We will get started soon...

31 March 2020
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Lecture 04:
Unit 3: definition of normal and shear stress

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General information

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Stress: Definition: intensity of internal force: acting on a specific plane passing through a point

\[
F_R = \sum F
\]

\[
M_{Ro} = \sum M_O
\]

Equivalent force and moment at the section

(b)

(c)
Stress: Definition: intensity of internal force: acting on a specific plane passing through a point

Finite force $\Delta F$ acting on a finite area $\Delta A$

Normal stress:

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$
Stress: Definition: intensity of internal force: acting on a specific plane passing through a point

Finite force $\Delta F$ acting on a finite area $\Delta A$

Definitions

Shear stresses:

\[
\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}
\]

\[
\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}
\]
Stress: Definition: intensity of internal force: acting on a specific plane passing through a point

Finite force $\Delta F$ acting on a finite area $\Delta A$
General state of stresses

Further sectioning leads to a “stress cube”

State of stresses of a finite size cube
State of stresses: normal stress equilibrium (1D)

Finite size cube: stress cube

One-dimensional normal stress

\[ \sum F = 0; \quad \Rightarrow \]

\[ \sigma (\Delta A) = \sigma' (\Delta A) \]

Therefore,

\[ \sigma = \sigma' \]
State of stresses: shear stress equilibrium

Finite size cube: stress cube

**Forces:**
\[ \sum F_y = 0 \quad \Rightarrow \]
\[ \tau_{zy} (\Delta x \ \Delta y) = \tau'_{zy} (\Delta x \ \Delta y) \]
Therefore, \[ \tau_{zy} = \tau'_{zy} \]

**Moments:**
\[ \sum M_x = 0 \quad \Rightarrow \]
\[ \tau_{zy} (\Delta x \ \Delta y) \cdot \Delta z = \tau_{yz} (\Delta x \ \Delta z) \cdot \Delta y \]
Therefore, \[ \tau_{zy} = \tau_{yz} \]
Average normal stress in an axially loaded bar

Tensile test
Average normal stress in an axially loaded bar

Tensile test

Typical results

Ductile material

Brittle material

Necking
Average normal stress in an axially loaded bar

Bar subjected to axial load

Internal distribution of forces
Average normal stress in an axially loaded bar

\[ \sum \Delta F = \sigma \Delta A \]

\[ \int dF = \int \sigma \, dA \]

\[ P = \sigma A \]

Average normal stress:

\[ \sigma = \frac{P}{A} \]
Average normal stress in an axially loaded bar

\[ \sigma = \frac{P}{A} \]

- Tensile average normal stress
- Compressive average normal stress
Average normal stress in an axially loaded bar

Figure: 02-01-A-UN
Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension. The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

Figure: 02-01-B-UN
Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension. The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.
Average direct shear stress

\[ \tau_{avg} = \frac{V}{A_V} \]

Bar subjected to shear load

(Area under shear)

(Internal loading)

\( V_{avg} \)
The casting shown is made of steel having a specific weight of $\gamma_{st} = 490 \text{ lb/ft}^3$. Determine the average compressive stress acting at points A and B.

**Approach:**

1) Define free-body diagrams
2) Determine internal loadings
3) Compute average stresses
Average normal stress: example A

The casting shown is made of steel having a specific weight of $\gamma_{st} = 490 \text{ lb/ft}^3$. Determine the average compressive stress acting at points $A$ and $B$.

Free-body diagram (FBD)

Compute internal loading

&

Average compressive stresses

(Stress cube)
Average shear stress: example B

Wood joints 150 mm deep (perpendicular to the plane) are loaded as shown. Determine the average shear stress developed along planes \( a-a \) and \( b-b \).

Approach:
1) Define free-body diagrams
2) Determine internal loadings
3) Compute average stresses
Average shear stress: example B

Wood joints 150 mm deep (perpendicular to the plane) are loaded as shown. Determine the average shear stress developed along planes \( a-a \) and \( b-b \).
Wood joints 150 mm deep (perpendicular to the plane) are loaded as shown. Determine the average shear stress developed along planes $a-a$ and $b-b$. 

\[ \tau_a = 200 \text{ kPa} \]

\[ \tau_b = 160 \text{ kPa} \]

Compute internal loading & Average shear stresses
Reading assignment

• Chapter 1 of textbook
• Review notes and text: ES2001, ES2501
Homework assignment

• As indicated on webpage of our course