# What is a good question?

Brigitte Servatius

March 11, 2019

# Gombaud

Antoine Gombaud (1607-1684)

Chevalier de Méré asked Pascal in 1654 : What is more likely: To get at least one 6 rolling a die four times or to get a double 6 rolling two dice 24 times.

This lead to correspondence between Pascal and Fermat and 1654 is considered the birth year of probability theory.


Pierre de Fermat (1607-1665) Blaise Pascal (1623-1662)

 Augustin Cauchy (1789-1857) Cauchy proved that two convex polyhedra with congruent corresponding faces must be congruent to each other. Cauchy asked whether or not convexity was necessary. (1813)

After many attempts to prove that convexity was not needed, Connelly, in 1979, gave a counterexample. Steffen's example is to this day the smallest and easiest to build Connely sphere.

A symmetric flexible Connelly sphere with only nine vertices

by Klaus Steffen (I.H.E.S.)

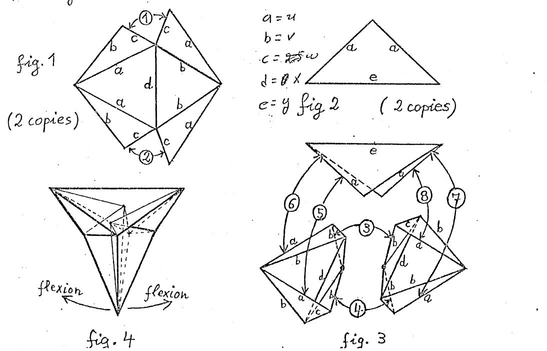
1.) Make 14 rigid triangles and attach them to each other in a flexible fashion as indicated in fig. 1,2 (two copies!); a good choice of parameters is e.g. $a:=6$, $b:=5$, $c:=2.5$, $d:=5.5$, $e:=8.5$.

2.) Connect (in a flexible way!) the two edges marked ① in fig. 1 by rotating the corresponding triangles upward and the two edges marked ② by rotating the corresponding triangles downward (in either copy!).

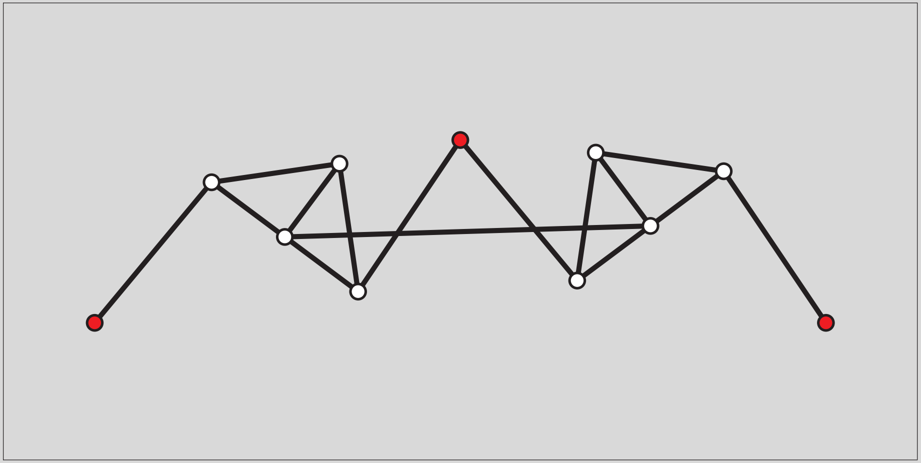
3.) Attach the two aggregates of 6 triangles to each other as indicated by ③, ④ in fig. 3.

4.) Connect the two remaining single triangles (fig. 2) along edge e thereby making a "roof" which is attached to the configuration of 12 triangles from step 3.) as indicated by ⑤, ⑥, ⑦, ⑧ in fig. 3.

5.) If you did not mess up everything the resulting sphere looks like fig. 4 and flexes by about 30° as indicated by the arrows.
(It is a good idea to cut a "window" in the "roof" to make the inside visible.)



Connelly in turn asked if there exists a mechanism with a cusp in the configuration space.



H. Servatius answered by double Watt.

 Carl Pomerance, in his 2016 AMS lecture stated:

> I began my career at the University of Georgia in 1972. In the spring of my second year there a fortuitous event happened: On April 8, 1974, Hank Aaron of the Atlanta Braves hit his 715th career homerun, thus finally eclipsing the supposedly unbeatable total of 714 set some 40 years earlier by Babe Ruth. I was watching the game on television, and I noticed that $714 \times 715 = 2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17$

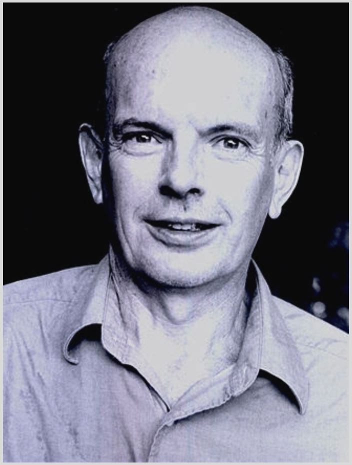
so we have two consecutive integers whose product is also the product of the first $k$ primes for some $k$. It seemed to me that this was not likely to occur ever again.

The next day I challenged my colleague David Penney to find an interesting property of 714 and 715, and he quickly saw the same thing. He asked a numerical analysis class he was teaching, and one of the students came up with another property: The sum of prime divisors of 714 equals the sum of prime divisors of 715.

With Penney and a student, Carol Nelson, I quickly wrote a paper for the Journal of Recreational Mathematics, which was accepted by return mail and was published that same spring.

C. Nelson, D.E. Penney and C. Pomerance, '714 and 715', J. Recreational Mathematics, 7(1974), 87-89

# OEIS



Neil J. A. Sloane

A039752 Ruth-Aaron numbers (2): sum of prime divisors of n = sum of prime divisors of n+1 (both taken with multiplicity).
5, 8, 15, 77, 125, 714, 948, 1330, 1520, 1862, 2491, 3248, 4185, 4191, 5405, 5560, 5959, 6867, 8280, 8463, 10647, 12351, 14587, 16932, 17080, 18490, 20450, 24895, 26642, 26649, 28448, 28809, 33019, 37828, 37881, 41261, 42624, 43215

# Calculus homework

Here is a standard calculus text question: Given $f(x) = ax^3 + bx^2 + cx + d$ and the point $Q$ with coordinates $(x_1, y_1)$, find the equation of the tangent line to $f$ that contains $Q$.

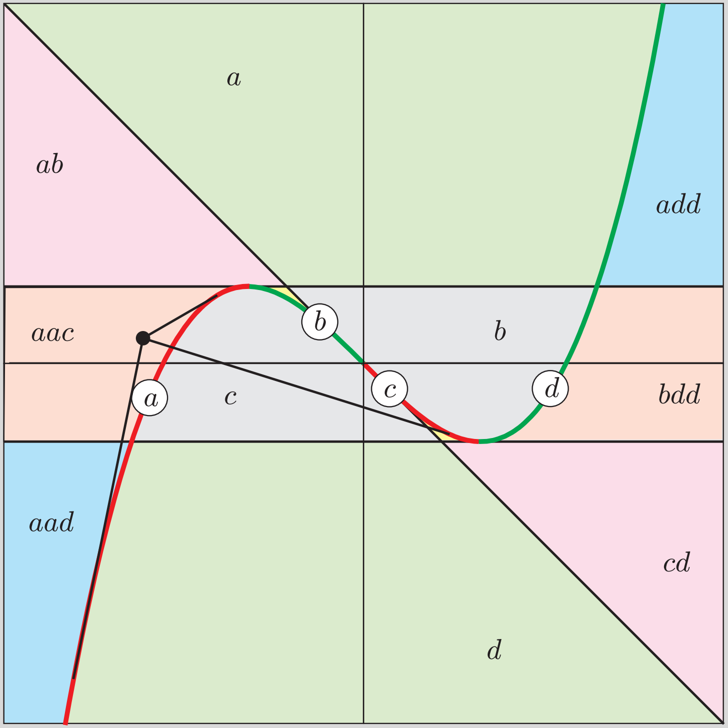
# student complaint

There is only one answer in the book.

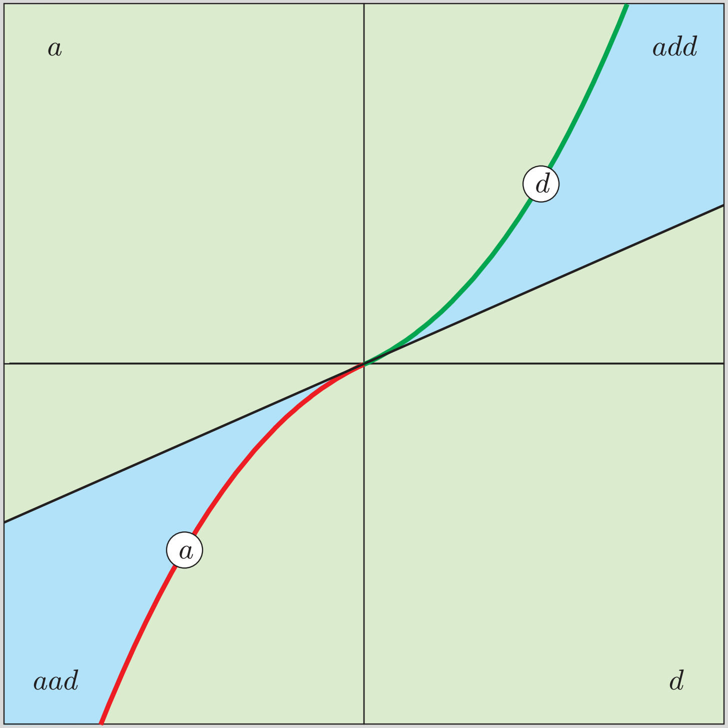
# challenge

Turn your complaint into a question.
Can you formulate a question? Can you answer that question?

Given the graph of a cubic $f(x)$, determine the sets $P_k$ of all points $p_k$ with the property that exactly $k$ tangents to $f$ intersect in $p_k$. How many (non-empty) regions are there? What are the possible values of $k$?




The multiplicity of symbols in a region label indicates the number of tangents to the arcs of the cubic with the same label. The tiny yellow regions should be labeled $bb$ and $cc$.




The multiplicity of the numbers in a region label indicates the number of tangents to the arc of the cubic with the same label.

# Suggested readings

The College Mathematics Journal MAA

The Pi Mu Epsilon Journal

# Solve problems

# Write up solutions

## 1247

*Proposed by Arthur Holshouser, Charlotte, NC and Patrick Vennebush, NCTM, Reston, VA.*

Let $P$ be a $2n$-sided regular polygon. Suppose $k \geq 3$ points are randomly and uniformly selected from the boundary of $P$. Find the probability that the convex hull of the $k$ points includes the center of $P$.

## Solution by Mark A. Guest II, student, Auburn University at Montgomery, Montgomery, AL.

Let $O$ be the center of $P$ and let $p_1, p_2, \ldots, p_k$ be $k$ points selected randomly and uniformly from the boundary of $P$. For $1 \leq i \leq k$, let $H_i$ be the convex hull of $\{p_1, p_2, \ldots, p_i\}$. We will find $P(O \notin H_k)$, the probability that $H_k$ does not contain $O$.

By this construction, $H_{i-1} \subseteq H_i$ for $1 < i \leq k$. Therefore,

$$P(O \notin H_i) = P(O \notin H_i \cap O \notin H_{i-1}) = P(O \notin H_i \mid O \notin H_{i-1})P($$

Applying this relation repeatedly yields

$$P(O \notin H_k) = P(O \notin H_1) \prod_{n=2}^{k} P(O \notin H_n \mid O \notin H_{n-1}). \quad (1)$$

Clearly $P(O \notin H_1) = 1$. We now determine $P(O \notin H_n \mid O \notin H_{n-1})$.

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*Selected Matches for: Items authored by or related to Devaney, Robert L.*

**MR0258004 (41 #2652)** Reviewed

Devaney, Robert L.

**Lens spaces as coset spaces.**

*Pi Mu Epsilon J.* **5** 1969 7–11.

54.80

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From References: 0
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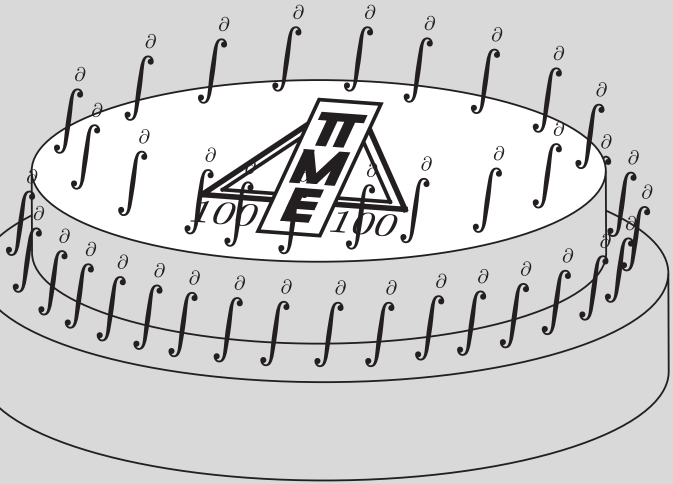
This paper gives a proof of the fact that the usual action of $Z_m$ on $S^3$ to obtain $L_m(r)$ arises from a subgroup of $S^3$ only in case of $r \equiv \pm 1 \bmod m$. No topological distinction of lens spaces is considered; the result already is contained in the work of W. Threlfall and H. Seifert [Math. Ann. **104** (1930), 1–70; ibid. **107** (1932), 543–586].

Reviewed by W. Metzler

# Problem

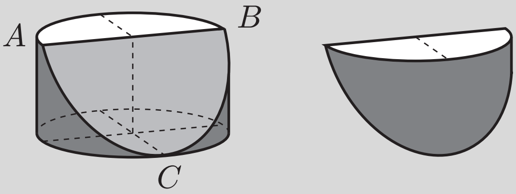
At Denksport on March 11, 2019, there is a cylindrical cake which is 18” high and 12” in radius. When asked how much cake she wants, Professor Servatius demands a piece of cake exactly 1 cubic foot in volume, and furthermore she demands that all the cutting planes be constructible by ruler and compass. How can this be done with as few cuts as possible. You can use a plumb bob to determine vertical.




Wrong decorations, and I said a cylindrical cake!

# Solution

You can cut the cake with a single cut.
You have a cylinder of height $H$ and radius $R$. Consider the plane passing through opposite points on the upper circle $\{A, B\}$, and one of the points of intersection, $C$, of the lower circle with the plane which is the perpendicular bisector of $AB$. See Figure.




If we use cylindrical coordinates with the $x$-axis through point $C$, the volume of the solid is given by

$$\begin{aligned}\int_{-\pi/2}^{\pi/2}\int_0^R (H/R)r\cos(\theta)r\ dr d\theta &= \int_{-\pi/2}^{\pi/2}(H/R)(R^3/3)\cos(\theta)\ d\theta \\ &= (H/R)(R^3/3)(2) = 2HR^3/3\end{aligned}$$

which is 1 cubic foot if $H = 3/2$ and $R = 1$.