

### Exam problem E06-1. ES2502, Furlong

$$\phi_{AC} := 0.75 \quad \text{in} \quad \text{LAC} := 4 \quad \text{LCD} := 8$$

$$\phi_{CB} := 1.5 \quad \text{in} \quad \text{LDB} := 10$$

$$J_{AC} := \frac{\pi}{2} \cdot \left( \frac{\phi_{AC}}{2} \right)^4 = 0.031 \quad \text{in}^4$$

$$J_{CB} := \frac{\pi}{2} \cdot \left( \frac{\phi_{CB}}{2} \right)^4 = 0.497 \quad \text{in}^4$$

$$G_{br} := 5.4 \cdot 10^6 \quad \text{psi}$$

$$G_{st} := 11 \cdot 10^6 \quad \text{psi}$$

$$T_{applied} := 600 \cdot 12 = 7.2 \times 10^3 \quad \text{lb-in}$$

Solve system of equations for reaction torques: equilibrium + compatibility

Guess:  $T_A := 100 \quad T_B := 100$

Given

$$T_A + T_B = T_{applied} \quad \text{Equilibrium}$$

$$\frac{T_A \cdot \text{LAC}}{J_{AC} \cdot G_{br}} + \frac{T_A \cdot \text{LCD}}{J_{CB} \cdot G_{st}} = \frac{T_B \cdot \text{LDB}}{J_{CB} \cdot G_{st}} \quad \text{Compatibility}$$

$$a := \text{Find}(T_A, T_B) = \begin{pmatrix} 485.272 \\ 6.715 \times 10^3 \end{pmatrix}$$

$$\underline{T_A} := a_0 \quad T_A = 485.272 \quad \text{lb-in}$$

$$\underline{T_B} := a_1 \quad T_B = 6.715 \times 10^3 \quad \text{lb-in}$$

Check:  $\frac{T_A + T_B}{12} = 600 \quad (\text{Correct computation of torques})$

$$T_A + T_B - T_{\text{applied}} = 9.095 \times 10^{-13}$$

(zero, which means that  $T_A$  and  $T_B$  have same direction and are opposite to applied torque)

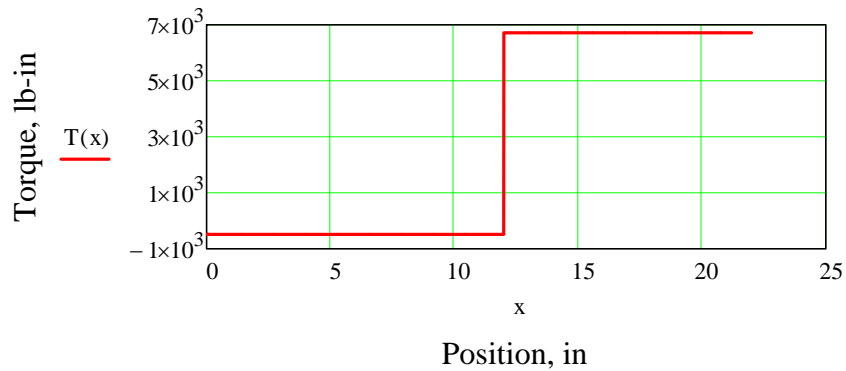
$$-T_A + T_{\text{applied}} = 6.715 \times 10^3$$

Plot torque-diagram:

Define step function:  $S(x, z) := \text{if}(x \geq z, 1, 0)$

$$x := 0, 0.001 \dots (\text{LAC} + \text{LCD} + \text{LDB})$$

Torque function:  $T(x) := -T_A + S(x, \text{LAC} + \text{LCD}) \cdot T_{\text{applied}}$



Part (a):  $\tau_{CB} := \frac{T_B \cdot \left(\frac{\phi_{CB}}{2}\right)}{J_{CB}} = 1.013 \times 10^4 \text{ psi}$

Part (b):  $\tau_{AC} := \frac{T_A \cdot \left(\frac{\phi_{AC}}{2}\right)}{J_{AC}} = 5.858 \times 10^3 \text{ psi}$

Part (c):  $\alpha := \frac{T_A \cdot \text{LAC}}{J_{AC} \cdot G_{br}} \cdot \frac{180}{\pi} = 0.663 \text{ degrees}$

## Exam problem E06-2. ES2502, Furlong

$$P := 20 \cdot 550 \cdot 12 \quad P = 1.32 \times 10^5 \quad \text{lb in/sec}$$

$$\omega_A := 1750 \cdot \frac{2 \cdot \pi}{60} \quad \omega_A = 183.26 \quad \text{rad/sec}$$

$$r_A := 2 \quad \text{in}$$

$$r_{BC} := 5 \quad \text{in}$$

$$\omega_{BC} := \frac{\omega_A \cdot r_A}{r_{BC}} \quad \omega_{BC} = 73.304$$

Torques:

$$T_A := \frac{P}{\omega_A} \quad T_A = 720.29 \quad \text{lb in}$$

$$T_{BC} := \frac{P}{\omega_{BC}} \quad T_{BC} = 1.801 \times 10^3 \quad \text{lb in}$$

Allowed stress:  $\tau_{\text{allow}} := 10000 \quad \text{psi}$

Diameters:

$$sd_A := \sqrt[3]{\frac{2 \cdot T_A}{\pi \cdot \tau_{\text{allow}}}} \cdot 2 \quad sd_A = 0.716 \quad \text{Round to: } 3/4" \quad (\text{at A})$$

$$sd_{BC} := \sqrt[3]{\frac{2 \cdot T_{BC}}{\pi \cdot \tau_{\text{allow}}}} \cdot 2 \quad sd_{BC} = 0.972 \quad \text{Round to: } 1" \quad (\text{at BC})$$

Check: compute stresses with new diameters

$$\tau_{A\text{check}} := \frac{2 \cdot T_A}{\pi \cdot \left(\frac{0.75}{2}\right)^3} = 8.695 \times 10^3$$

$$\tau_{BC\text{check}} := \frac{2 \cdot T_{BC}}{\pi \cdot \left(\frac{1}{2}\right)^3} = 9.171 \times 10^3$$

Both of the stresses are within allowed limit.