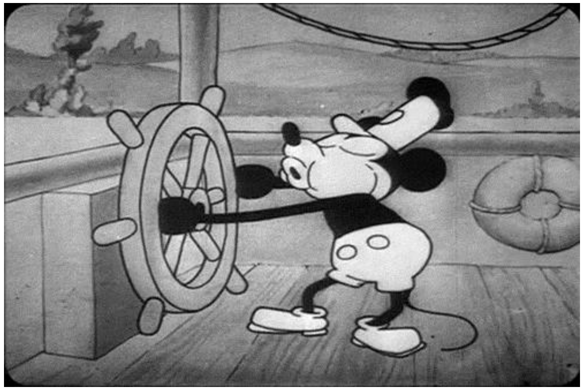
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, D'2020

We will get started soon...



31 March 2020

WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, D'2020

We will get started soon...

Lecture 04:
Unit 3: definition of normal and shear stress

31 March 2020

General information

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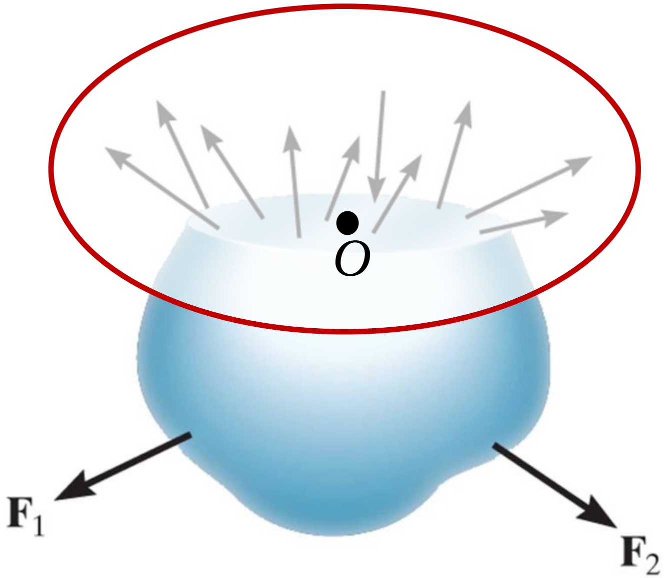
Stress: Definition: intensity of internal force: acting on a specific plane passing through a point

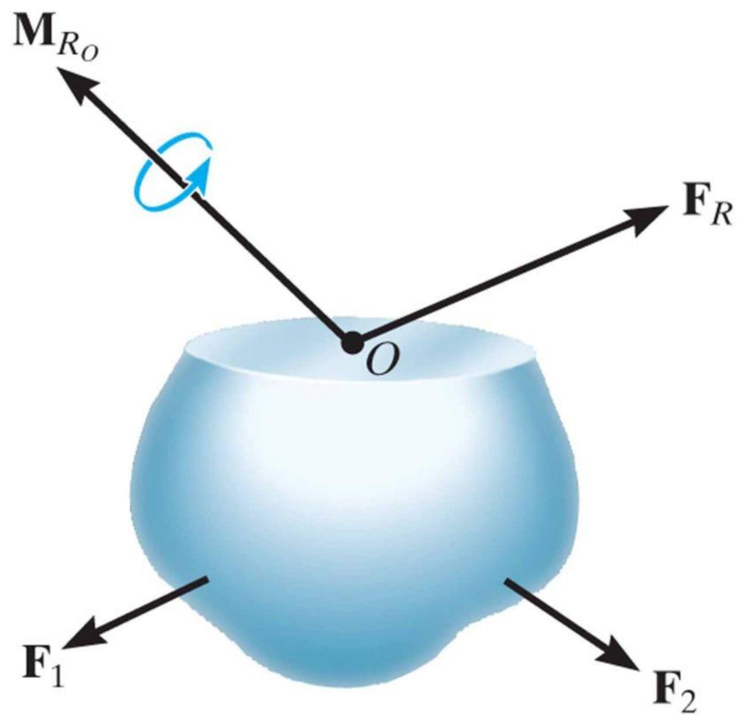
$$\mathbf{F}_R = \sum \mathbf{F}$$

$$\mathbf{M}_{R_O} = \sum \mathbf{M}_O$$



(b)

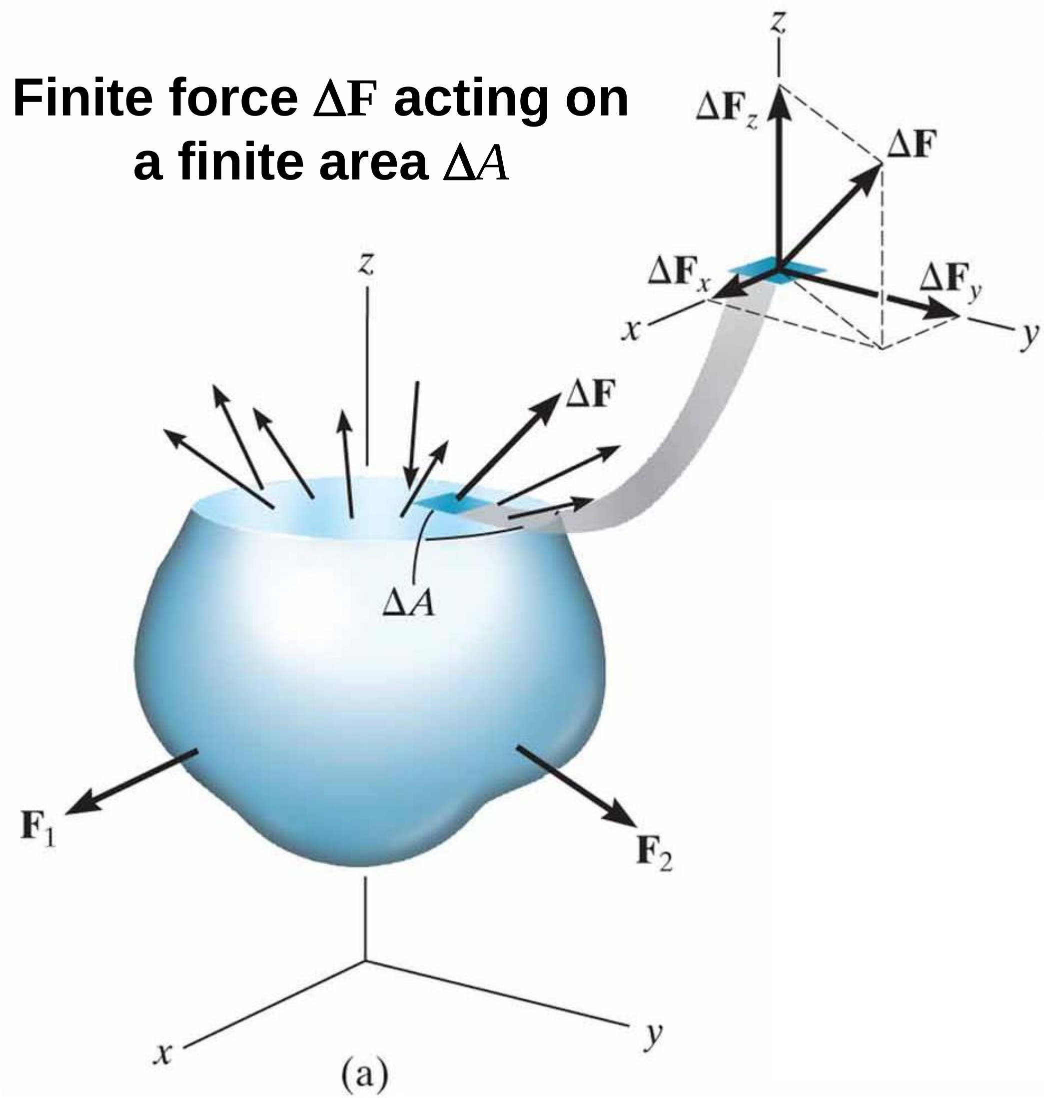
Equivalent force and moment at the section



(c)

Stress: Definition: intensity of internal force: acting on a specific plane passing through a point

Finite force ΔF acting on a finite area ΔA



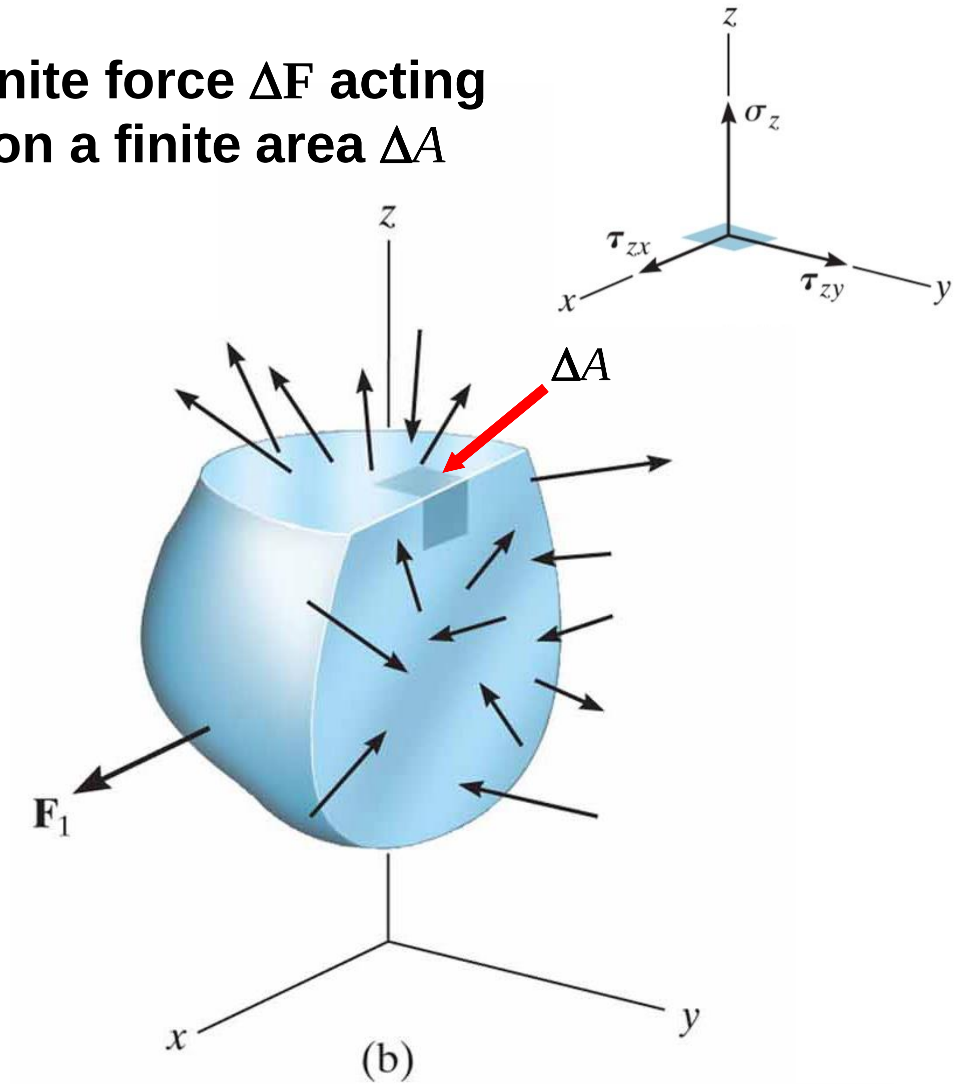
Definition

Normal stress:

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

Stress: Definition: intensity of internal force: acting on a specific plane passing through a point

Finite force ΔF acting on a finite area Δ*A*



Definitions

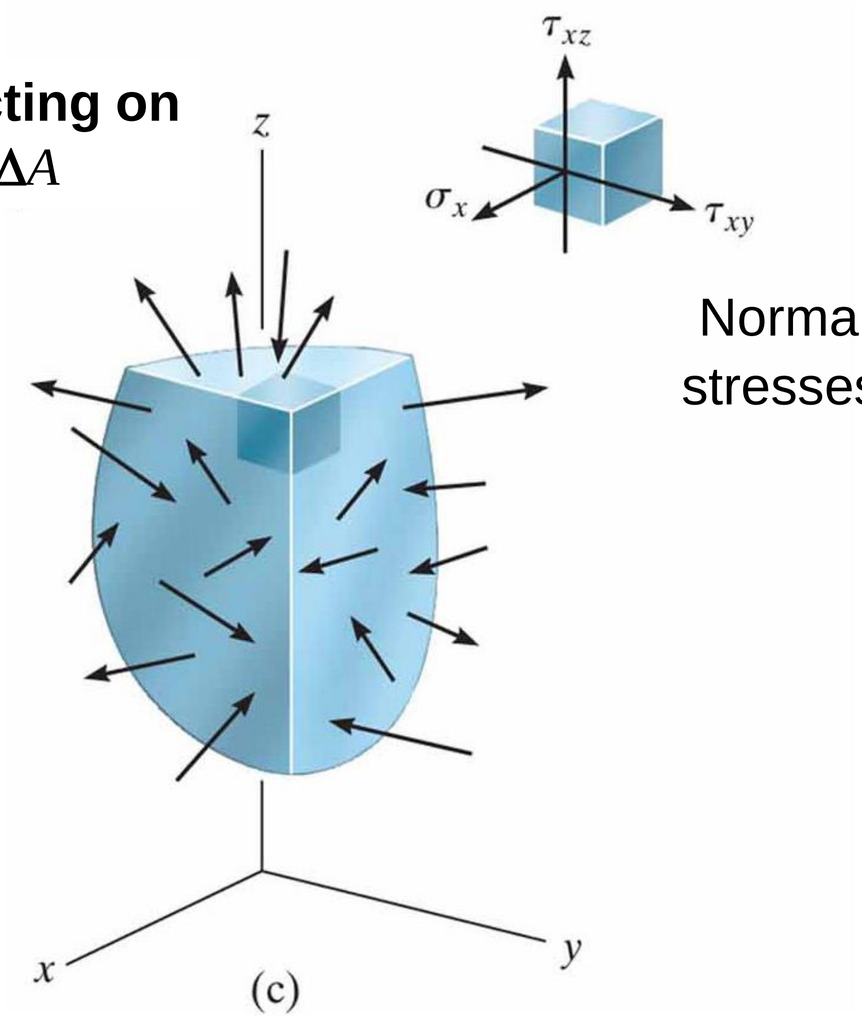
Shear stresses:

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

Stress: Definition: intensity of internal force: acting on a specific plane passing through a point

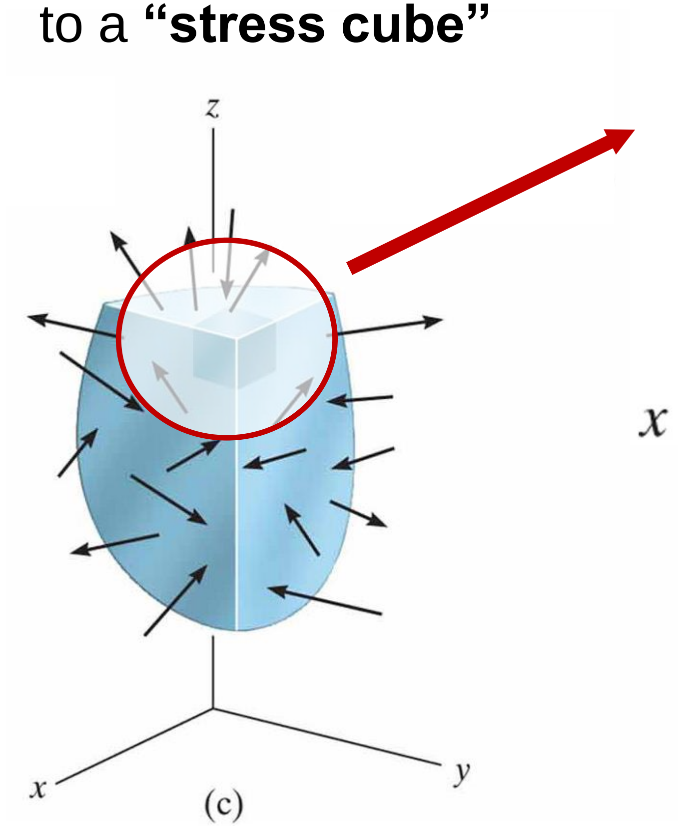
Finite force ΔF acting on a finite area ΔA



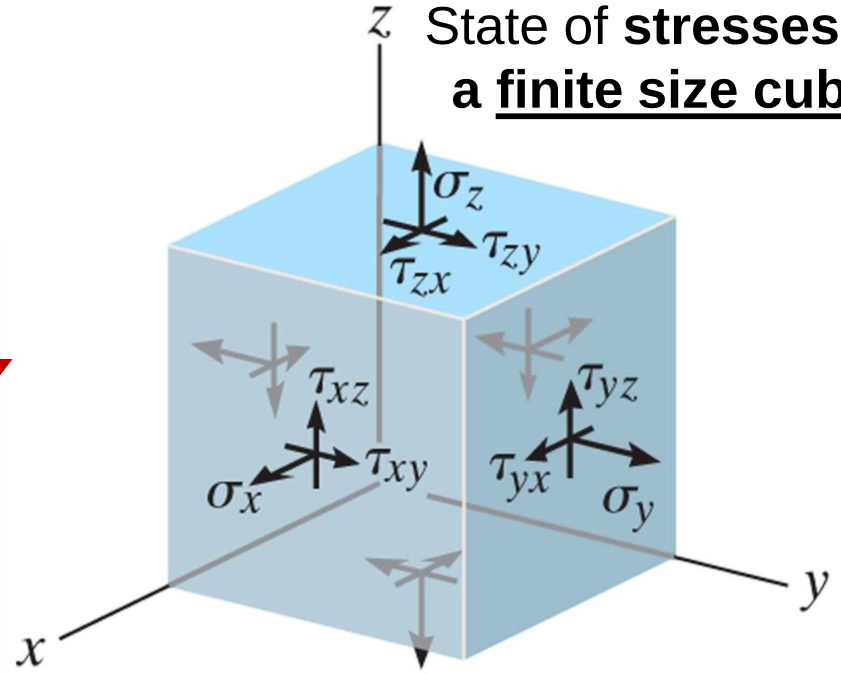
Normal and shear stresses on plane x

General state of stresses

Further sectioning leads to a **"stress cube"**



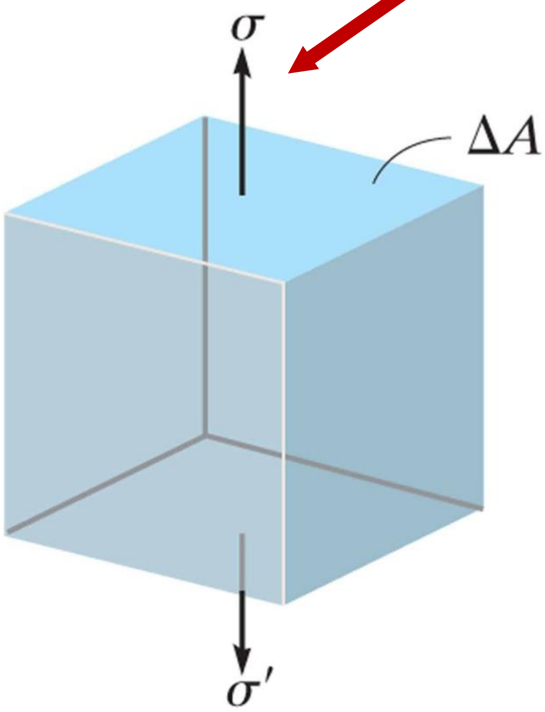
State of **stresses of a <u>finite size cube</u>**



State of stresses: normal stress equilibrium (1D)

Finite size cube:
stress cube

One-dimensional normal stress



$$\sum \mathbf{F} = \mathbf{0}; \quad \Rightarrow$$

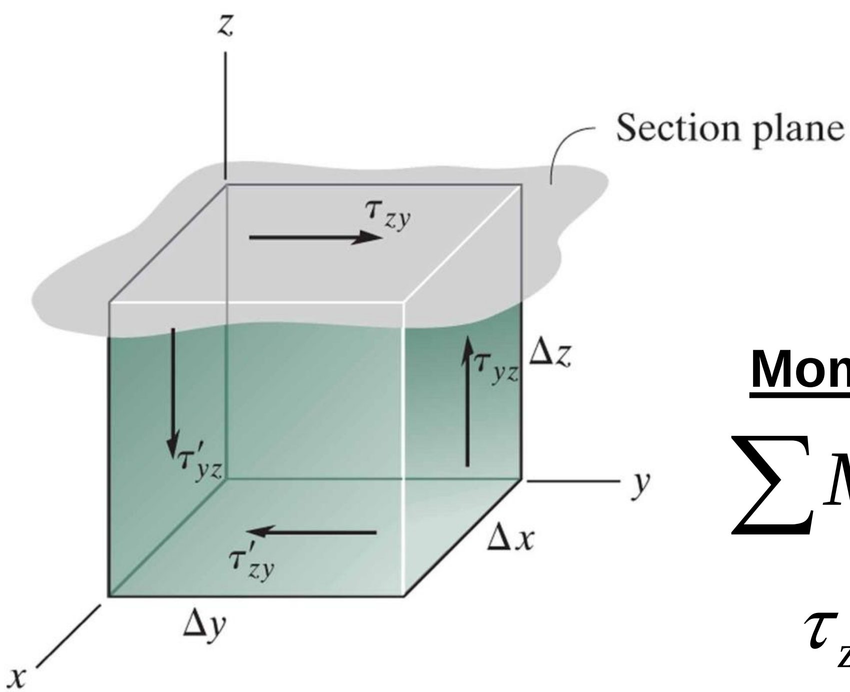
$$\sigma(\Delta A) = \sigma'(\Delta A)$$

Therefore,

$$\sigma = \sigma'$$

State of stresses: shear stress equilibrium

Finite size cube:
stress cube



Forces:

$$\sum F_y = 0; \quad \Rightarrow$$

$$\tau_{zy}(\Delta x\, \Delta y) = \tau'_{zy}(\Delta x\, \Delta y)$$

Therefore, $\tau_{zy} = \tau'_{zy}$

Moments:

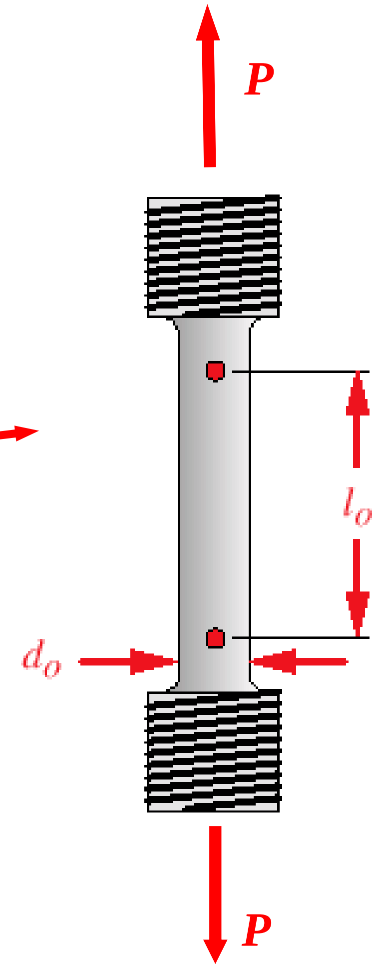
$$\sum M_x = 0; \quad \Rightarrow$$

$$\tau_{zy}(\Delta x\, \Delta y)\cdot \Delta z = \tau_{yz}(\Delta x\, \Delta z)\cdot \Delta y$$

Therefore, $\tau_{zy} = \tau_{yz}$

Average normal stress in an axially loaded bar

Tensile test



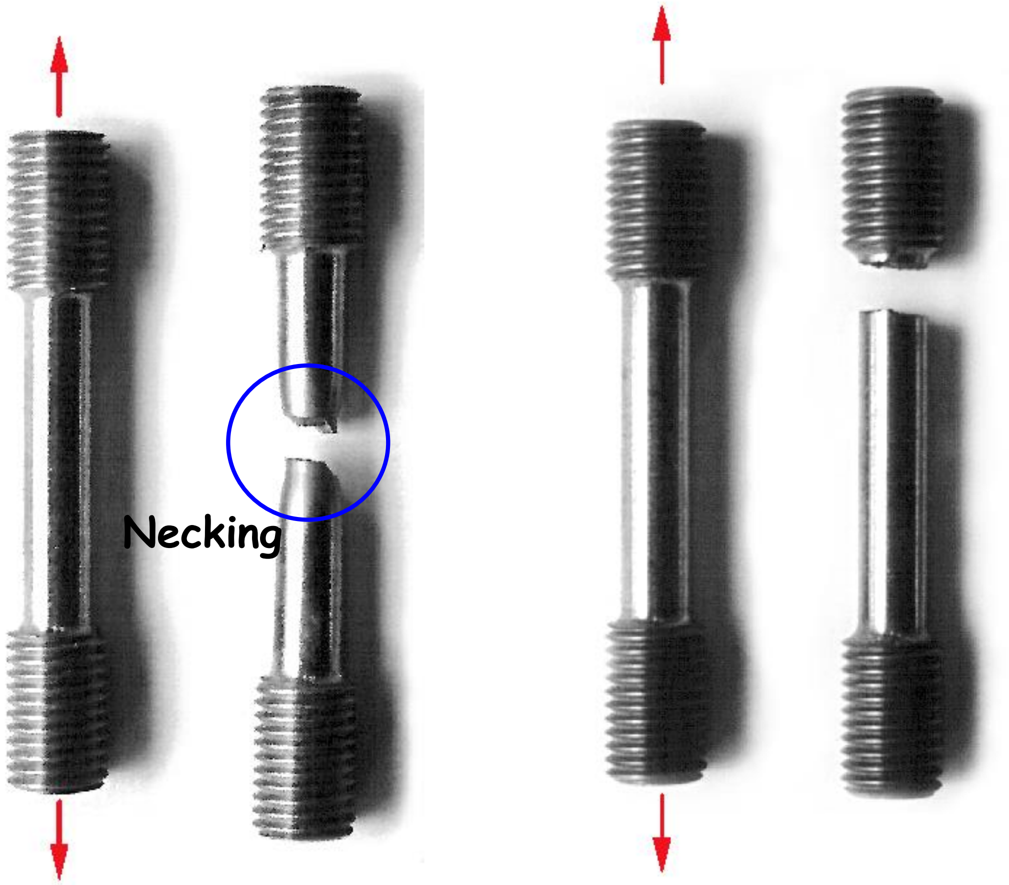
Average normal stress in an axially loaded bar

Tensile test

Typical results

Ductile material

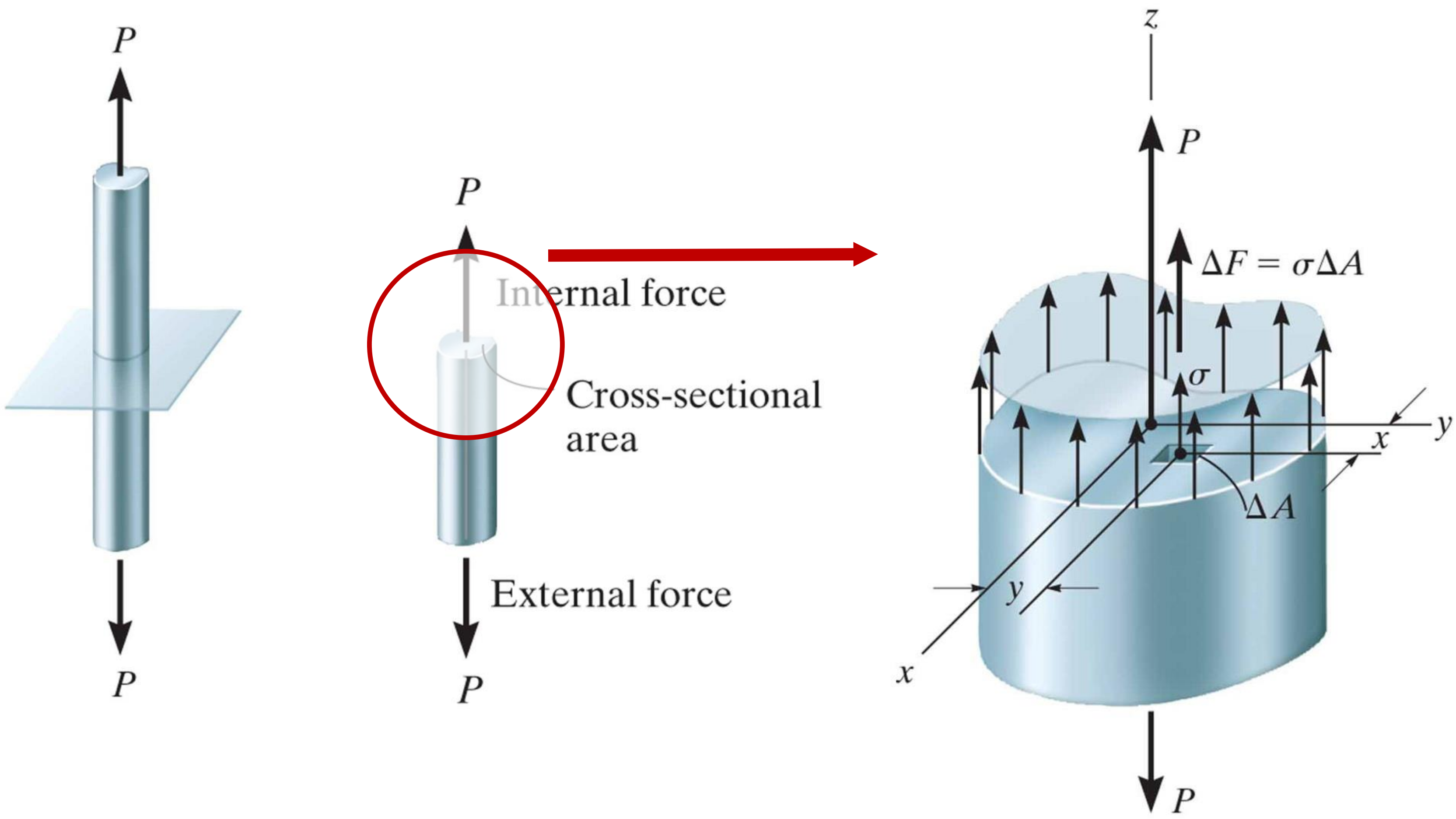
Brittle material



Average normal stress in an axially loaded bar

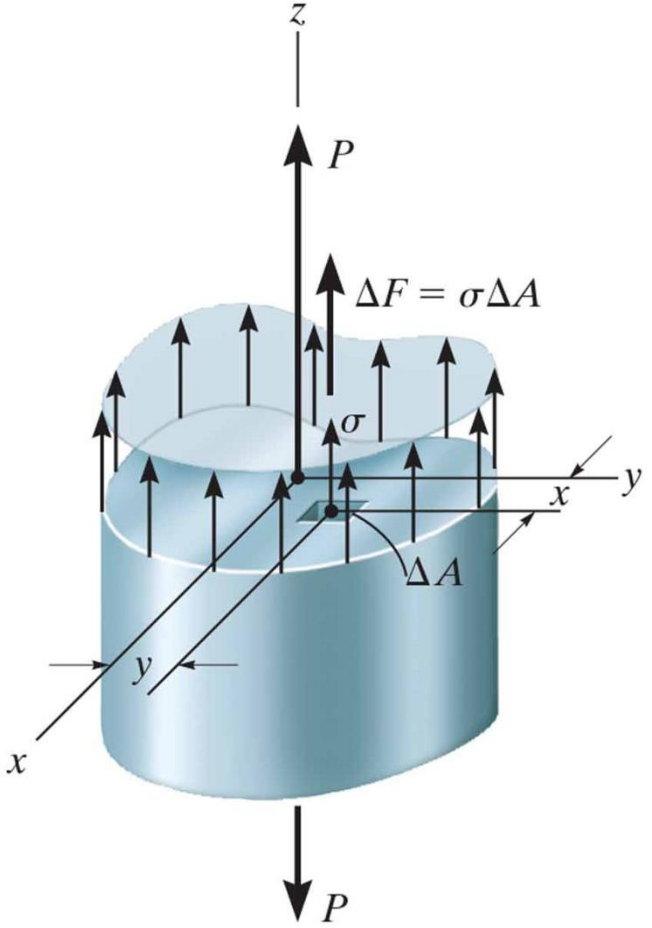
Bar subjected to axial load

Internal distribution of forces



Average normal stress in an axially loaded bar

Internal distribution of forces



$$+\uparrow F_{Rz} = \sum F_z$$

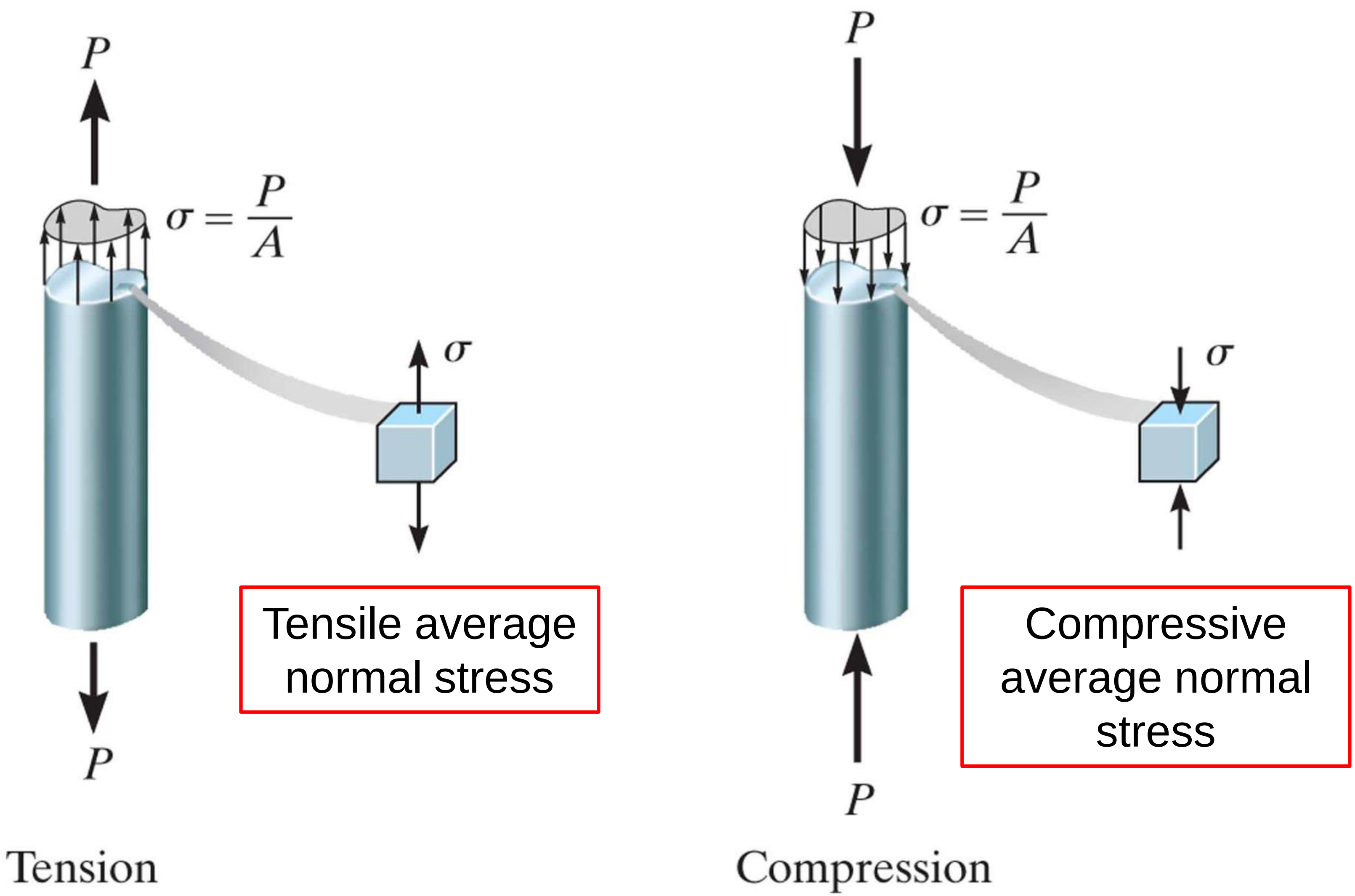
$$\int dF = \int_A \sigma \, dA$$

$$P = \sigma A$$

Average normal stress:

$$\sigma = \frac{P}{A}$$

Average normal stress in an axially loaded bar



Average normal stress in an axially loaded bar

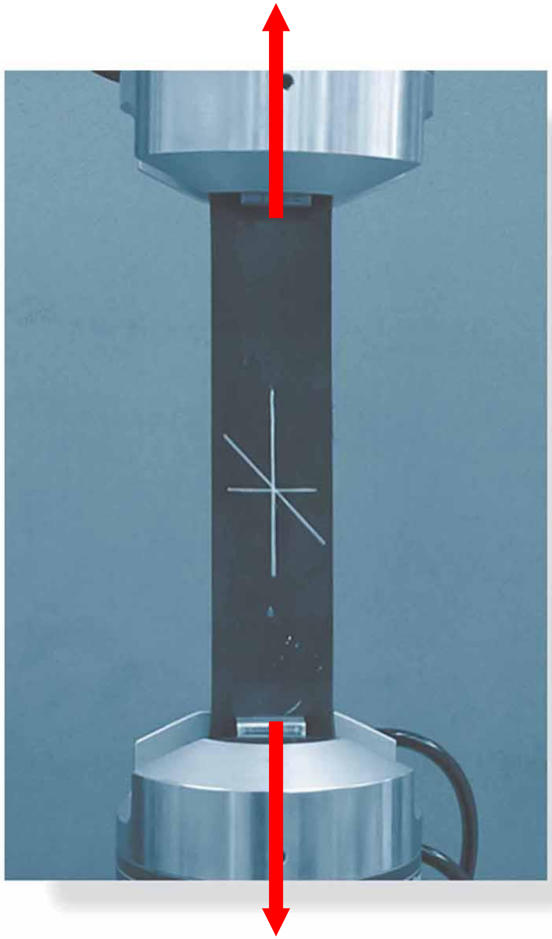


Figure: 02-01-A-UN

Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension.The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

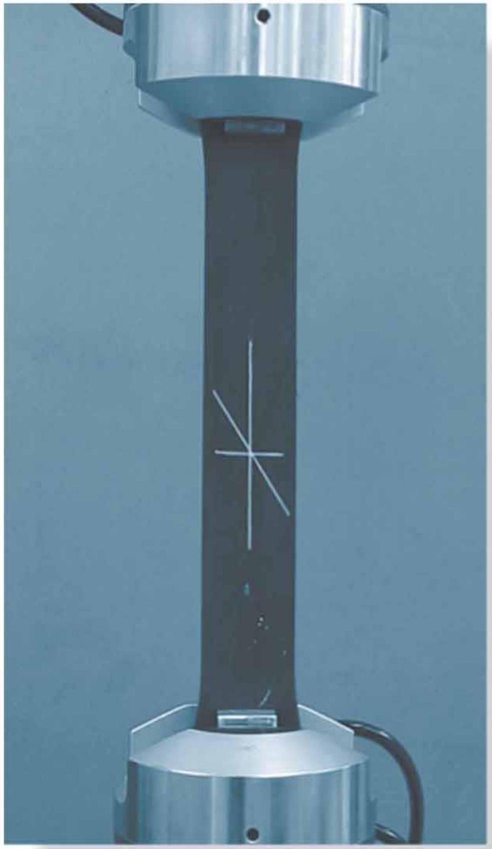


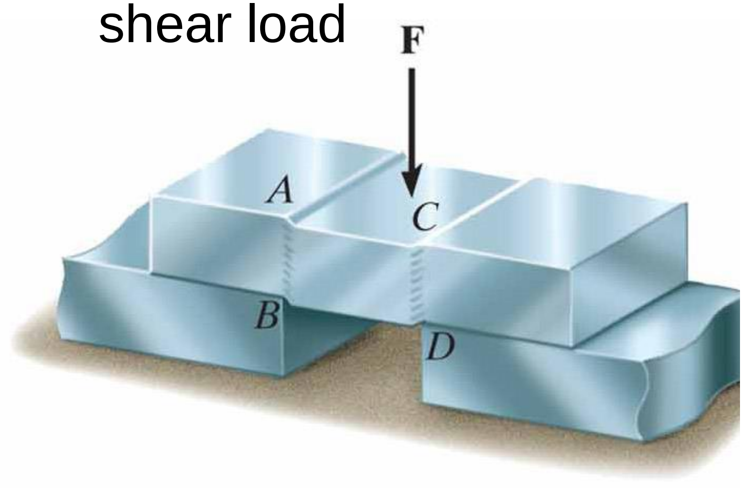
Figure: 02-01-B-UN

Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension.The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

Average *direct shear* stress

$$\tau_{avg} = \frac{V}{A_V}$$

Bar subjected to shear load



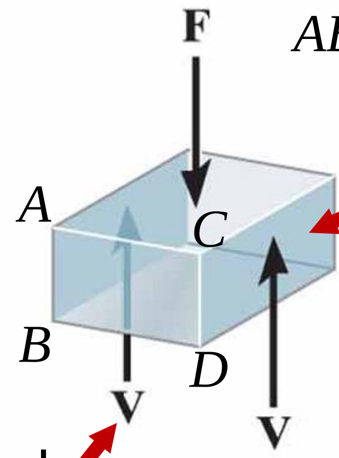
(a)

ABCD section

A_V

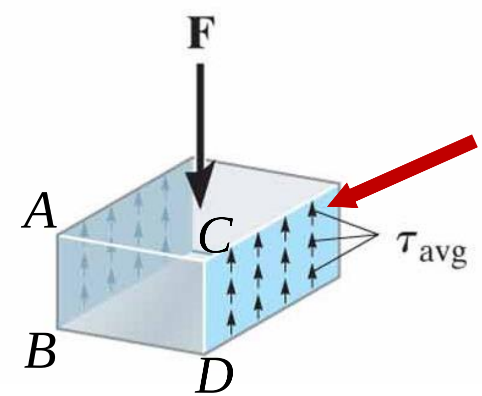
(Area under shear)

(Internal loading)



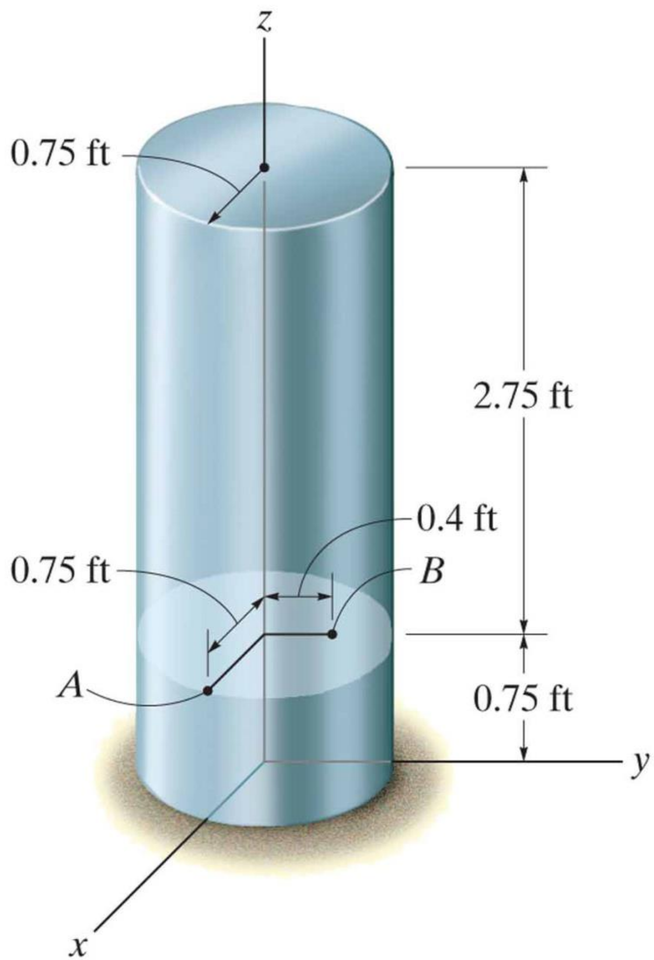
(b)

A_V



Average normal stress: example A

The casting shown is made of steel having a specific weight of $\gamma_{st}=490\ lb_f/ft^3$. Determine the average compressive stress acting at points A and B.



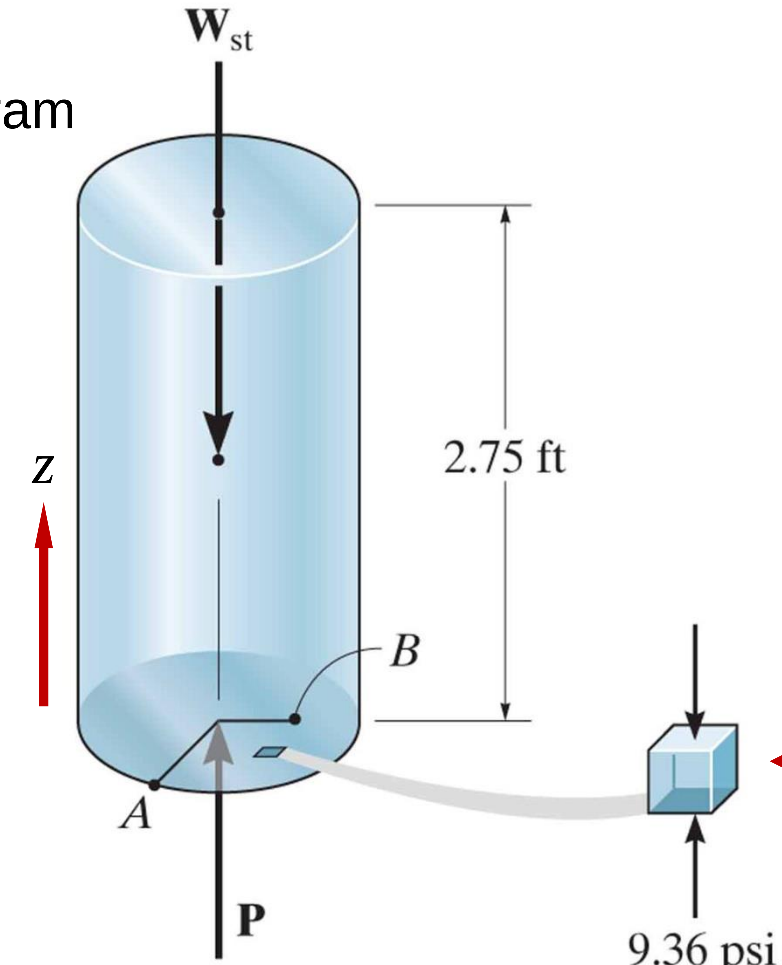
Approach:

1) Define free-body diagrams
2) Determine internal loadings
3) Compute average stresses

Average normal stress: example A

The casting shown is made of steel having a specific weight of $\gamma_{st}=490\ lb_f/ft^3$. Determine the average compressive stress acting at points A and B.

Free-body diagram (FBD)



Compute internal loading

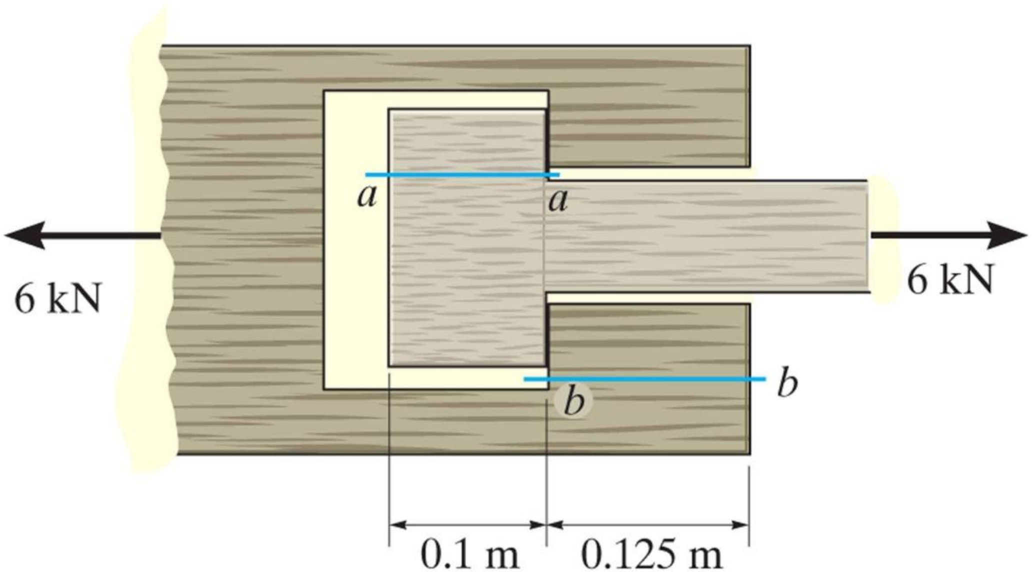
&

Average compressive stresses

(Stress cube)

Average shear stress: example B

Wood joints 150 mm deep (perpendicular to the plane) are loaded as shown. Determine the average shear stress developed along planes *a*-*a* and *b*-*b*.

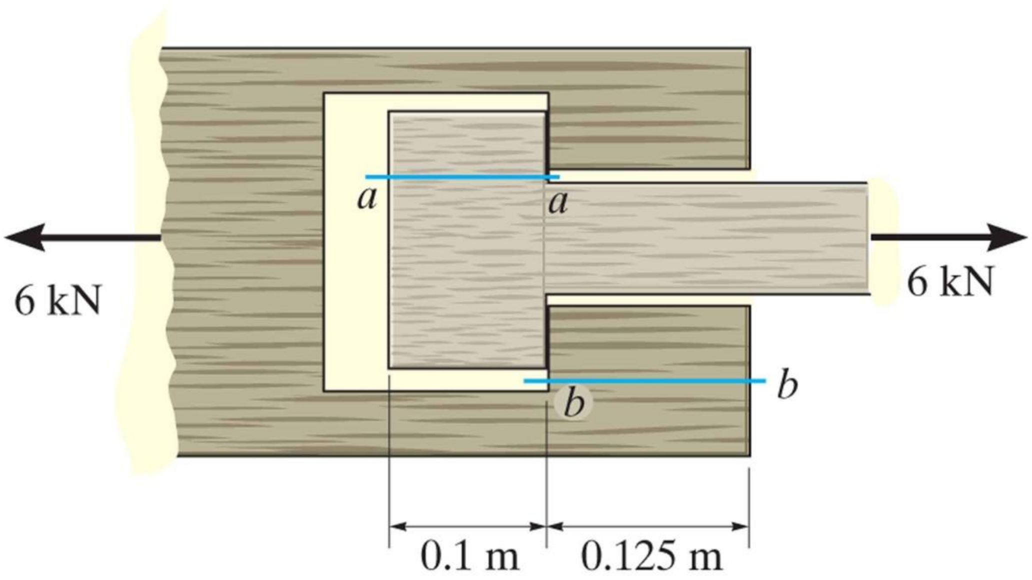


Approach:

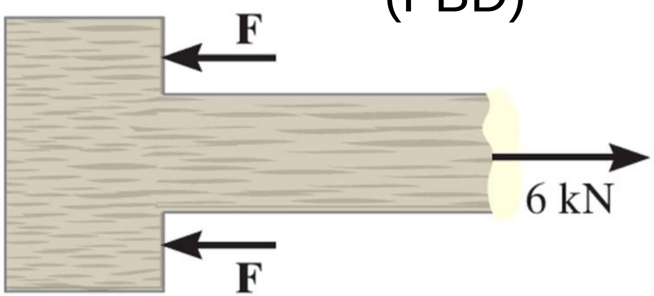
1) Define free-body diagrams
2) Determine internal loadings
3) Compute average stresses

Average shear stress: example B

Wood joints 150 mm deep (perpendicular to the plane) are loaded as shown. Determine the average shear stress developed along planes *a*-*a* and *b*-*b*.

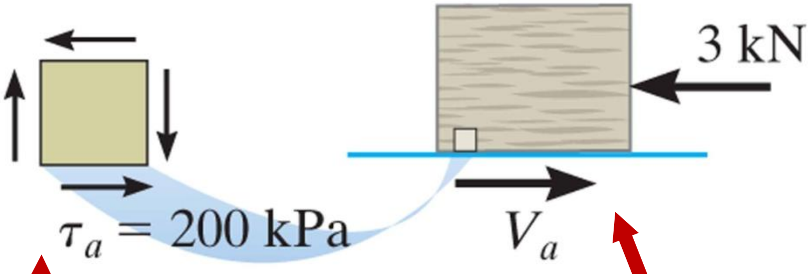


Free-body diagram (FBD)

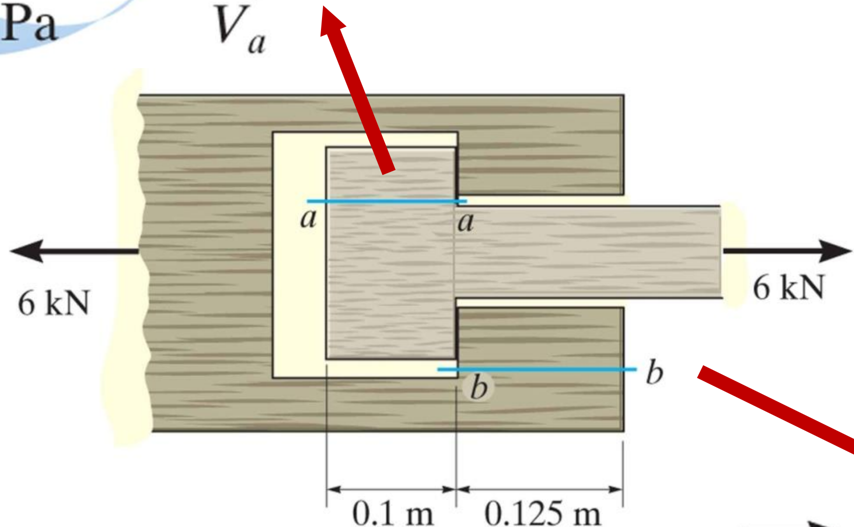


Average **shear** stress: example B

Wood joints 150 mm deep (perpendicular to the plane) are loaded as shown. Determine the average shear stress developed along planes *a*-*a* and *b*-*b*.



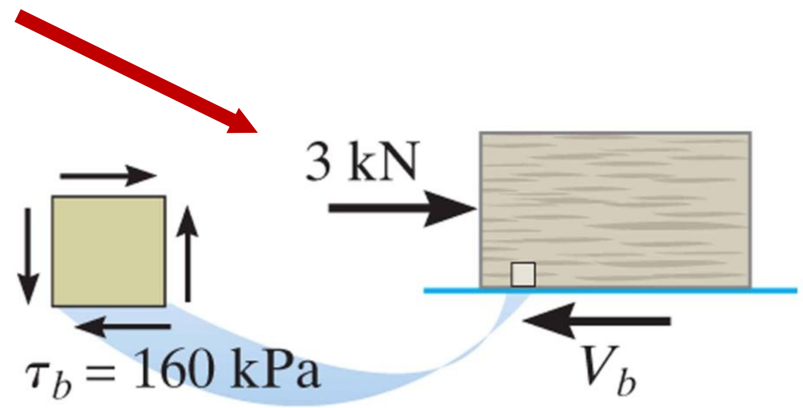
(Stress cube)



Compute internal loading

&

Average shear stresses



(Stress cube)

Reading assignment

- Chapter 1 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- **As indicated on webpage of our course**