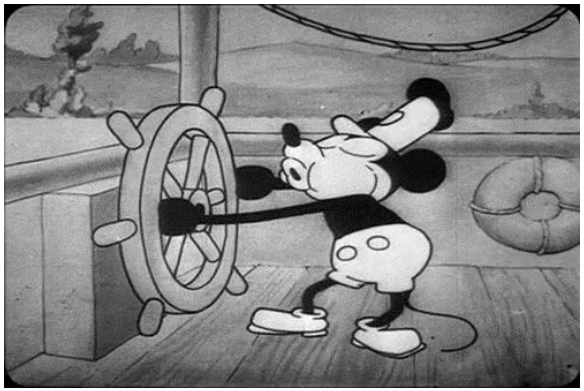
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, D'2020

We will get started soon...



27 March 2020

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STRESS ANALYSIS
ES-2502, D'2020

We will get started soon...

Lecture 02: Internal Forces

27 March 2020

General information

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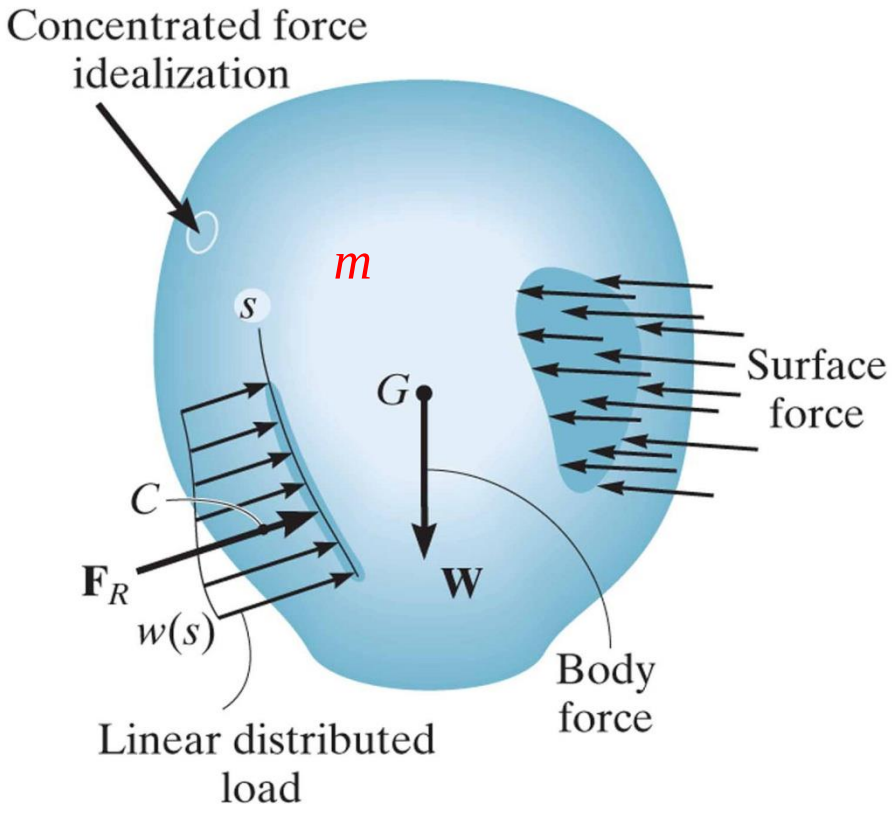
Force analysis. Free-body diagrams

Static equilibrium: forces

Arbitrary component under load



Static equilibrium condition (vectorial representation) :

$$\sum \mathbf{F} = \mathbf{0}$$

Force analysis. Free-body diagrams

Static equilibrium: forces

Forces acting on a particle or body (vectorial representation):

$$\sum \mathbf{F} = \mathbf{0} = \sum F_x\,\mathbf{i} + \sum F_y\,\mathbf{j} + \sum F_z\,\mathbf{k}$$

Null vector !!

Individual orthogonal components (scalar representation):

$$\sum F_x = 0$$

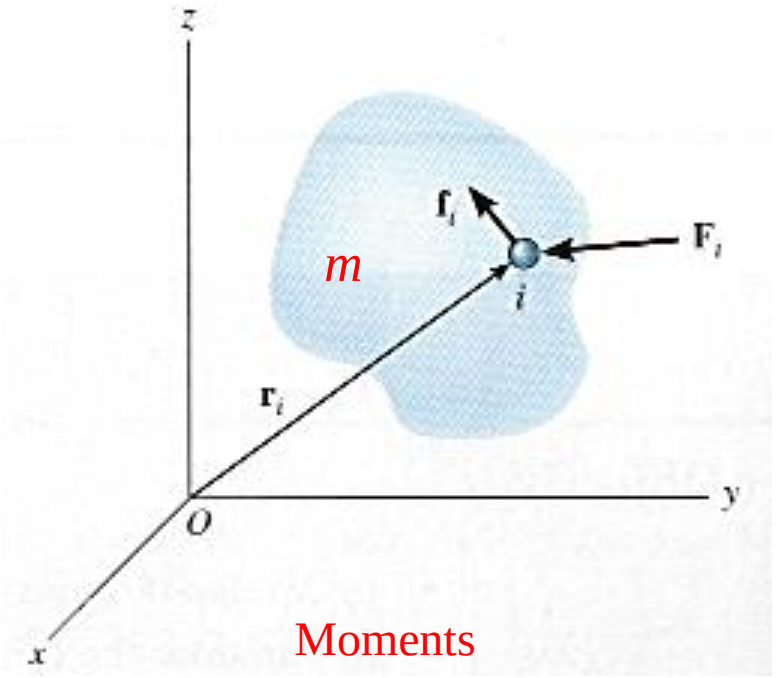
Null - Scalar !!

$$\sum F_y = 0$$

$$\sum F_z = 0$$

Force analysis. Free-body diagrams

Static equilibrium: moments



Moments

Static equilibrium condition (vectorial representation):

$$\sum \mathbf{M}_O = \mathbf{0}$$

Orthogonal components:

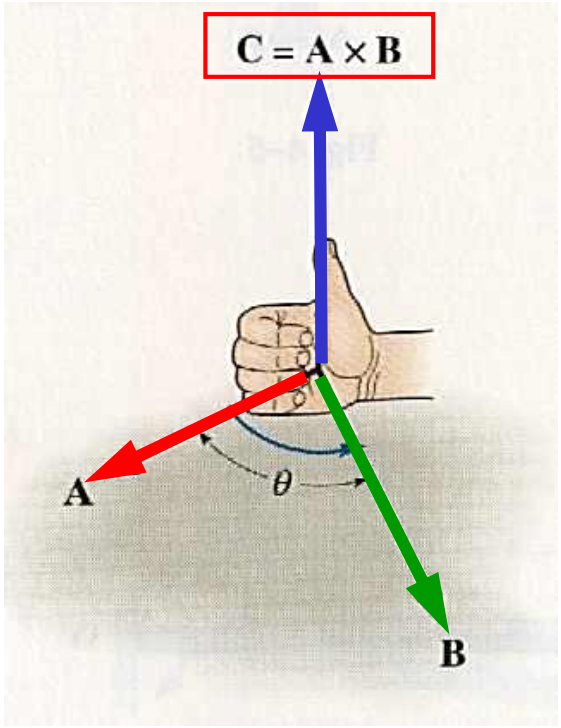
$$\sum M_{x_o}\,\mathbf{i} + \sum M_{y_o}\,\mathbf{j} + \sum M_{z_o}\,\mathbf{k} = 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$$

Vectorial evaluation:

$$\sum \mathbf{r}_i \times \mathbf{f}_i + \sum \mathbf{r}_i \times \mathbf{F}_i = \mathbf{0}$$

Force analysis. Free-body diagrams

Static equilibrium: moments

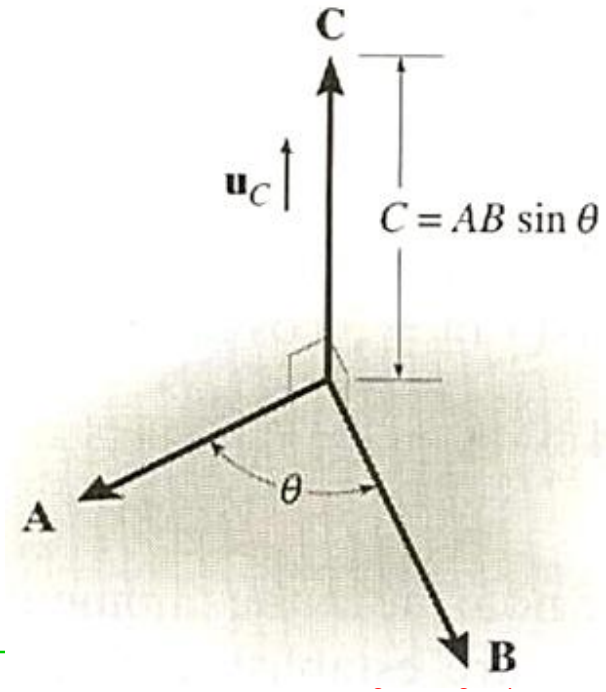


$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = A\,B\sin(\theta)\,\mathbf{u}_C$$

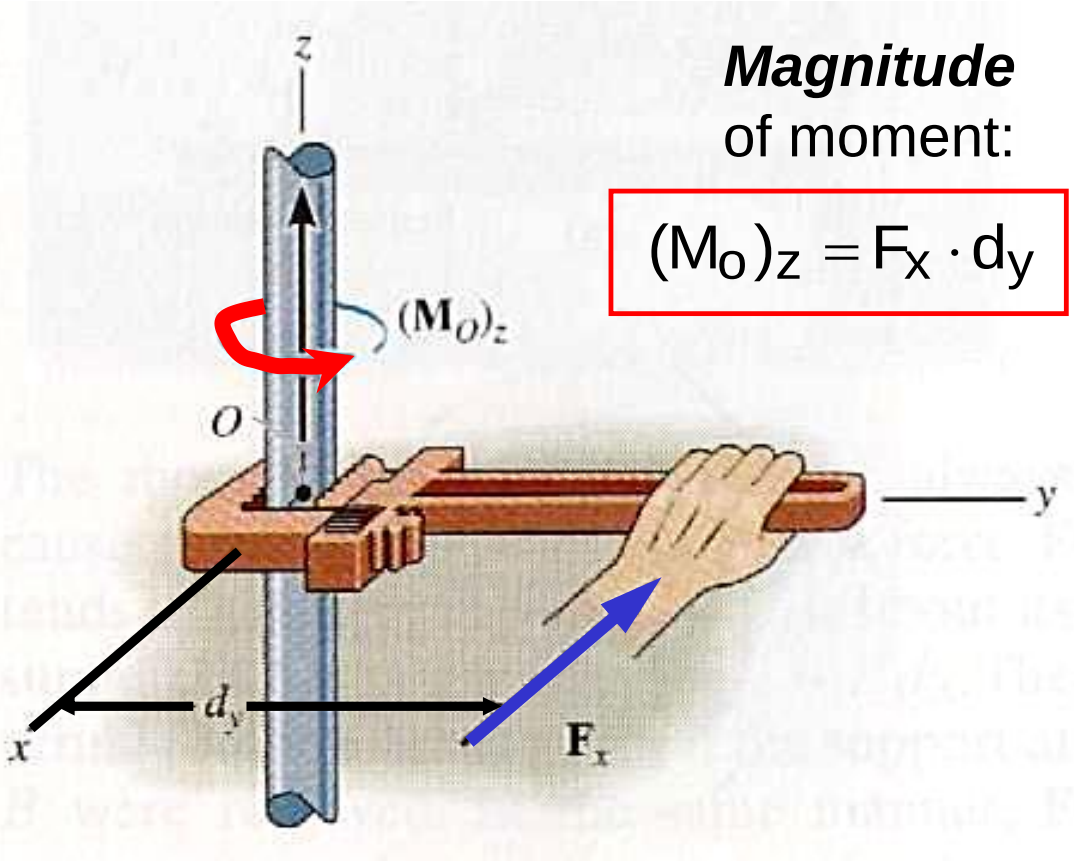
Vector

Scalar

Unit vector



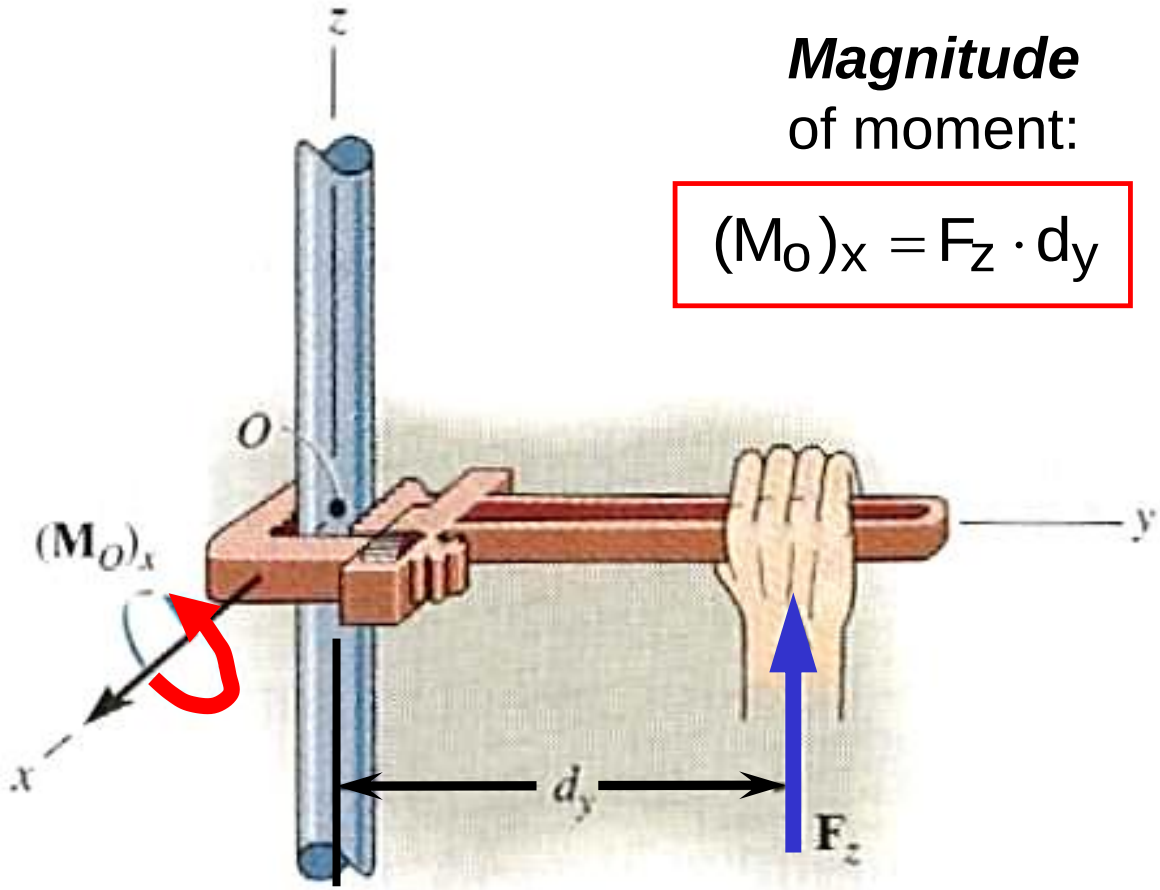
Moment of a force: about *z*-axis



Magnitude of moment:

$$(M_O)_z = F_x \cdot d_y$$

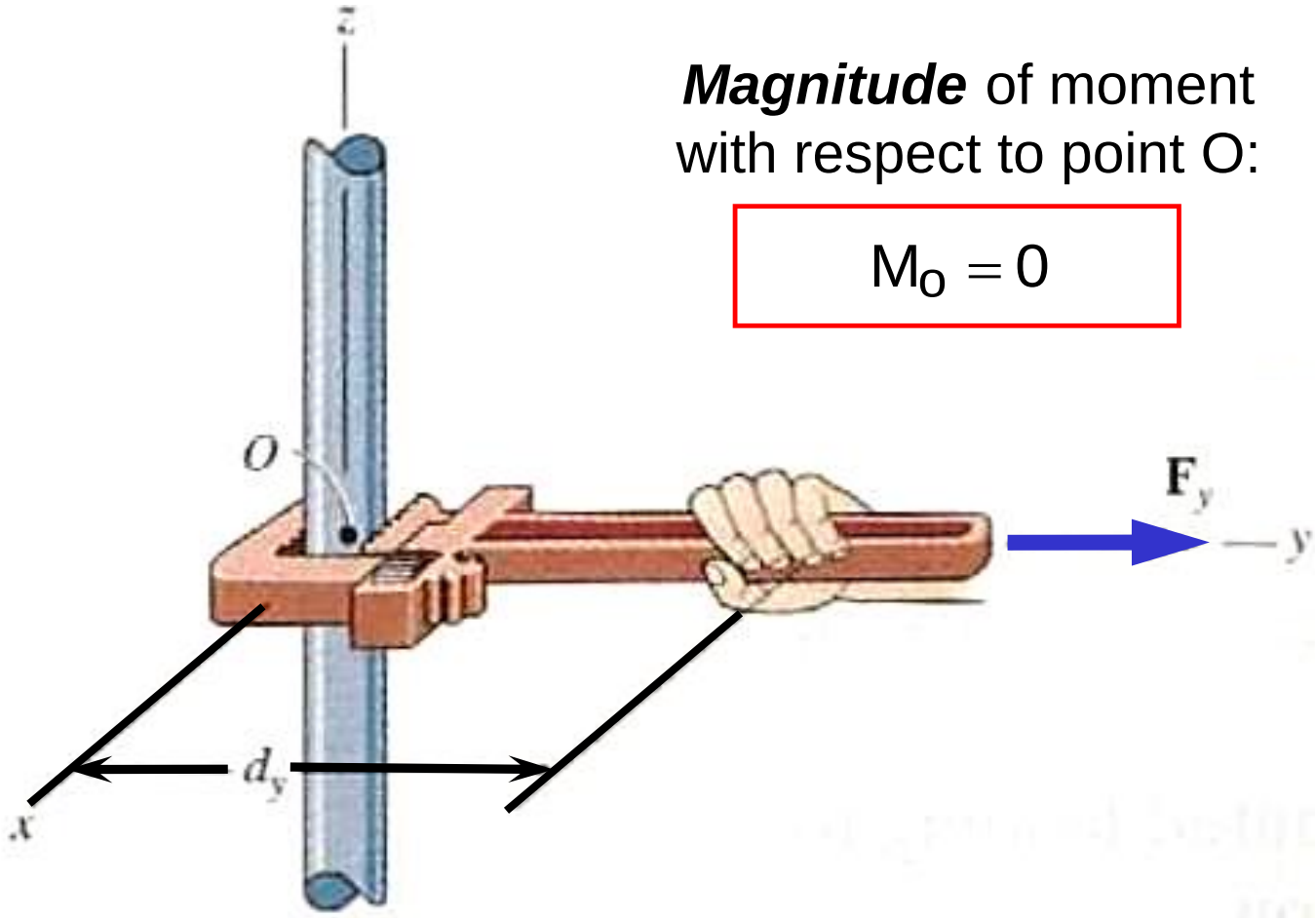
Moment of a force: about x-axis



Magnitude
of moment:

$$(M_o)_x = F_z \cdot d_y$$

Moment of a force: null effect

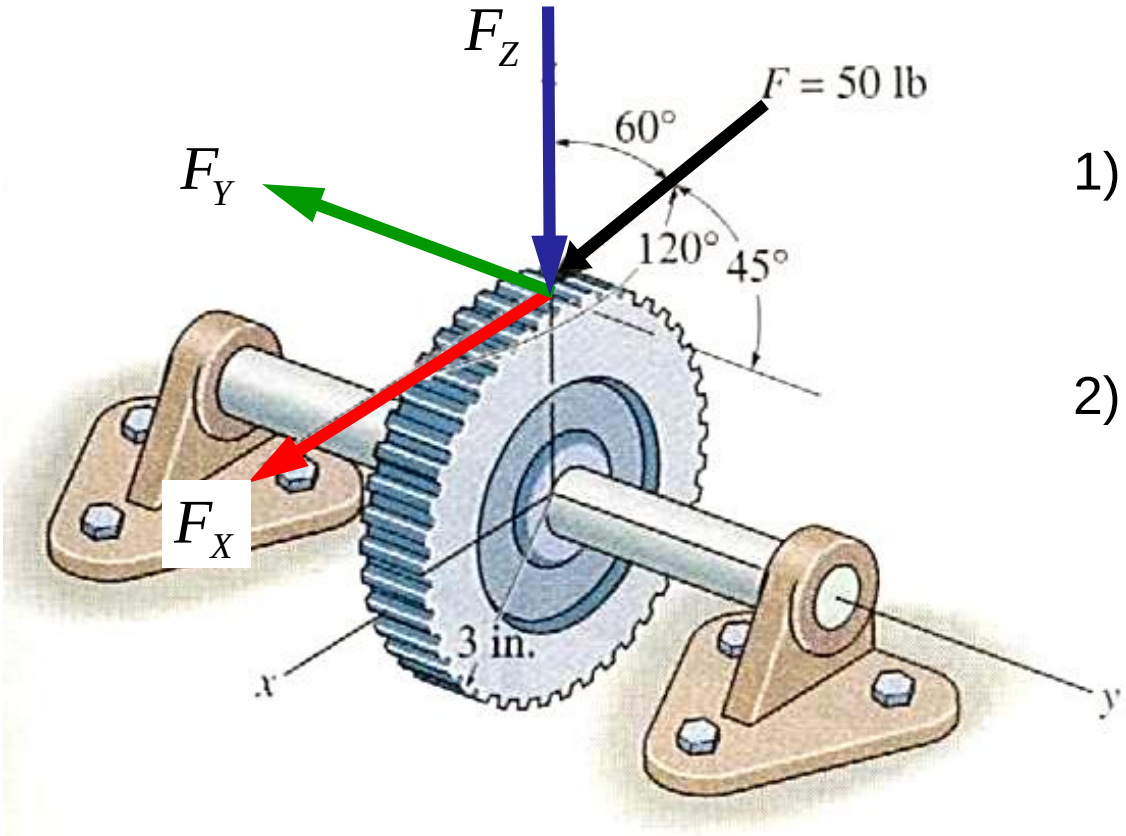


Magnitude of moment with respect to point O:

$$M_O = 0$$

Force analysis: example

The 50 lb force acts on the gear in the direction shown. Determine the moment of this force about the y-axis.

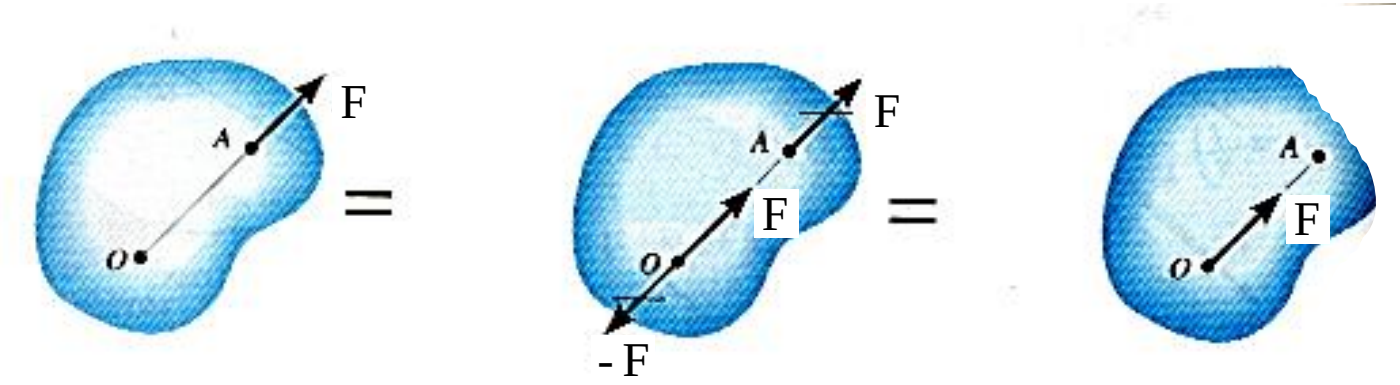


Approach:

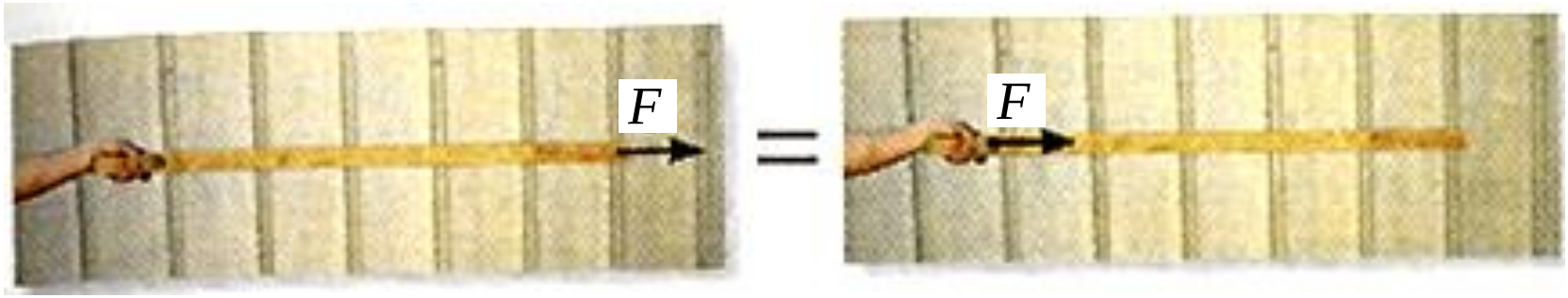
1) Determine x-y-z components of applied force
2) Compute moment about y-axis

Equivalent systems: are they really?

Force translated along its line of application:



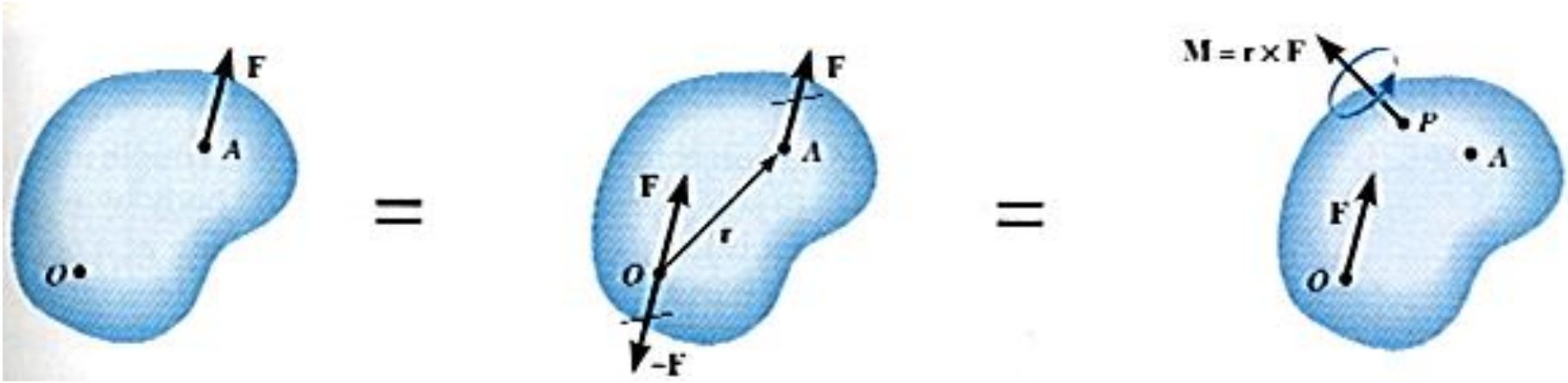
Example:



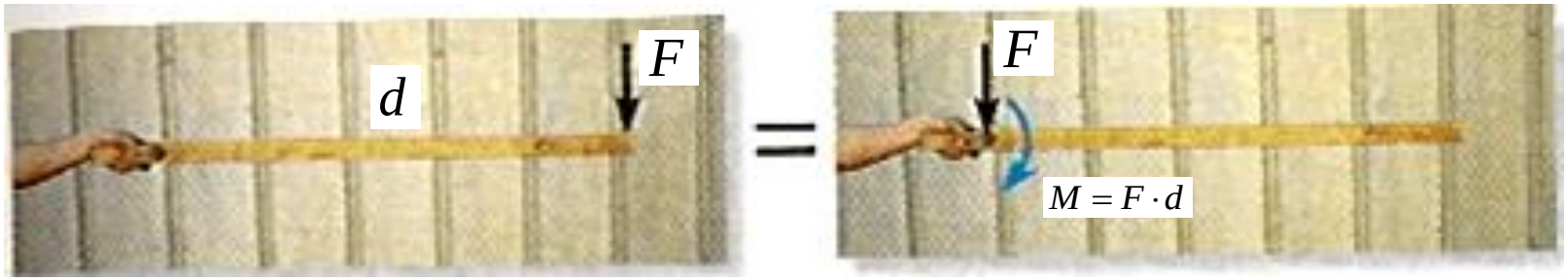
Idealization: similar effects at the support ??

Equivalent systems: are they really?

Equivalent force and couple/moment system:

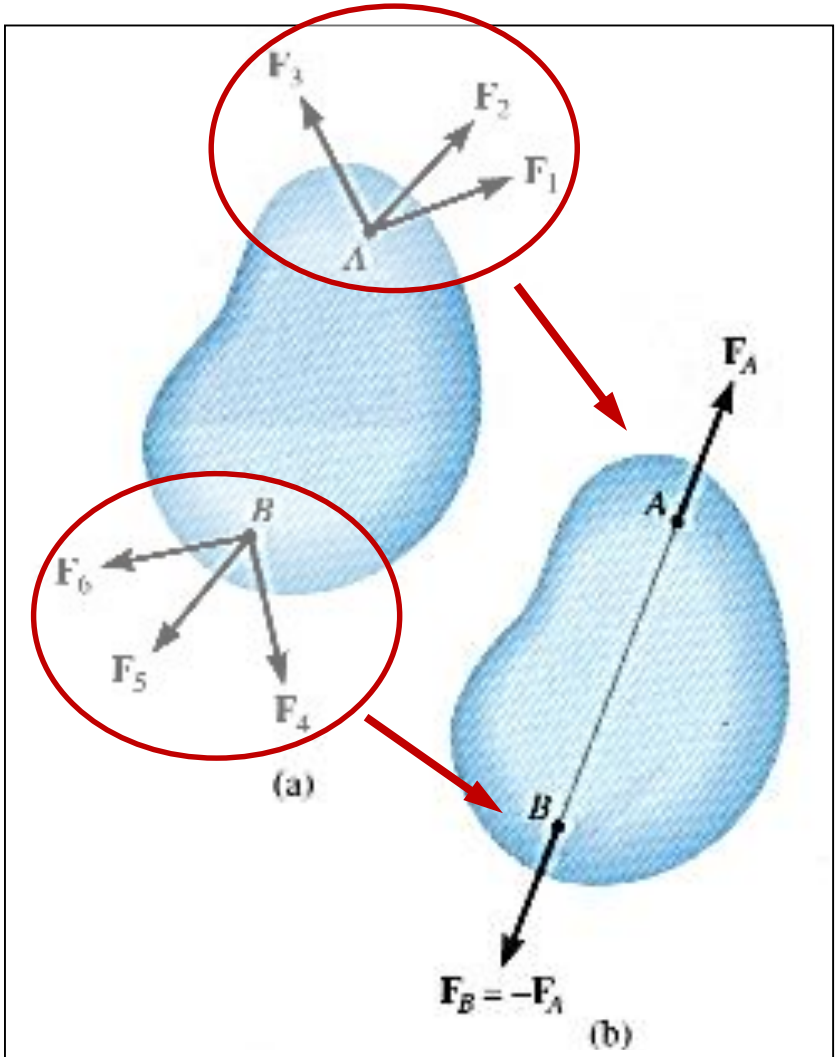


Example:

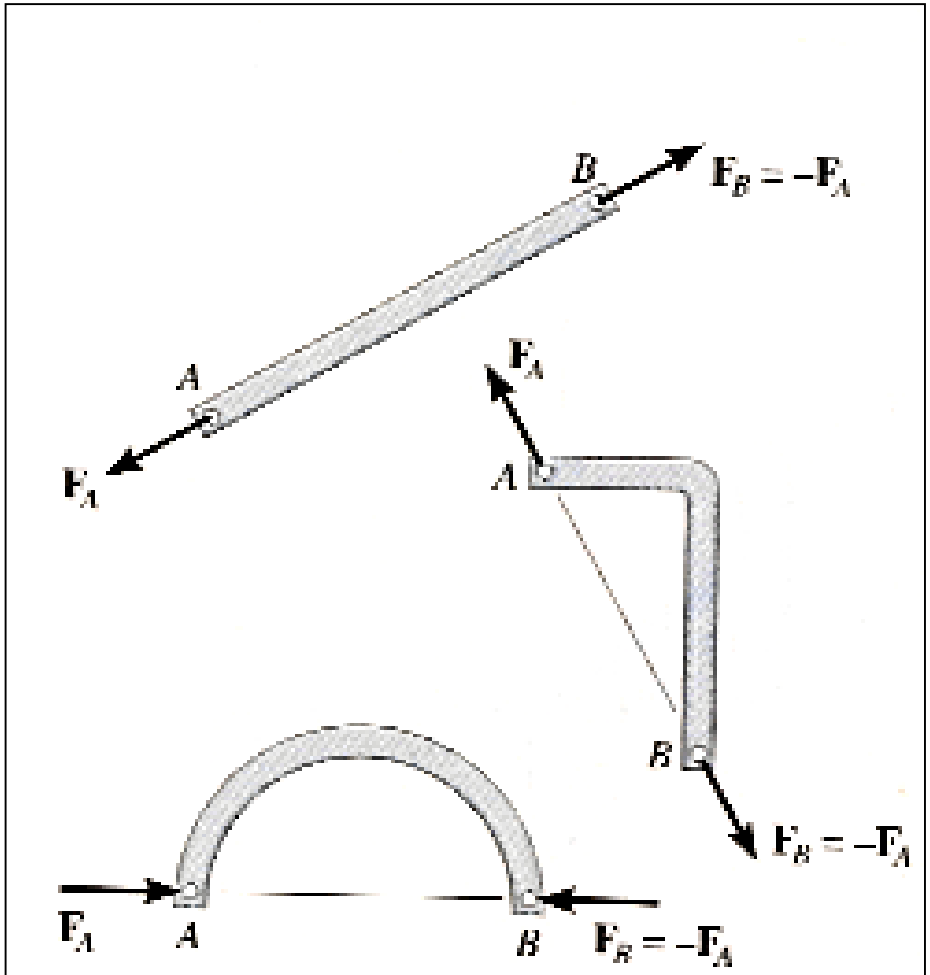


Idealization: similar effects at the support ??

Equilibrium of rigid bodies: equivalent systems



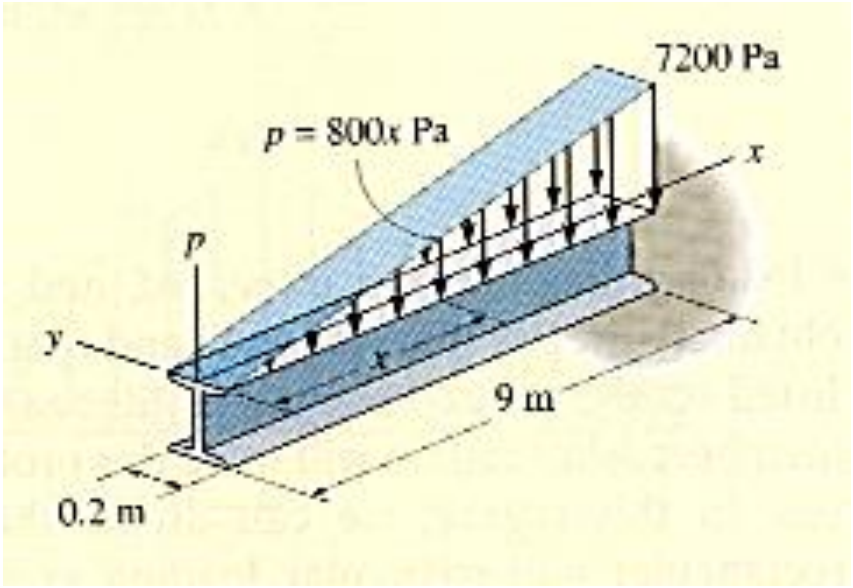
Two-force member

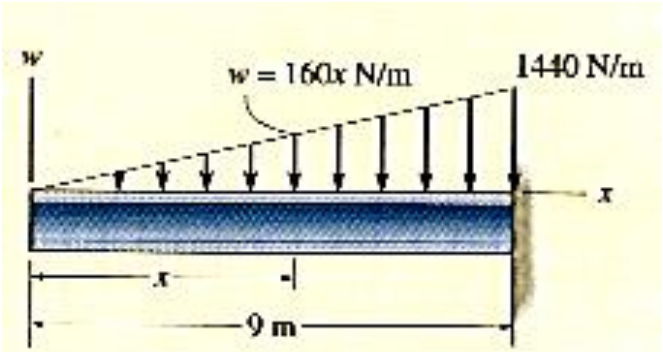


Two-force member

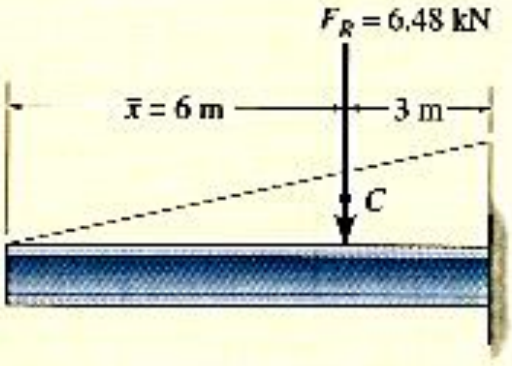
Distributed loads: equivalent systems

Original 3D problem





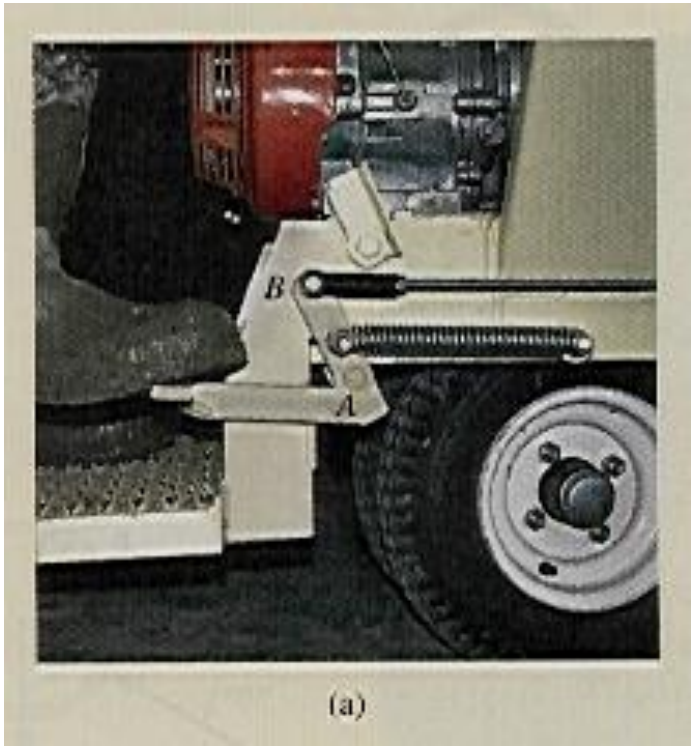
2D projection



Equivalent system

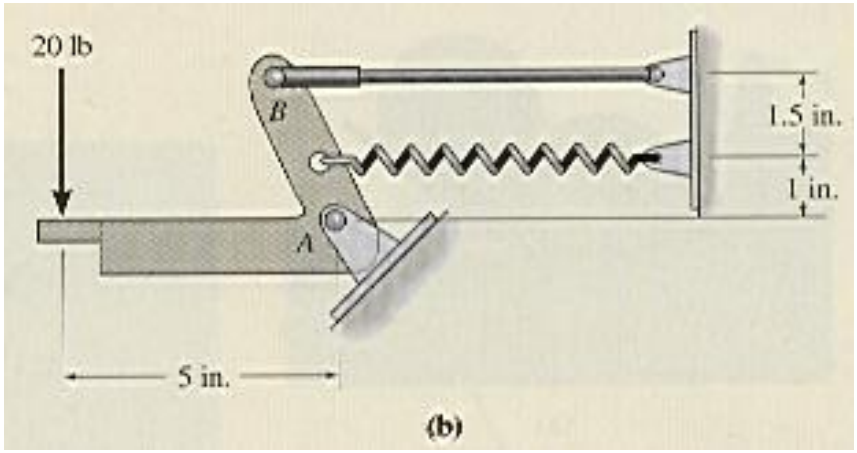
Free-body diagrams

Operator applies 20-lb to pedal stretching spring by 1.5 in.

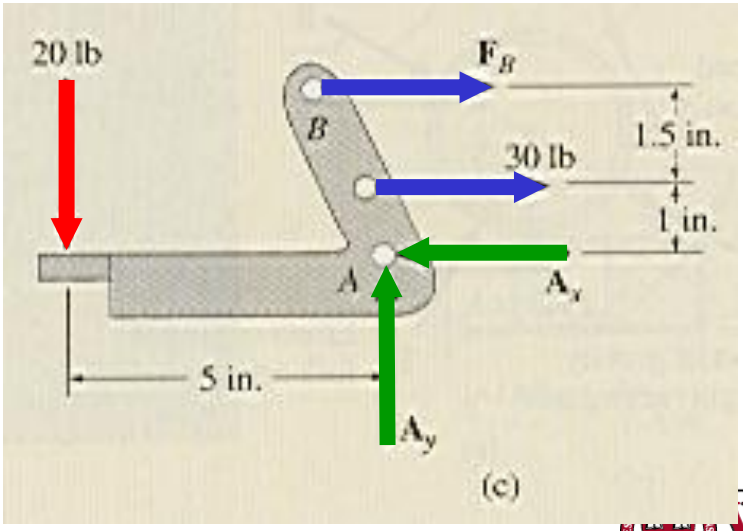


Actual mechanism

Schematic representation:



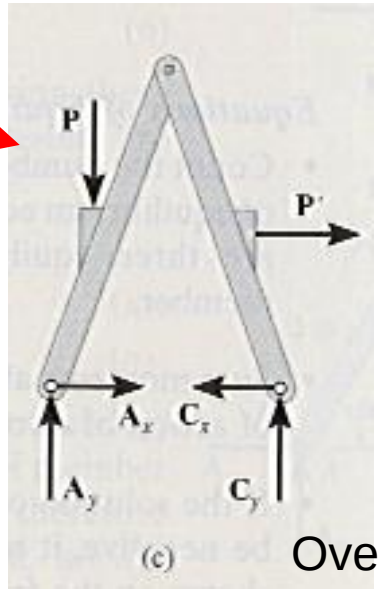
Free-body diagram:



Force analysis. Free-body diagrams

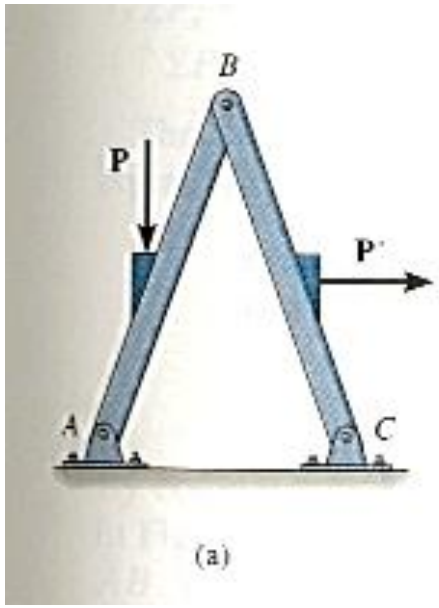
Number of unknowns ?

Equilib. Equations ?

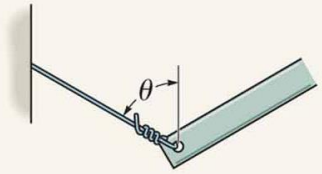
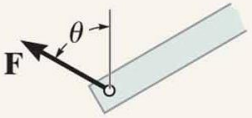
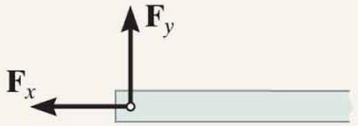
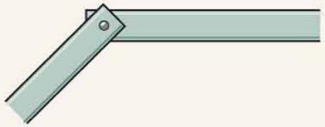
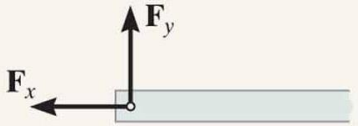
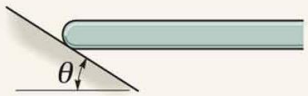
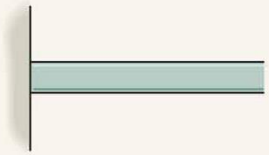
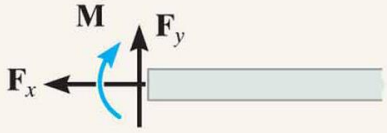


Overall FBD

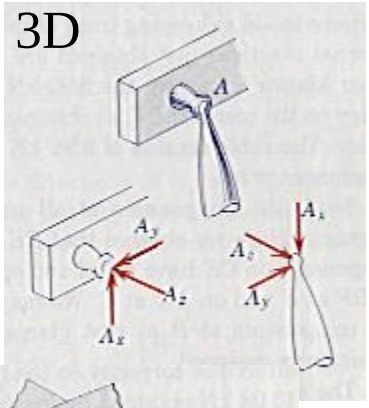
Is this a statically indetermined case ?

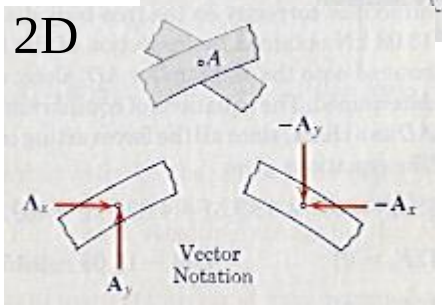


Force analysis. Free-body diagrams

Type of connection	Reaction	Type of connection	Reaction
 Cable	 One unknown: F	 External pin	 Two unknowns: F_x, F_y
 Roller	 One unknown: F	 Internal pin	 Two unknowns: F_x, F_y
 Smooth support	 One unknown: F	 Fixed support	 Three unknowns: F_x, F_y, M

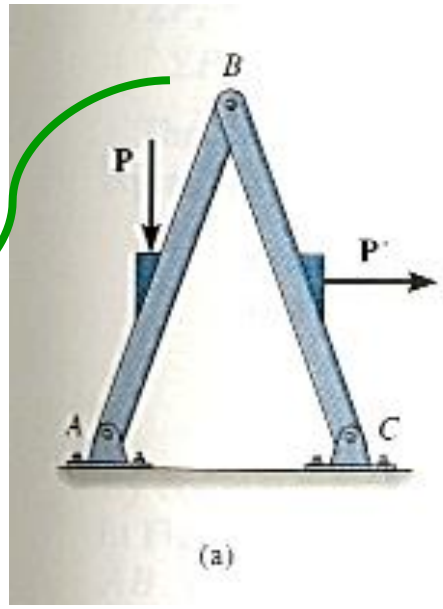
Force analysis. Free-body diagrams

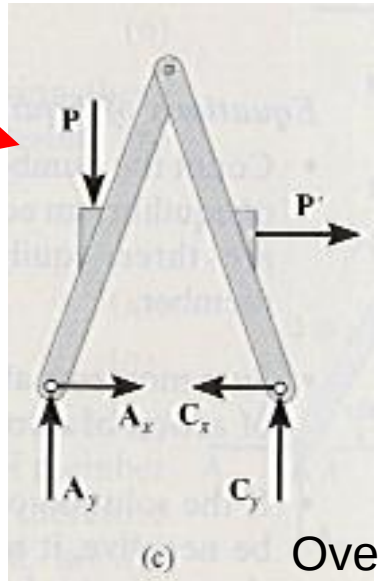




Number of unknowns ?

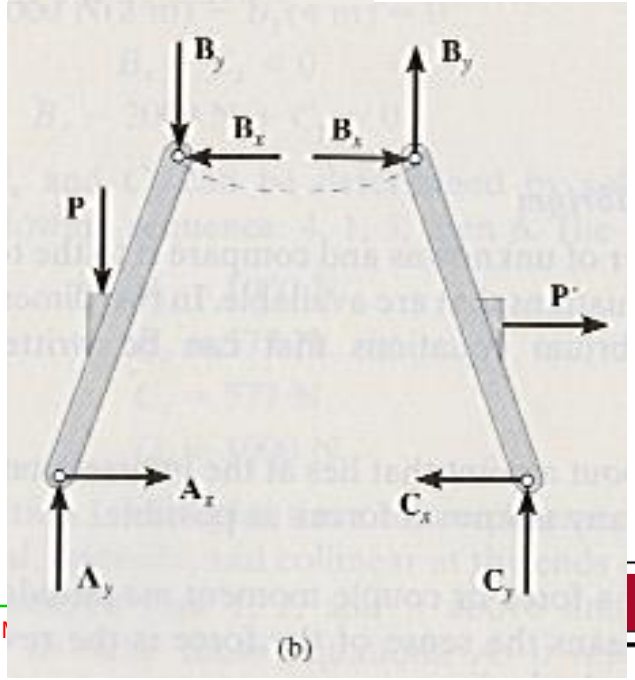
Equilib. Equations ?





Overall FBD

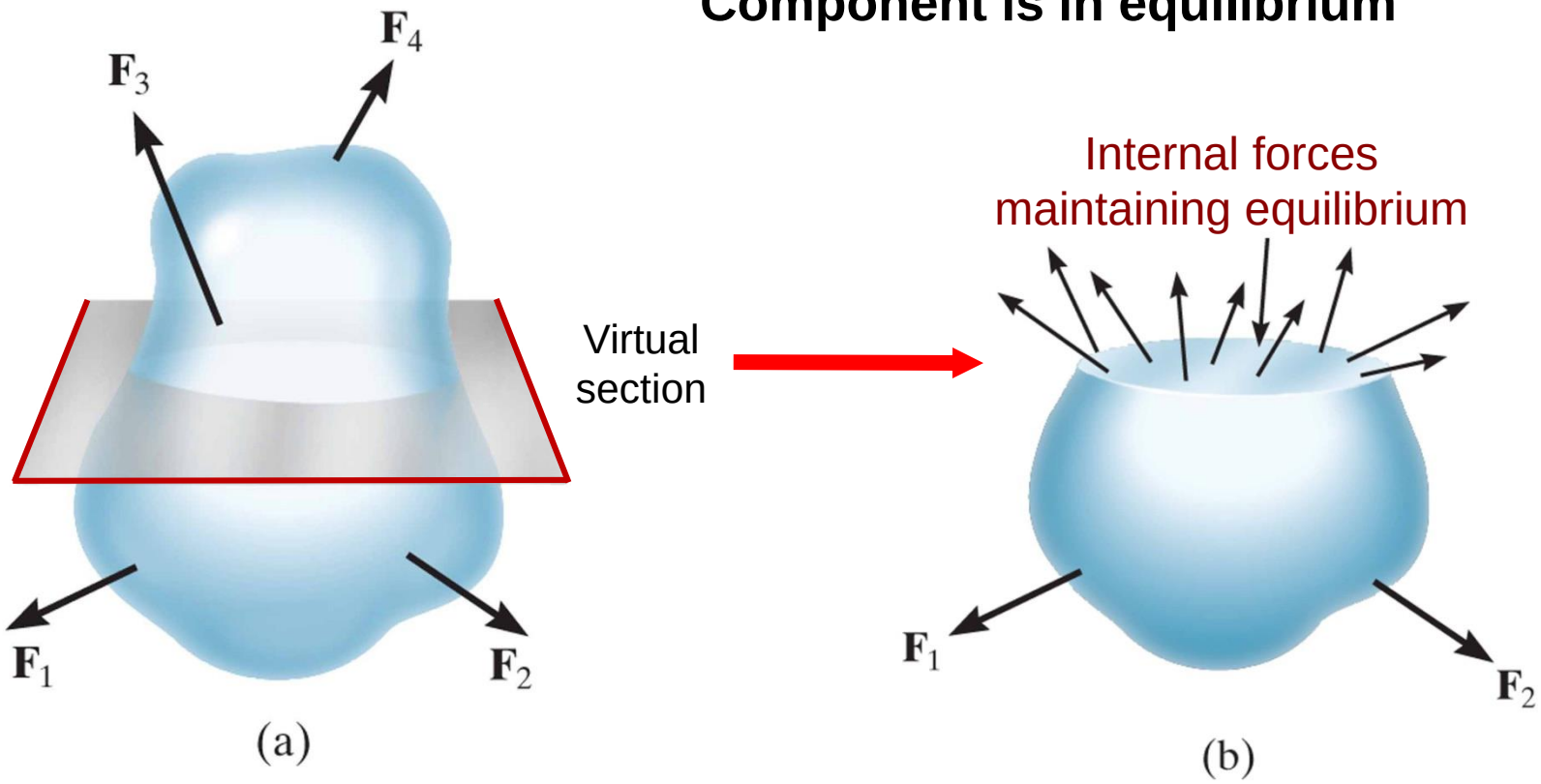
Individual FBD's



Internal resultant loading

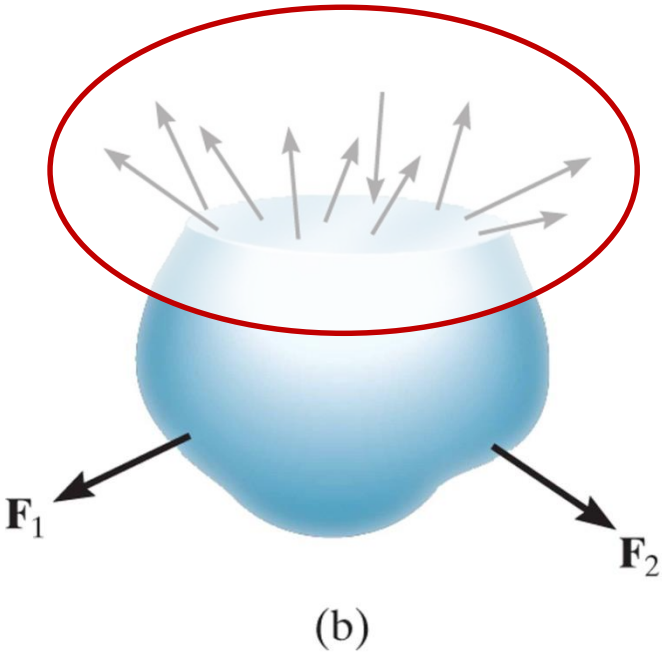
Arbitrary component under load

Component is in equilibrium



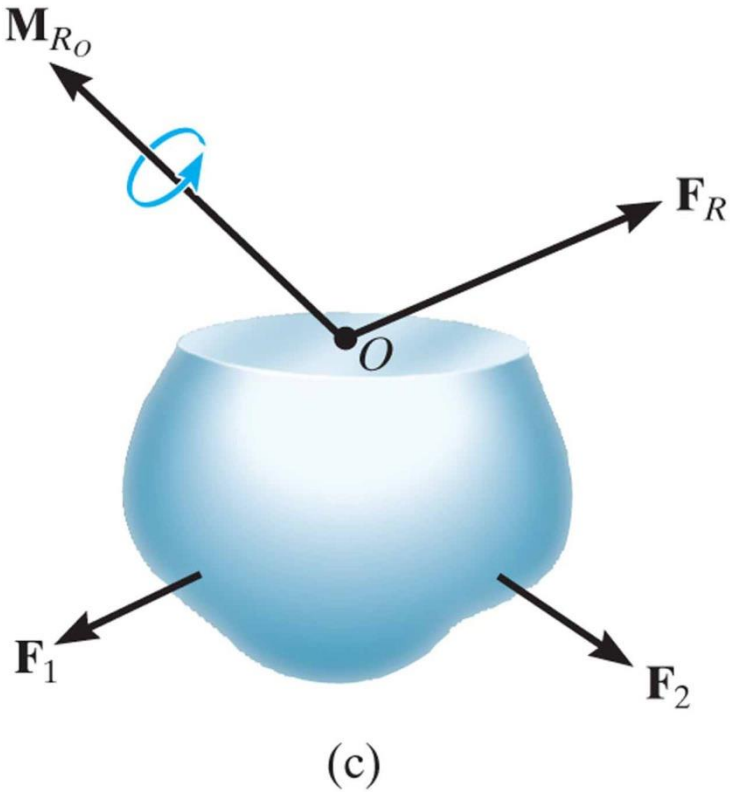
Internal resultant loading

Internal forces maintaining equilibrium



(b)

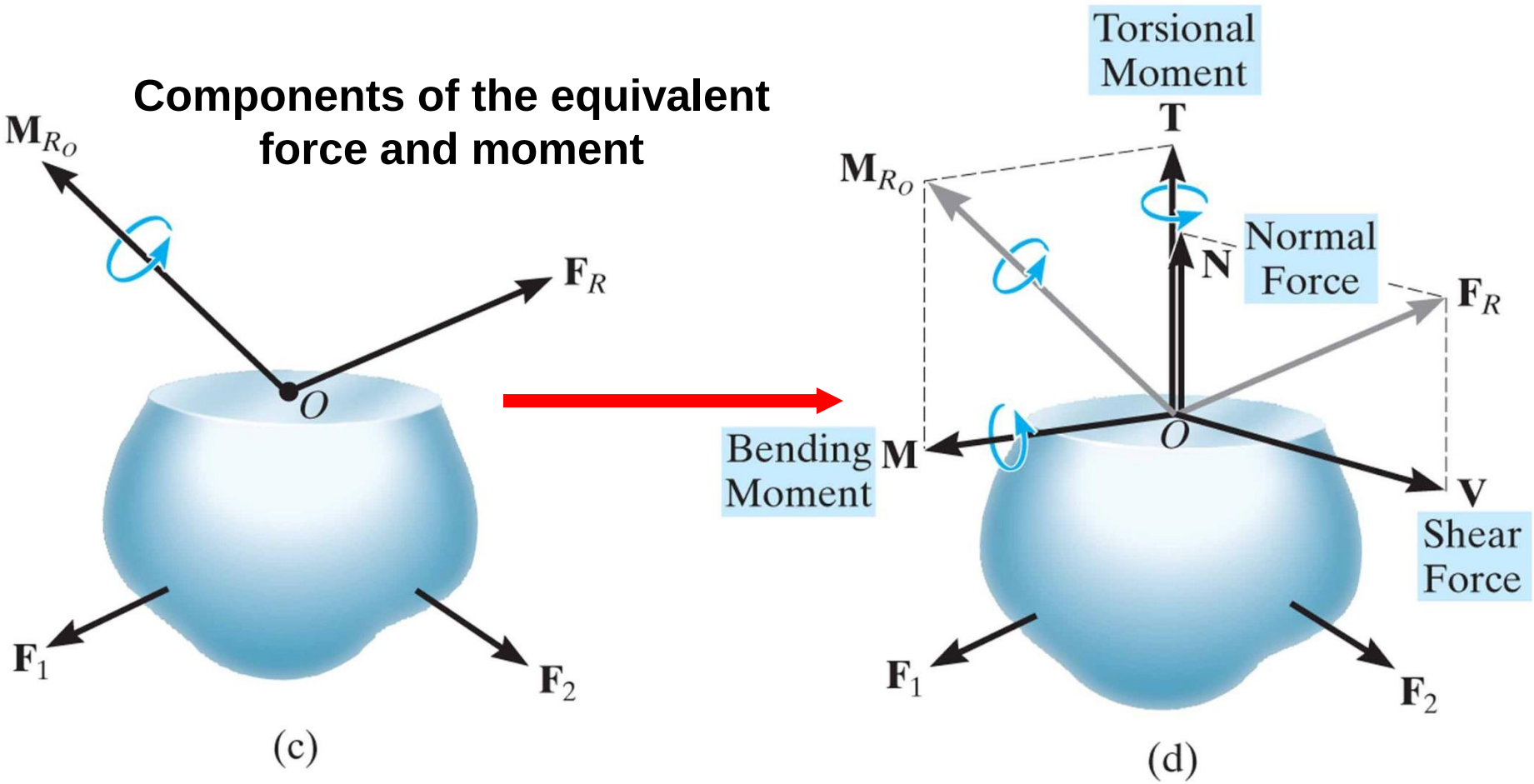
Equivalent force and moment at the section



(c)

Internal resultant loading

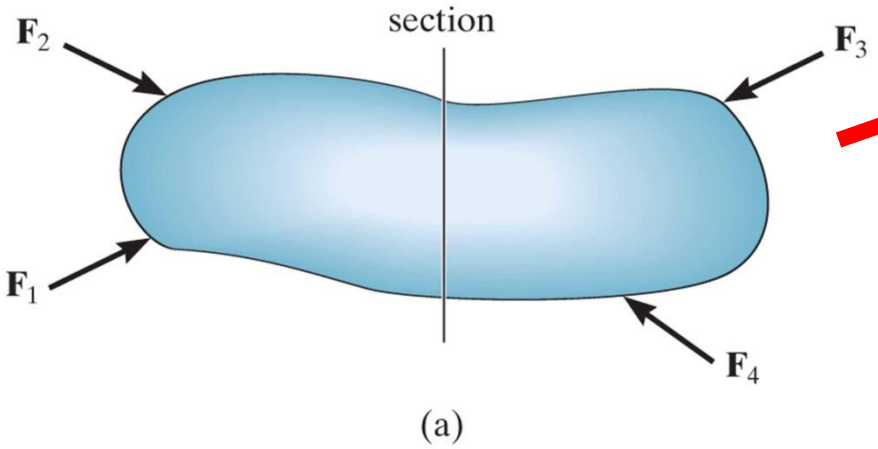
Components of the equivalent force and moment

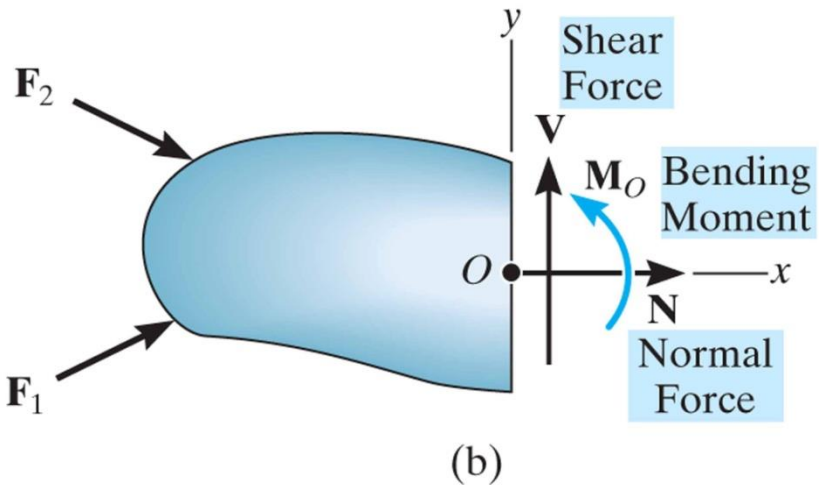


Internal resultant loading

Equivalent forces and moments at the section

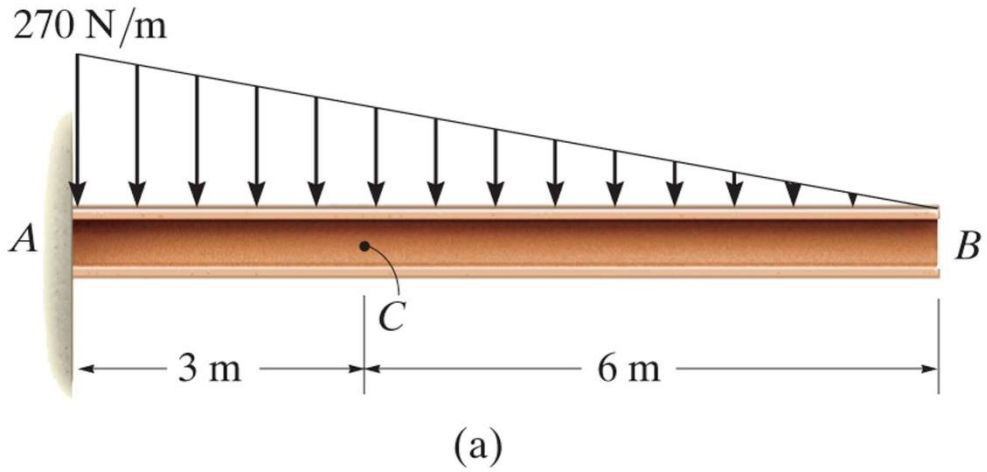
Component is in equilibrium





Internal resultant loading: example

Determine resultant internal loading acting on the cross section at C of the cantilever shown:

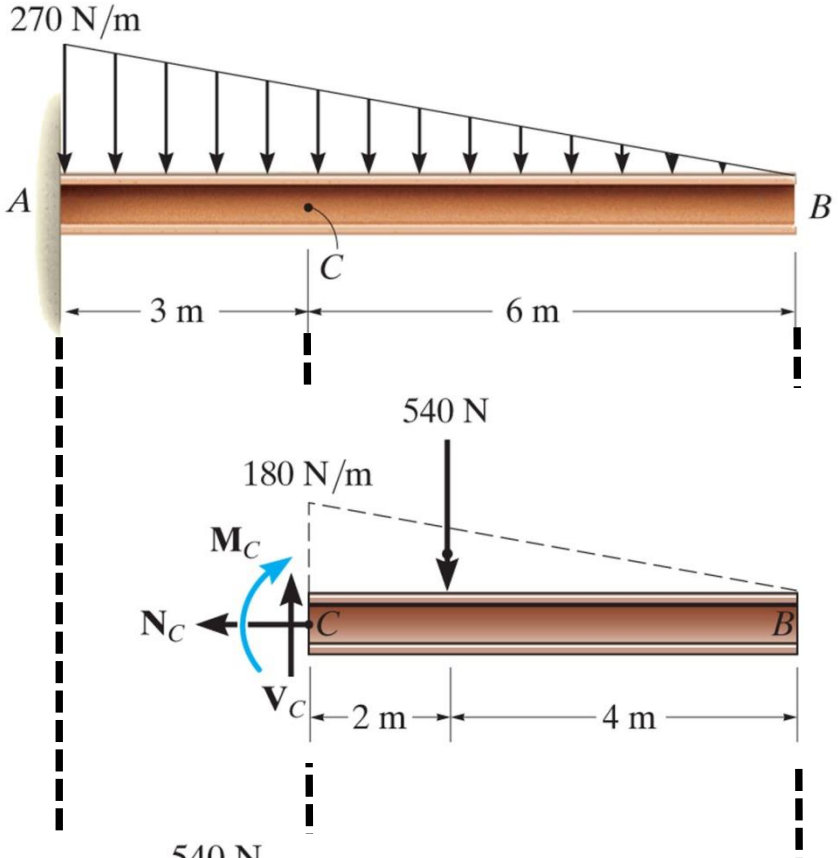


(a)

Approach:

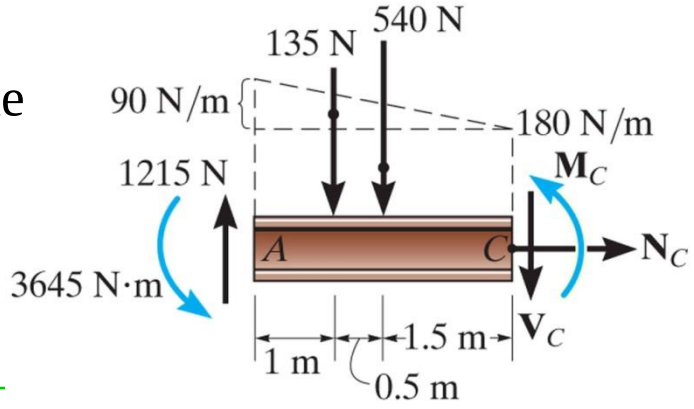
1) Define free-body diagrams
2) Apply equilibrium equations

Internal resultant loading: example



Free-body diagrams (FBDs):

Section to the right of *C*



Section to the left of *C*

Use either section to determine internal loadings

Reading assignment

- Chapter 1 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course