Generalized Configurations

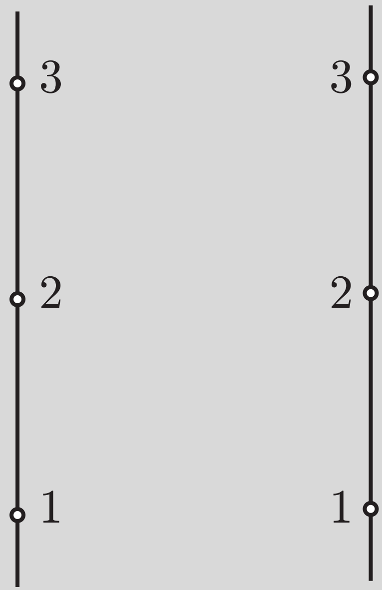
Brigitte Servatius

Zur Theorie der Netze und Configurationen von Konrad Zindler in Graz

- Elementary proof of a theorem of Möbius:
Given 4 points in the plane, one can, by ruler alone construct a point in the ϵ-neighborhood of a given 5'th point for any $\epsilon > 0$.

- Generalization of Configuration:
A system of points and lines in the plane such that on every line there are at least 3 points and through every point there are at least 3 lines.

1. Zindler's Construction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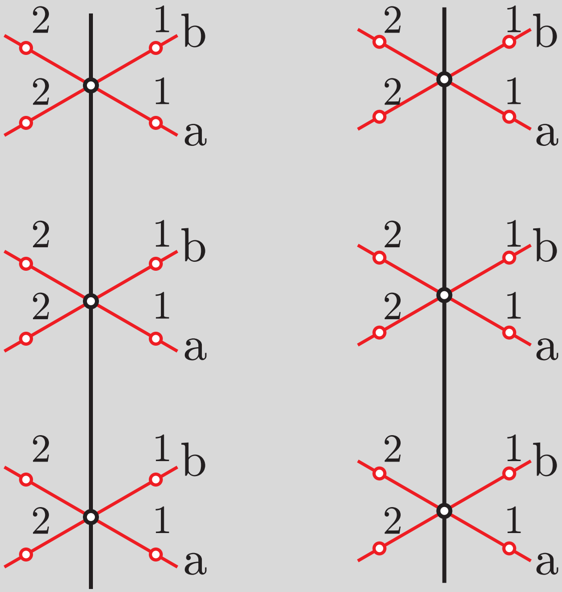
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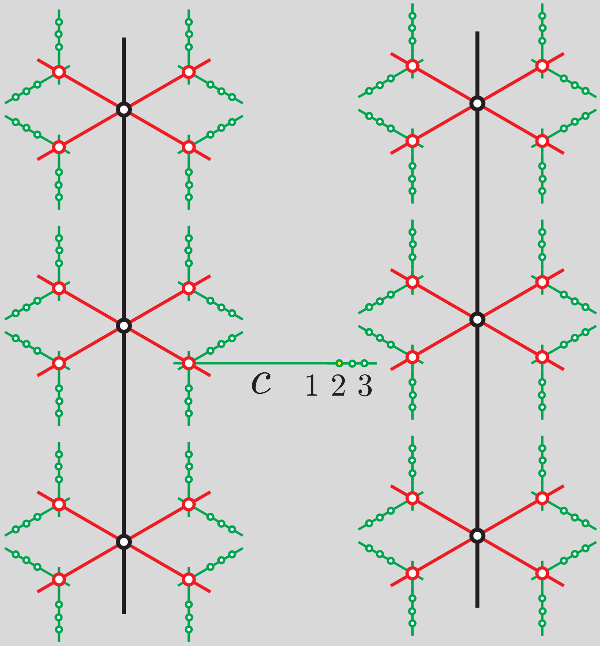
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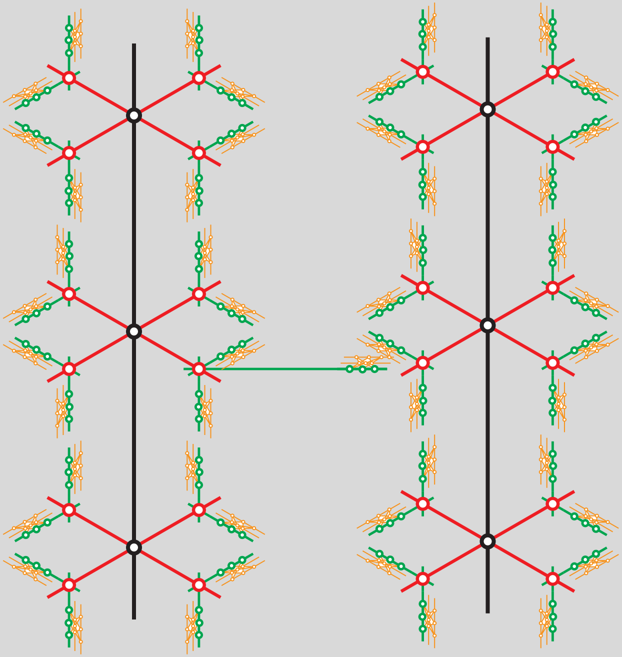

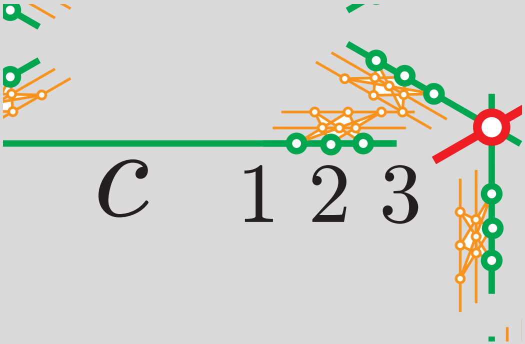
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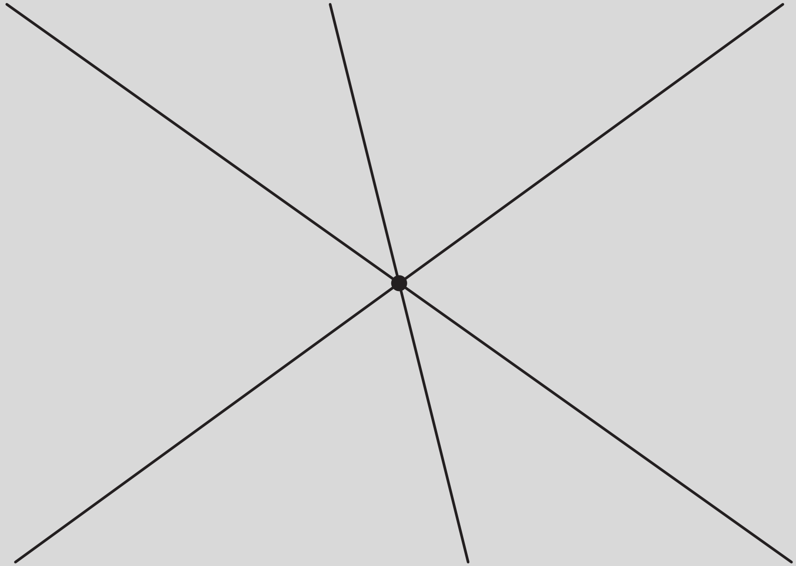

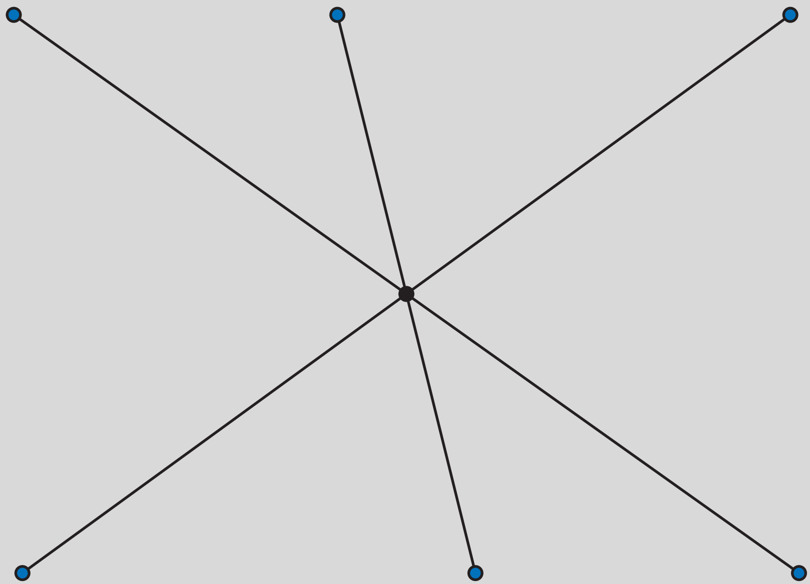

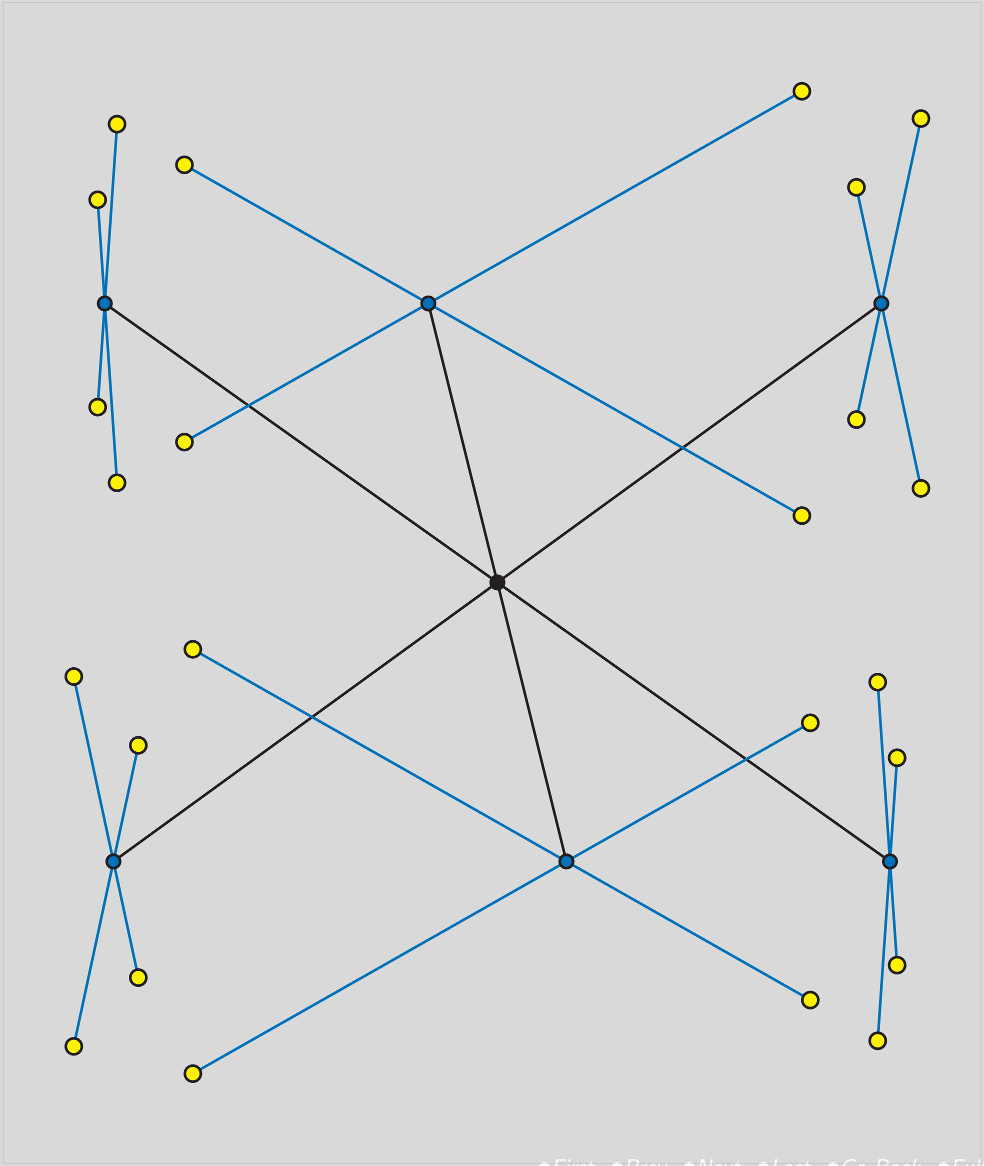

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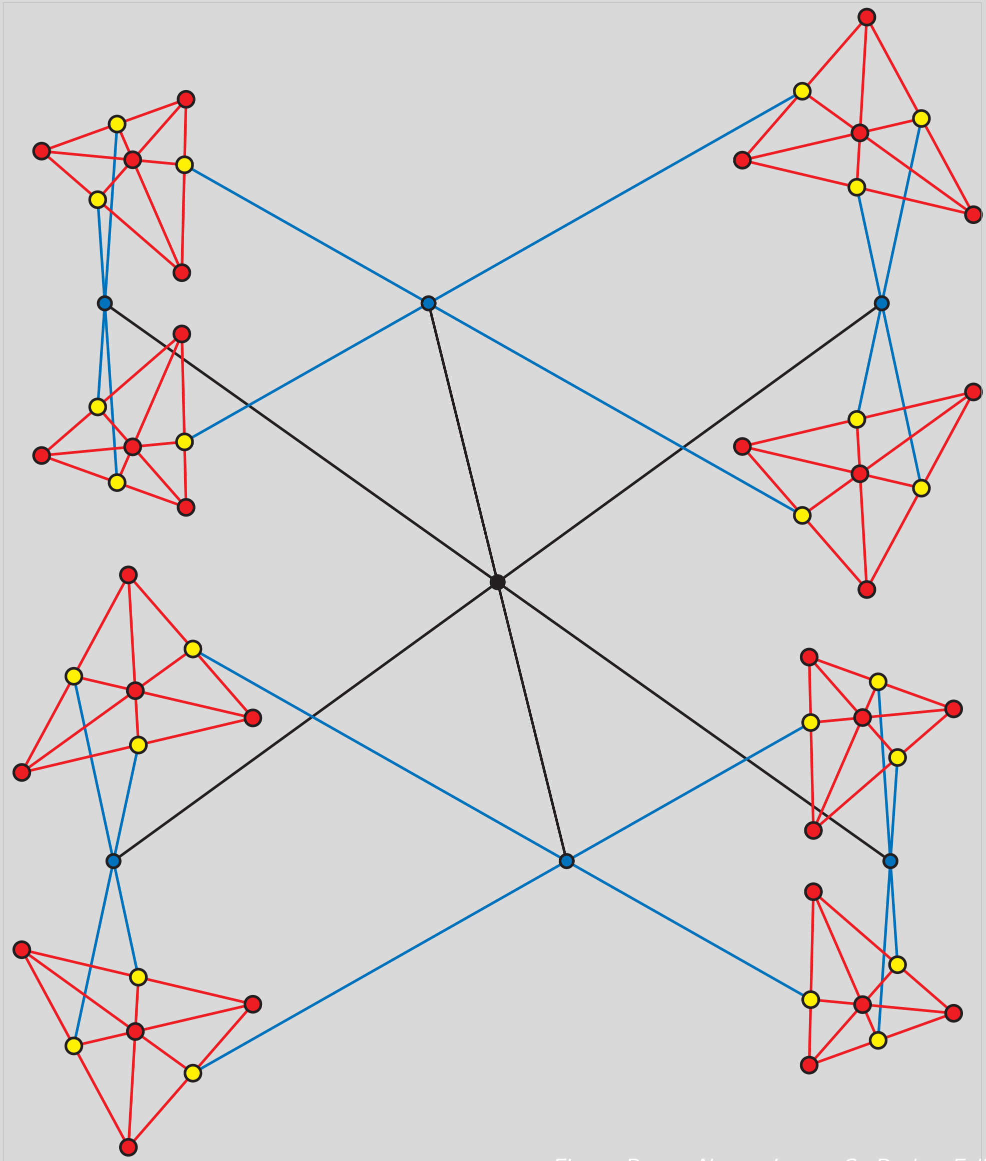
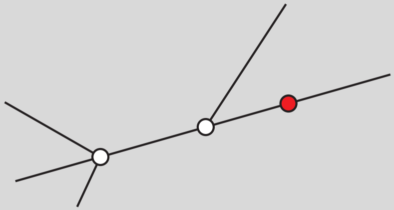
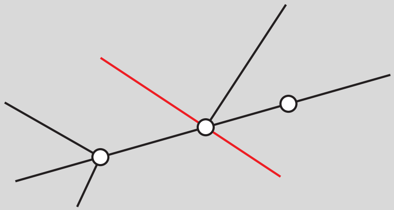


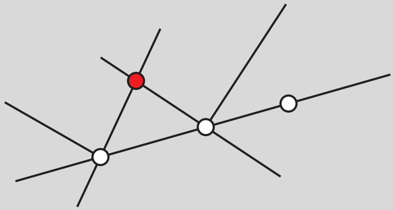
Realizable Moves



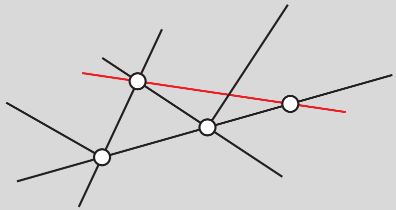
- Put a new point on a line.



- Put a new line through a point.



- Intersect two lines.



- Draw a line through two points.
- Join two components by putting a point of one component on a line of the other component.

Realizable Moves

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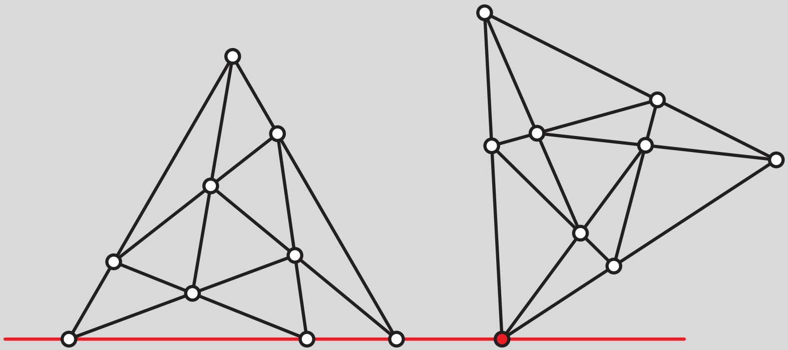
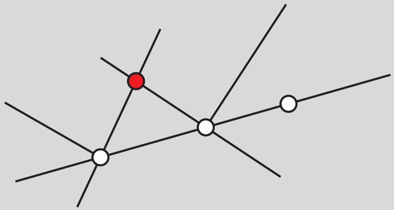


Dangerous Moves

- Intersect two lines.



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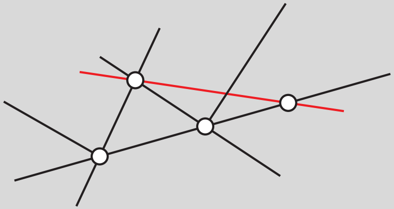
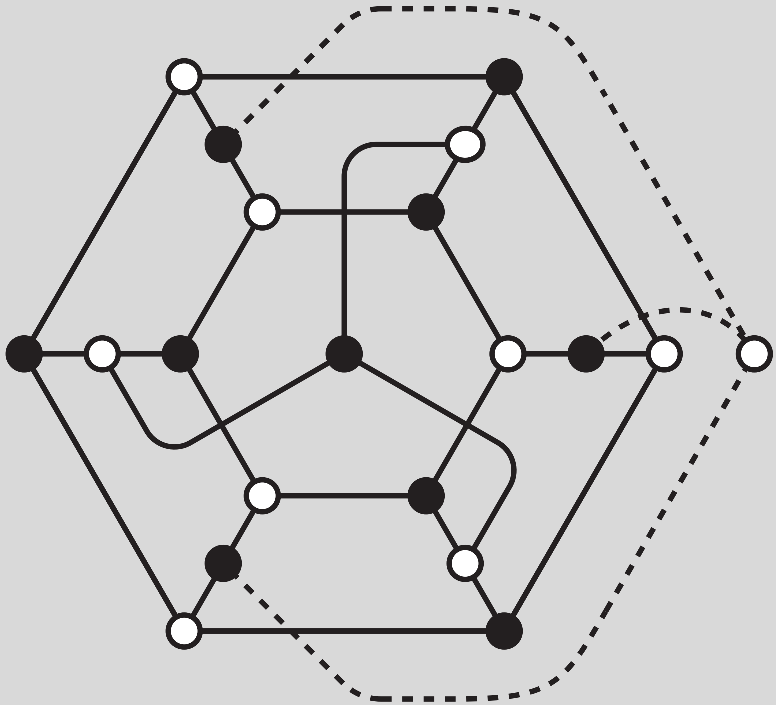


Realizable Moves on the Levi graph

- Add vertices of degree one.
- Add vertices of degree two such that bipartiteness and girth 6 are preserved.
(between points of the same color a distance at least 4 apart.)
- Add edges between connected components (bridges).

Given a bipartite graph G of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If G is 3-regular, then $G - v$ can be built up from a vertex by allowable moves



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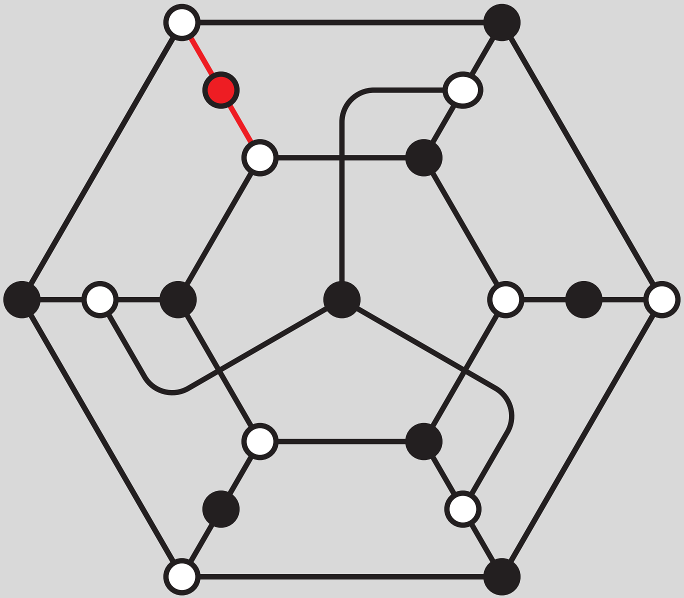

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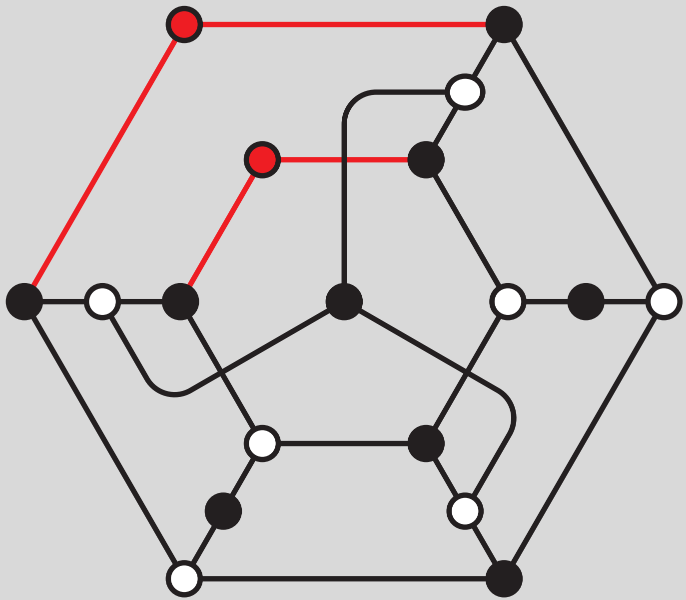

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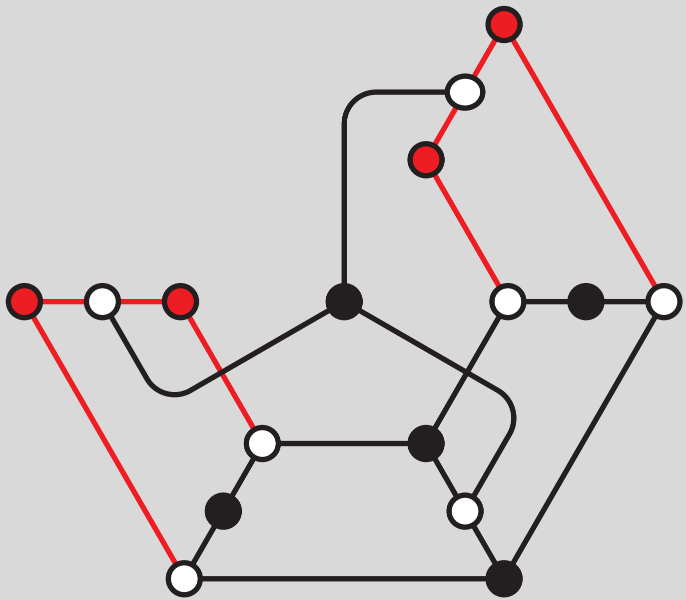

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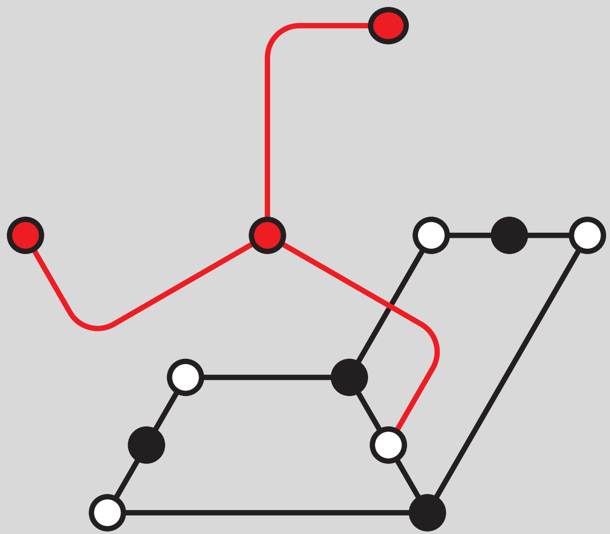

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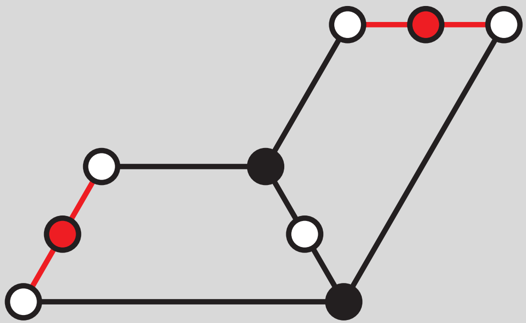

Zindler's . . .

Whiteley's Theorem

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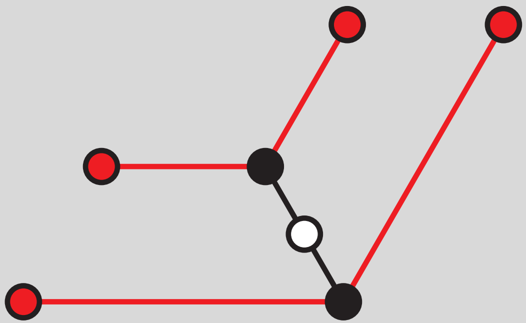

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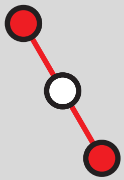
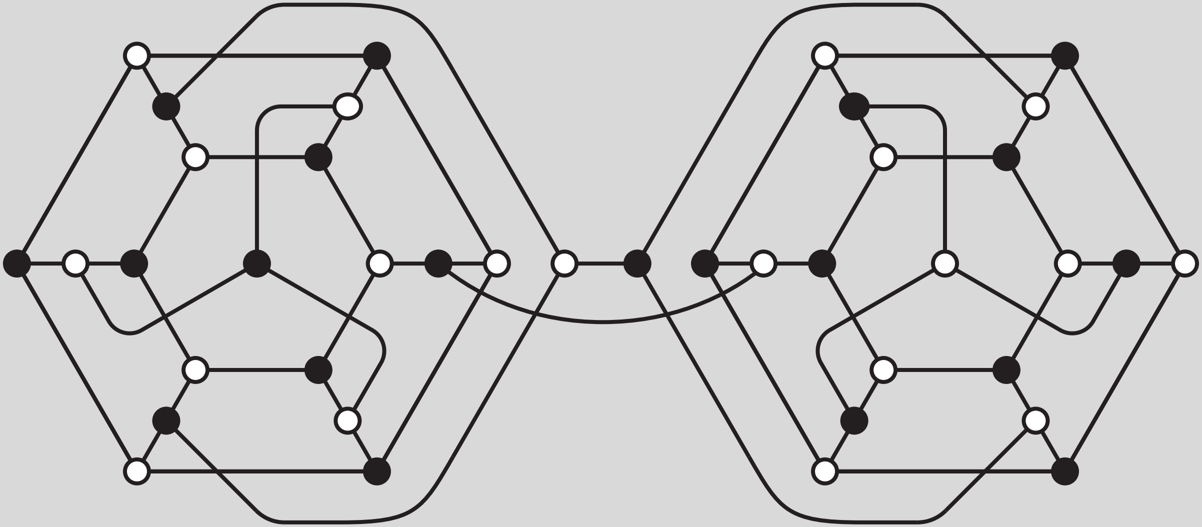
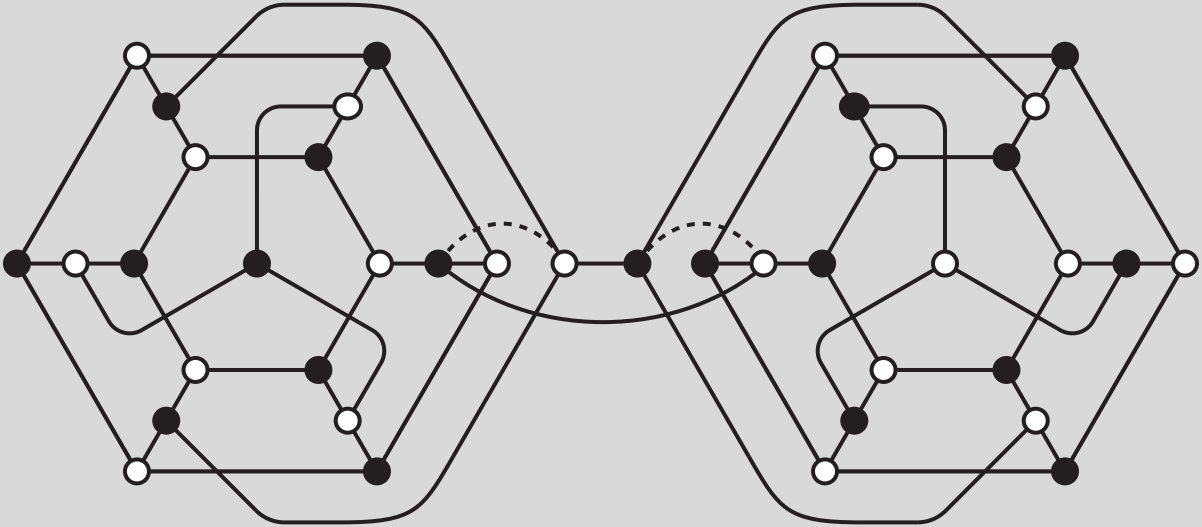
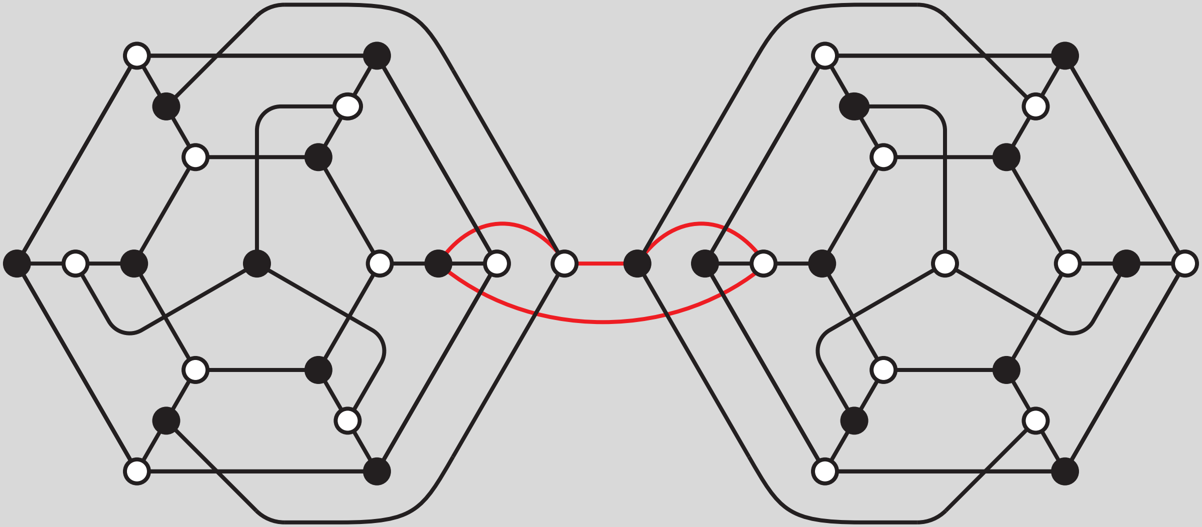






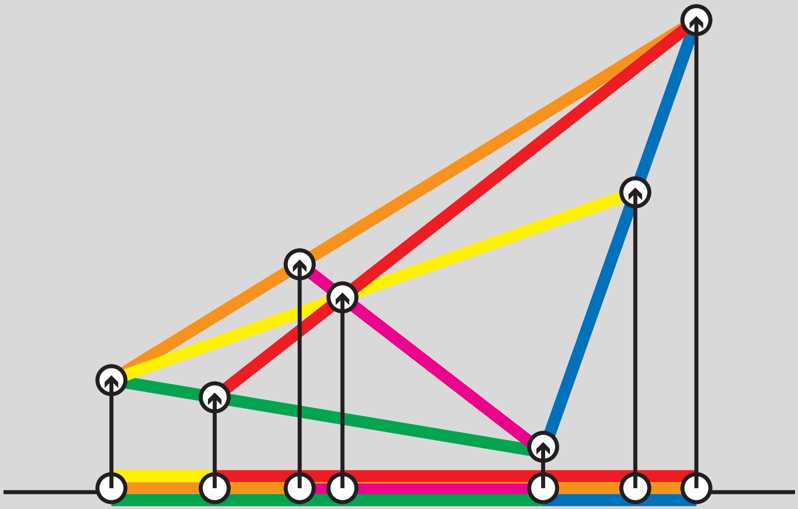


2. Whiteley's Theorem

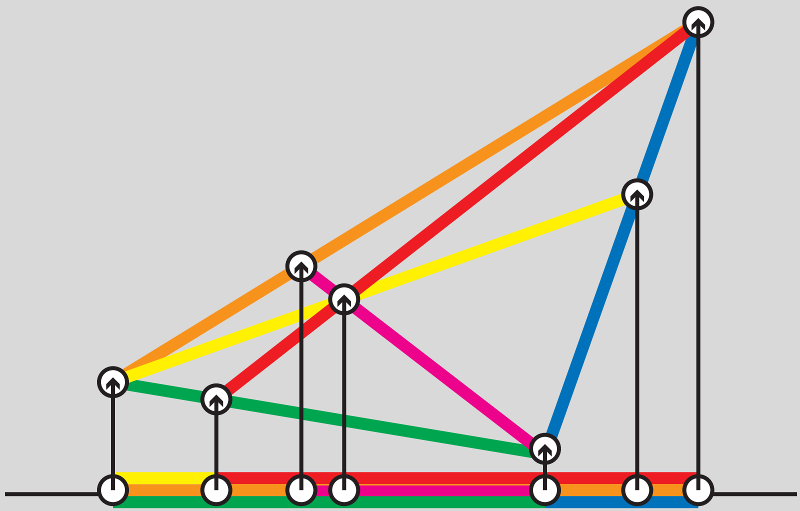
A generic picture in $k-1$ space of an incidence structure lifts to a sharp scene in k-space if and only if

$$i \leq a + kb - (k+1)$$

for all sub-incidence structures having at least two blocks.



For a 3-regular bipartite graph of girth six Whiteley's count is violated by three.



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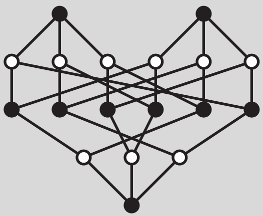
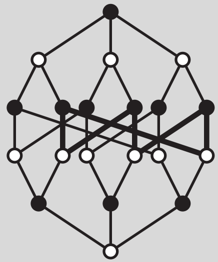


$i = 6$	$p = 2$	$l = 6$	$l + 2p - 2 = 8$	$2l + p - 2 = 12$
$i = 18$	$p = 8$	$l = 6$	$l + 2p - 2 = 20$	$2l + p - 2 = 18$
$i = 24$	$p = 8$	$l = 9$	$l + 2p - 2 = 23$	$2l + p - 2 = 24$
$i = 27$	$p = 9$	$l = 9$	$l + 2p - 2 = 25$	$2l + p - 2 = 25$



$i = 3$	$p = 1$	$l = 6$	$l + 2p - 2 = 8$	$2l + p - 2 = 12$
$i = 9$	$p = 8$	$l = 6$	$l + 2p - 2 = 20$	$2l + p - 2 = 18$
$i = 21$	$p = 8$	$l = 9$	$l + 2p - 2 = 23$	$2l + p - 2 = 24$
$i = 27$	$p = 9$	$l = 9$	$l + 2p - 2 = 25$	$2l + p - 2 = 25$
$i = 30$	$p = 9$	$l = 9$	$l + 2p - 2 = 25$	$2l + p - 2 = 25$

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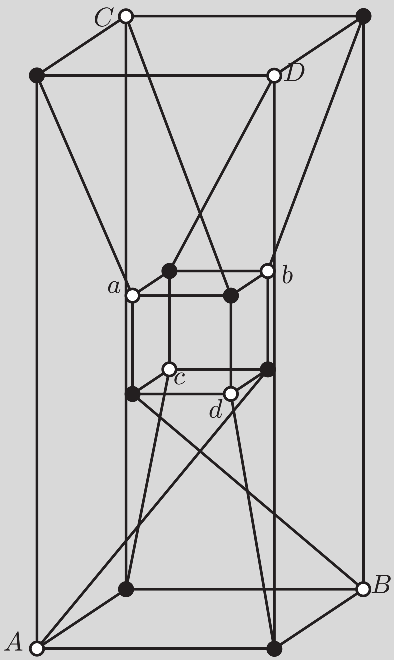
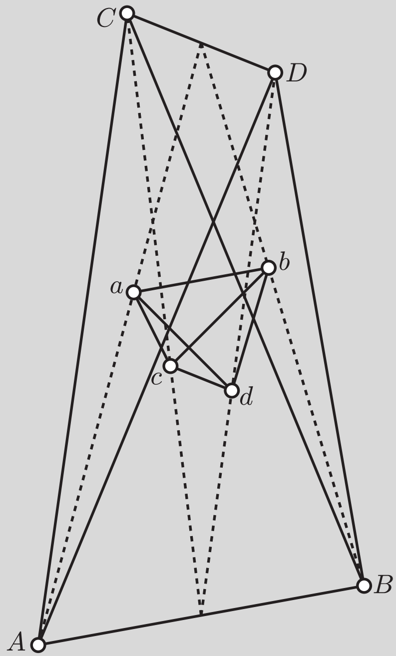
An (8_4) spatial configuration.

$a = 8$, $b = 8$, $i = 32$,

$$a + 3b - 4 = 28$$





A similar (8_4) spatial configuration.
Levi graph is a hypercube
$a = 8$, $b = 8$, $i = 32$,

$$a + 3b - 4 = 28$$