Generalized Configurations

Brigitte Servatius
Zur Theorie der Netze und Configurationen von Konrad Zindler in Graz

- Elementary proof of a theorem of Möbius:
  Given 4 points in the plane, one can, by ruler alone construct a point in the $\epsilon$-neighborhood of a given 5’th point for any $\epsilon > 0$.

- Generalization of Configuration:
  A system of points and lines in the plane such that on every line there are at least 3 points and through every point there are at least 3 lines.
1. Zindler’s Construction
Realizable Moves

- Put a new point on a line.

- Put a new line through a point.

- Intersect two lines.

- Draw a line through two points.
- Join two components by putting a point of one component on a line of the other component.
Realizable Moves

- Put a new point on a line.
- Put a new line through a point.
- Intersect two lines.
- Draw a line through two points.
- Join two components by putting a point of one component on a line of the other component.
Dangerous Moves

- Intersect two lines.

- Draw a line through two points.
Realizable Moves on the Levi graph

- Add vertices of degree one.
- Add vertices of degree two such that bipartiteness and girth 6 are preserved.
  (between points of the same color a distance at least 4 apart.)
- Add edges between connected components (bridges).
Given a bipartite graph $G$ of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If $G$ is 3-regular, then $G - v$ can be built up from a vertex by allowable moves.
Given a bipartite graph $G$ of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If $G$ is 3-regular, then $G - v$ can be built up from a vertex by allowable moves.
Given a bipartite graph $G$ of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If $G$ is 3-regular, then $G - v$ can be built up from a vertex by allowable moves.
Given a bipartite graph $G$ of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If $G$ is 3-regular, then $G - v$ can be built up from a vertex by allowable moves.
Given a bipartite graph $G$ of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If $G$ is 3-regular, then $G - v$ can be built up from a vertex by allowable moves.
Given a bipartite graph $G$ of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If $G$ is 3-regular, then $G - v$ can be built up from a vertex by allowable moves.
Given a bipartite graph $G$ of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If $G$ is 3-regular, then $G - v$ can be built up from a vertex by allowable moves.
Given a bipartite graph $G$ of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If $G$ is 3-regular, then $G - v$ can be built up from a vertex by allowable moves.
2. Whiteley’s Theorem

A generic picture in $k - 1$ space of an incidence structure lifts to a sharp scene in $k$-space if and only if

$$i \leq a + kb - (k + 1)$$

for all sub-incidence structures having at least two blocks.
For a 3-regular bipartite graph of girth six Whiteley’s count is violated by three.
For a 3-regular bipartite graph of girth six Whiteley’s count is violated by three.
An \((8_4)\) spatial configuration.
\[a = 8, \quad b = 8, \quad i = 32,\]
\[a + 3b - 4 = 28\]
A similar \((8_4)\) spatial configuration.  
Levi graph is a hypercube

\[ a = 8, \ b = 8, \ i = 32, \]

\[ a + 3b - 4 = 28 \]