zjzolotarevsky @ wpi.edu



# Design of simple connections

## Allowable stress: safety factor ( $SF$ )

$$SF = \frac{F_{fail}}{F_{allow}}$$

In terms of  
forces

or

$$SF = \frac{\sigma_{fail}}{\sigma_{allow}}$$

In terms of  
**normal** stresses

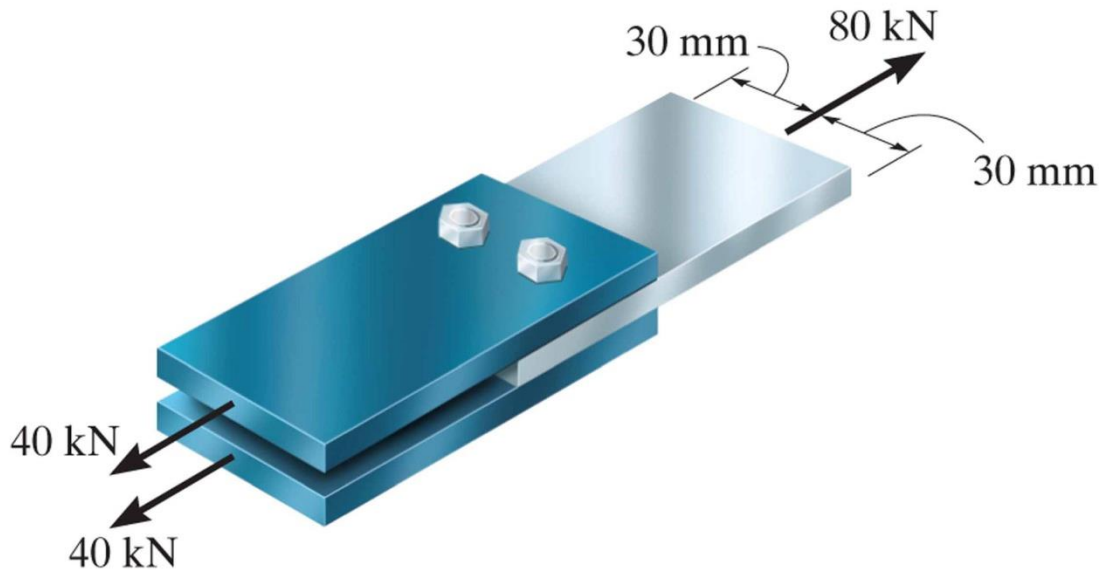
$$SF = \frac{\tau_{fail}}{\tau_{allow}}$$

In terms of  
**shear** stresses



# Design of simple connections: example D

The joint is fastened together using two bolts. Determine the required diameter of the bolts if the failure shear stress for the bolts is  $\tau_{fail} = 350 \text{ MPa}$ . Apply a safety factor for shear of 2.5.



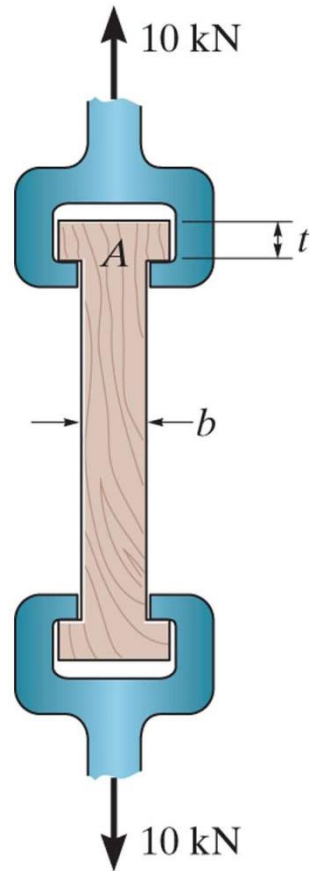
## Approach:

- 1) Define free-body diagrams
- 2) Determine internal loadings
- 3) Use safety factor
- 4) Compute diameter



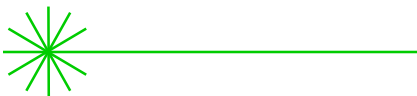
# Design of simple connections: example E

The wood specimen is subjected to the pull of  $10\text{ kN}$  in a tension testing machine. If the allowable normal stress for the wood is  $\sigma_{allow} = 12\text{ MPa}$  and the allowable shear stress is  $\tau_{allow} = 1.2\text{ MPa}$  determine the required dimensions  $b$  and  $t$  so that the specimen reaches these stresses simultaneously. The specimen has a depth of  $25\text{ mm}$ .



## Approach:

- 1) Define free-body diagrams
- 2) Determine internal loadings
- 3) Compute  $b$  and  $t$  using stress equations



# Strain

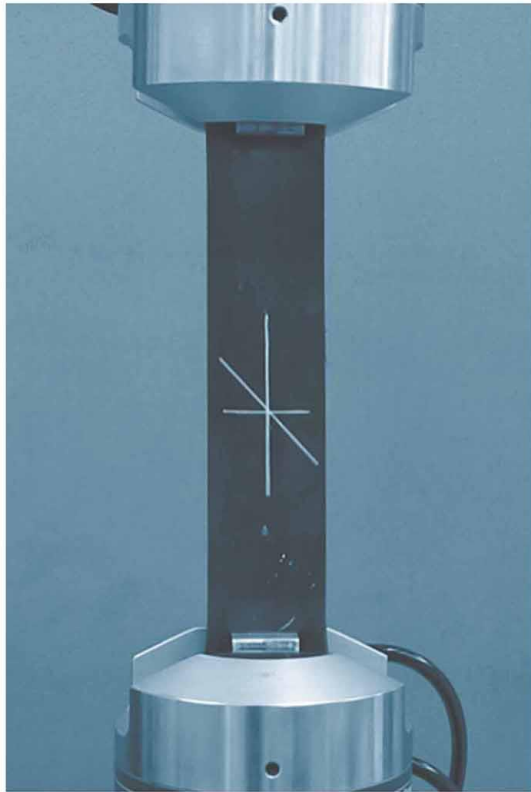


Figure: 02-01-A-UN

Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension. The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

Observe what happened to the white line segments in this tensile test experiment

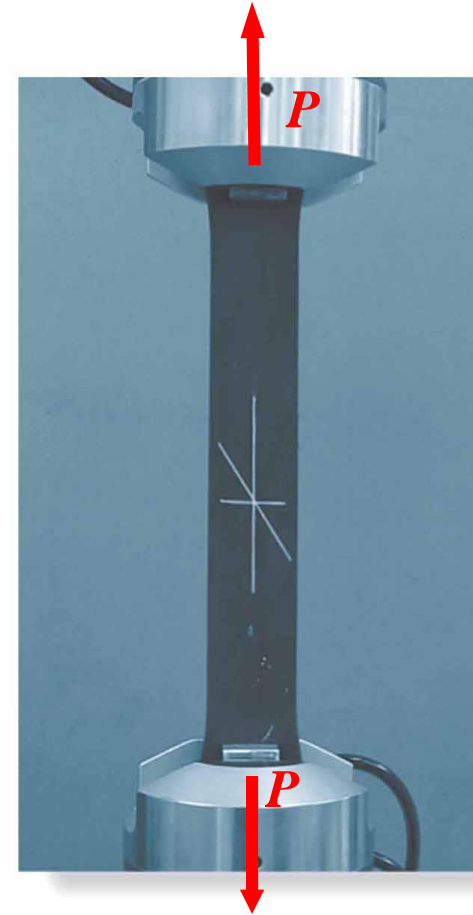


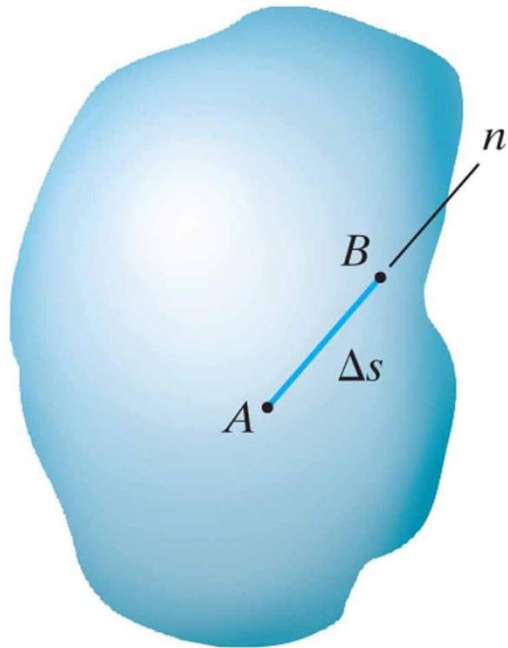
Figure: 02-01-B-UN

Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension. The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.



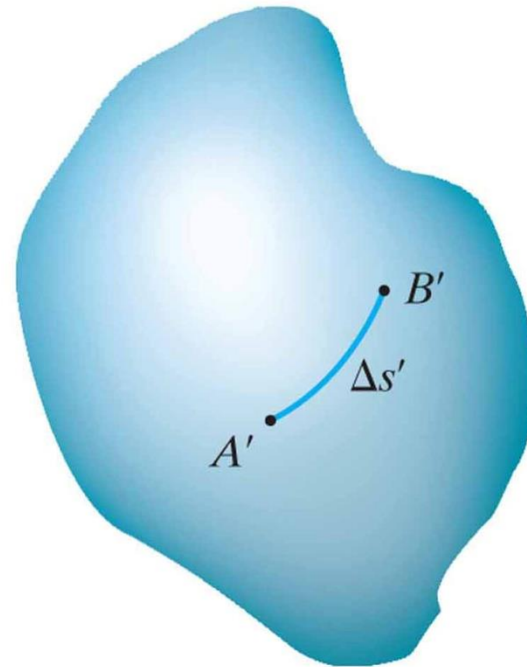
# Strain: definition: change in length per unit length

## Normal strain



Undeformed body

(a)



Deformed body

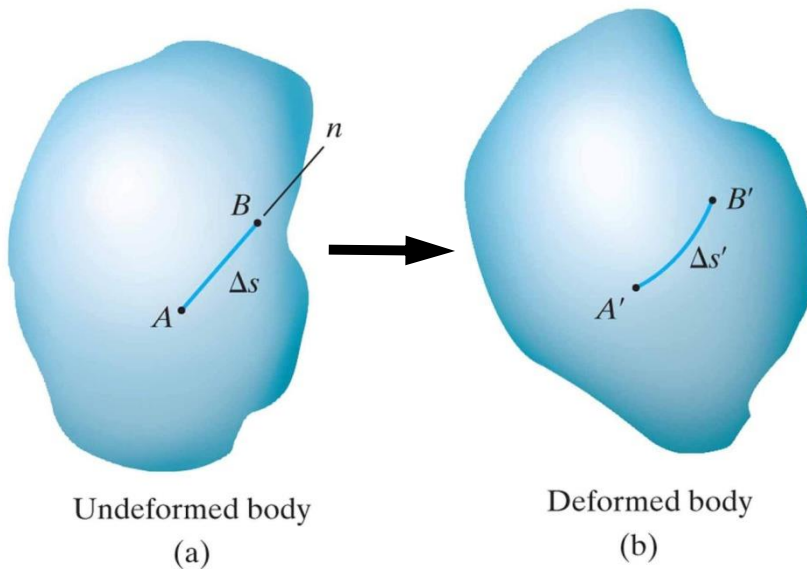
(b)





# Strain: definition: change in length per unit length

## Normal strain



Average normal strain:

$$\epsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}$$

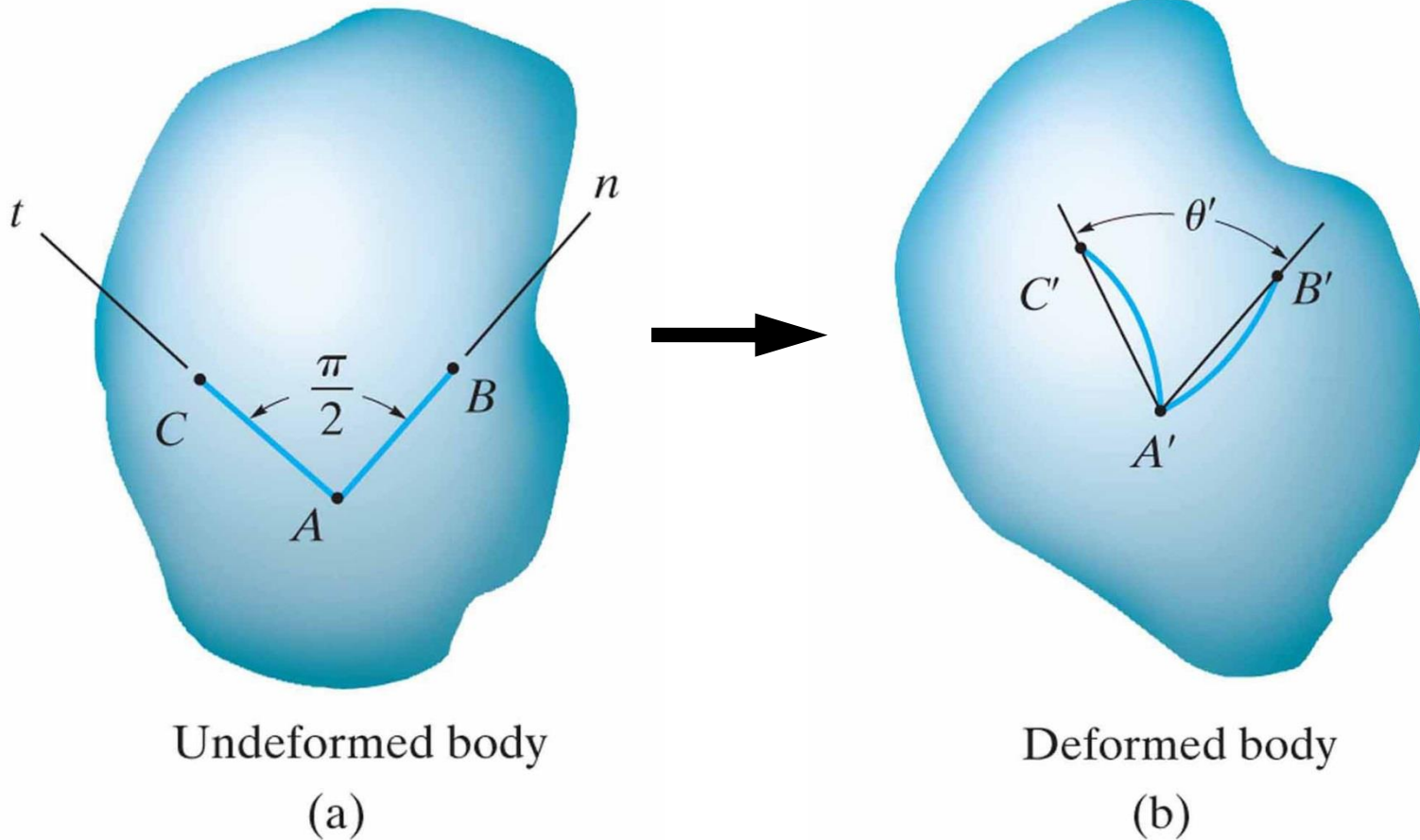
Normal strain:

$$\epsilon = \lim_{B \rightarrow A \text{ along } n} \frac{\Delta s' - \Delta s}{\Delta s}$$



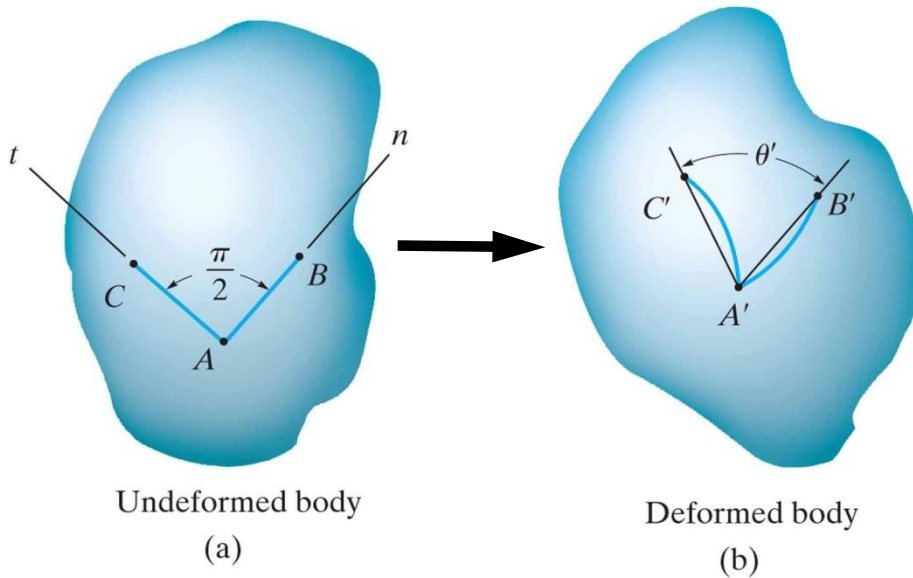
# Strain: definition: change in length per unit length

## Shear strain



# Strain: definition: change in length per unit length

## Shear strain



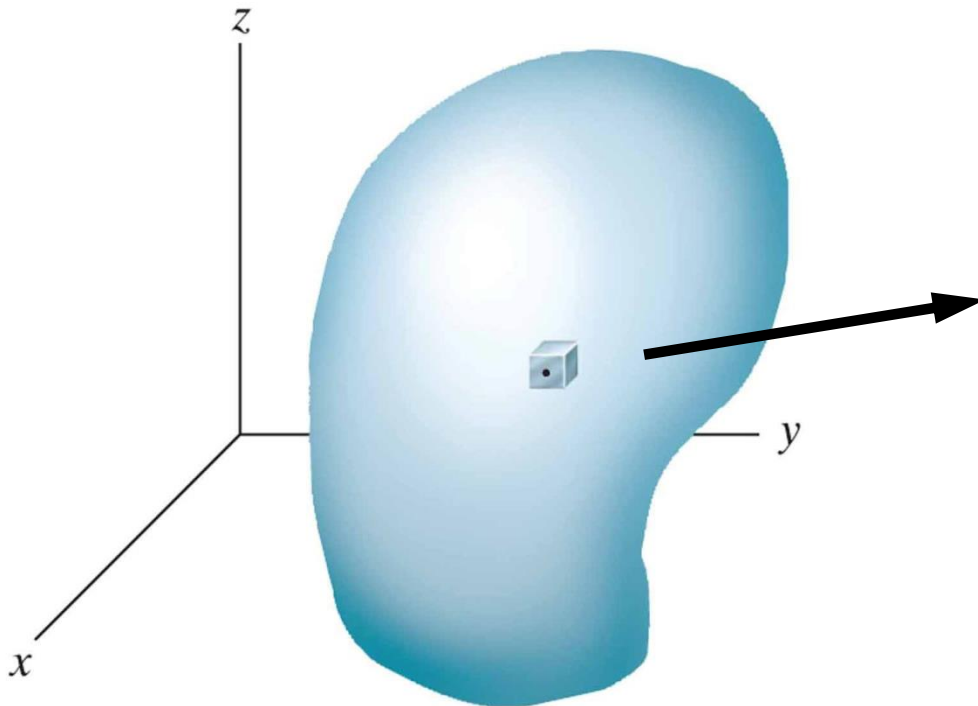
Shear strain:

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \rightarrow A \text{ along } n \\ C \rightarrow A \text{ along } t}} \theta'$$



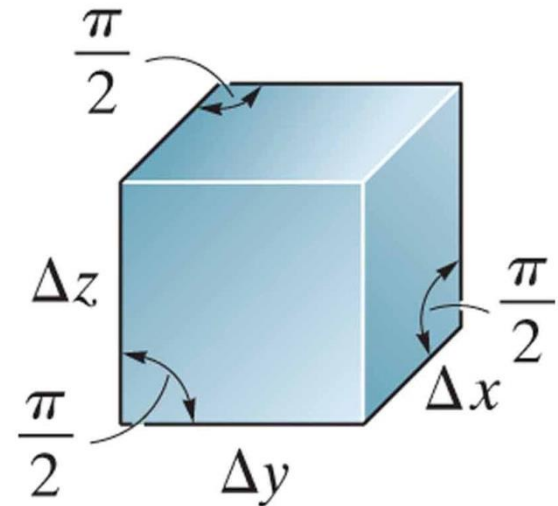
# Cartesian strain components

**Before  
deformations**



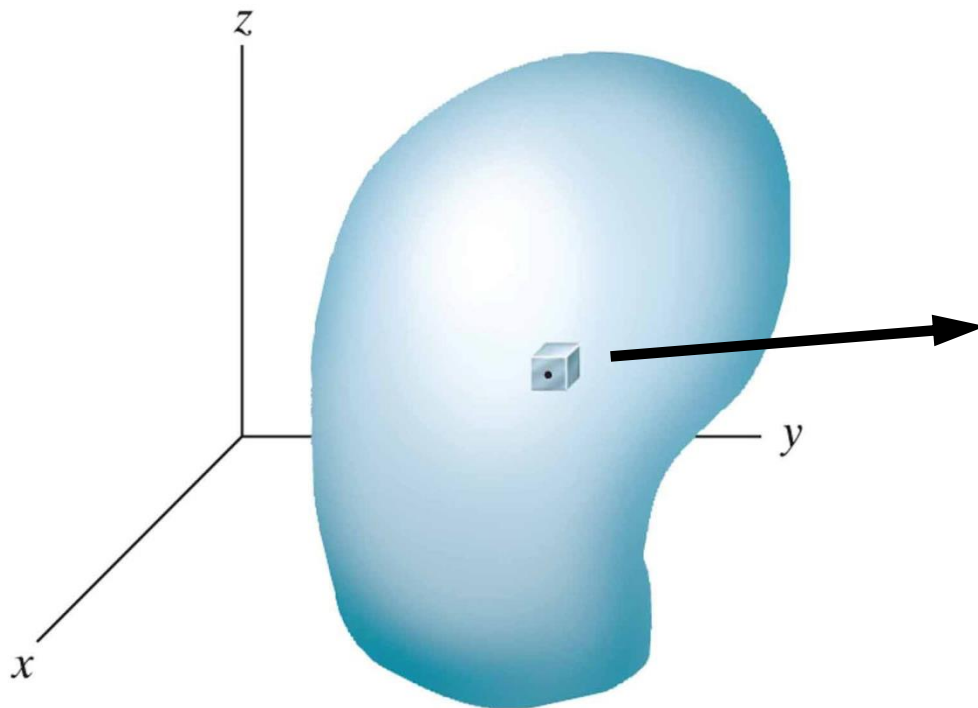
Differential  
element or cube  
(undeformed)

Dimensions:  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$



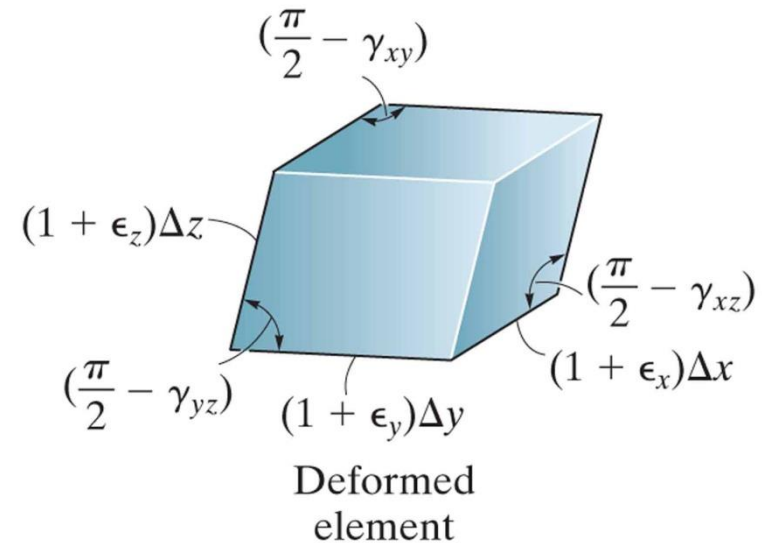
# Cartesian strain components

**After  
deformations**



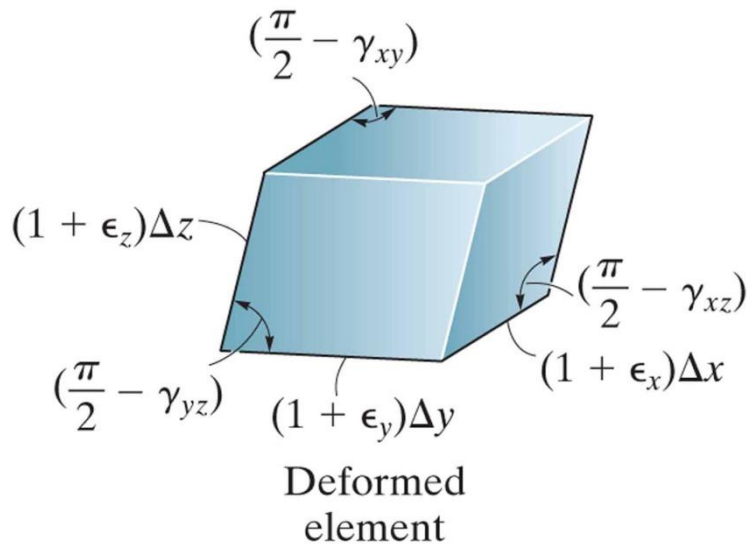
Differential  
element or cube  
(deformed)

New dimensions:  $\Delta x + \epsilon_x (\Delta x)$   
 $\Delta y + \epsilon_y (\Delta y)$   
 $\Delta z + \epsilon_z (\Delta z)$



# Cartesian strain components

Differential element or cube  
(deformed)



Original dimensions

Elongations

New dimensions:

$$\begin{array}{l} \Delta x + \epsilon_x (\Delta x) \\ \Delta y + \epsilon_y (\Delta y) \\ \Delta z + \epsilon_z (\Delta z) \end{array}$$

Approximate angles between sides:

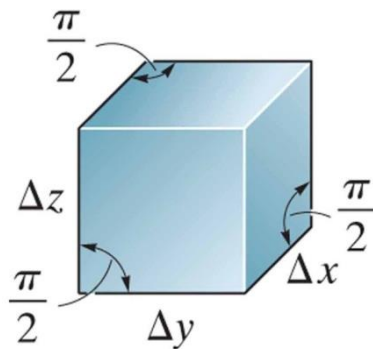
$$\frac{\pi}{2} - \gamma_{xy} , \quad \frac{\pi}{2} - \gamma_{yz} , \quad \frac{\pi}{2} - \gamma_{xz}$$



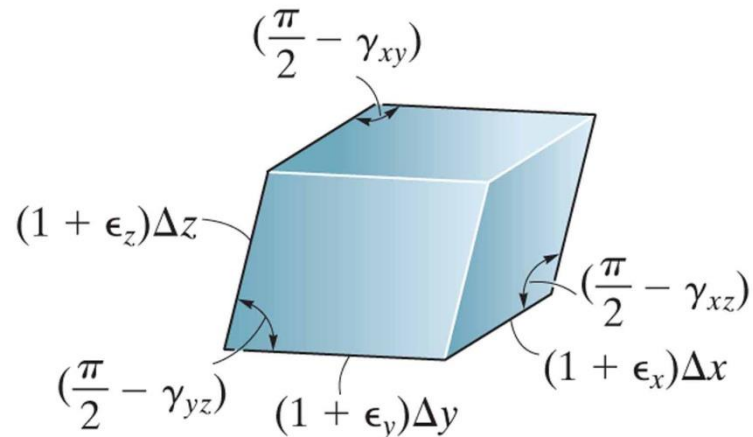
# Cartesian strain components

- 1) Normal strains cause a change in volume of the element
- 2) Shear strains cause a change in its shape
- 3) Normal and shear strains occurs simultaneously during deformation
- 4) State of strain at a point on a body requires:  $\epsilon_x, \epsilon_y, \epsilon_z$ , and

$$\gamma_{xy}, \gamma_{yz}, \gamma_{xz}$$



Undeformed  
element



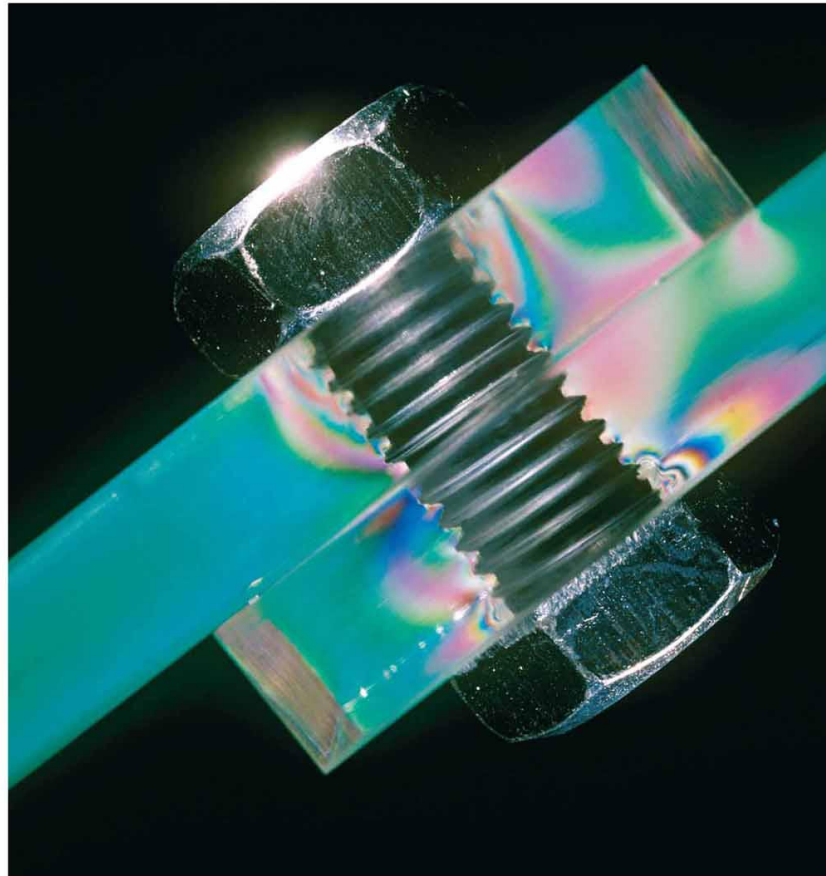
Deformed  
element



# Typical **strain** distributions generated inside a bolted assembly

Polarized light used in experiment shown: bolted assembly

Strains are related to stresses in the materials



Strains can be measured and stresses estimated from strains

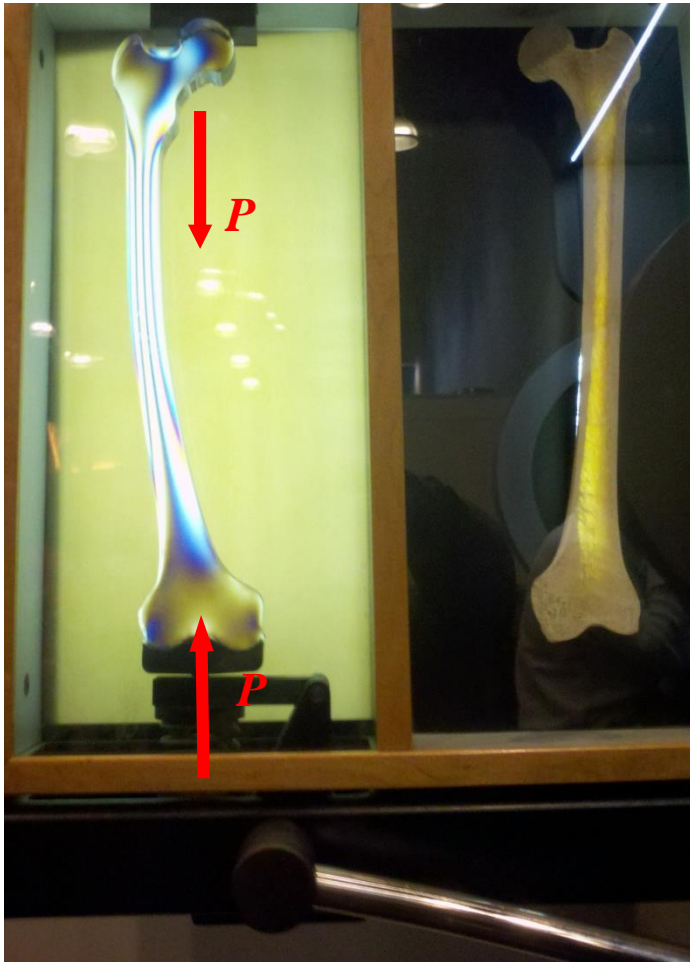




# Typical **strain** distributions generated inside a bolted assembly

Polarized light used in experiment shown: component in compression

Strains are related to stresses in the materials



Strains can be measured and stresses estimated from strains

# Reading assignment

- Chapter 1 of textbook
- Review notes and text: ES2001, ES2501



# Homework assignment

- As indicated on webpage of our course

