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Rigidity, connectivity and graph decompositions

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Clark Univerity

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Body/Simple Pin
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Outline of the Talk

- Beginning Stuff
- Middle Stuff
- Ending Stuff

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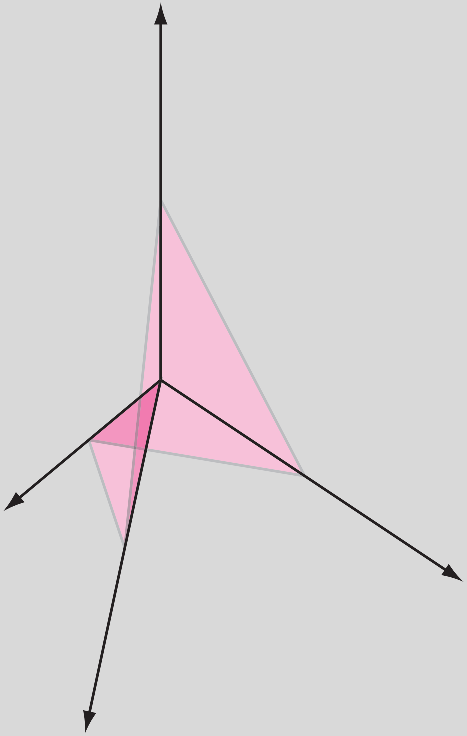
1. Angles

Suppose we have n vectors $\mathbf{r}_1, \ldots, \mathbf{r}_n$ of fixed length in $\mathbb{R}^3$ for which certain pairs of vectors are constrained by their angle from the origin.



Motions:

$$\begin{aligned} E_{i,j} &= \quad \mathbf{r}_i \cdot \mathbf{r}_j & &\text{for } (i,j) \in E \\ E_i &= (1/2)\mathbf{r}_i \cdot \mathbf{r}_i & &\text{for } i \in V, \end{aligned}$$

Motions require $E_{i,j}$ and E_i be constant.

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Infinitessimally:

$(|E| + |V|) \times 3|V|$ matrix

$$\begin{array}{c|ccccc|}
 & & i & & j & \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
(i,j) & \mathbf{0}\cdots\mathbf{0} & \mathbf{r}_j & \mathbf{0}\cdots\mathbf{0} & \mathbf{r}_i & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
i & \mathbf{0}\cdots\mathbf{0} & \mathbf{r}_i & \mathbf{0}\cdots\mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
j & \mathbf{0}\cdots\mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} & \mathbf{r}_j & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}$$

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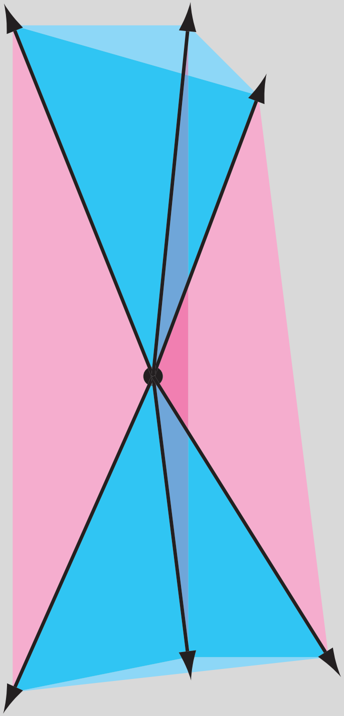
- 6 vectors - 18 degrees of freedom
- 3 length constraints
- 9 angle constraints
- 6 trivial degrees of freedom



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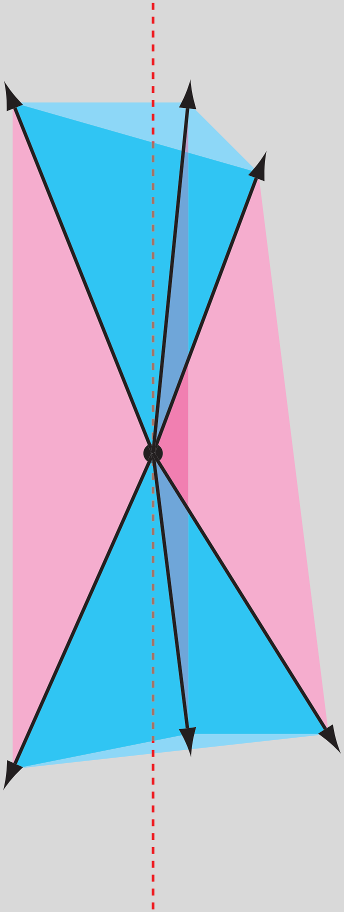

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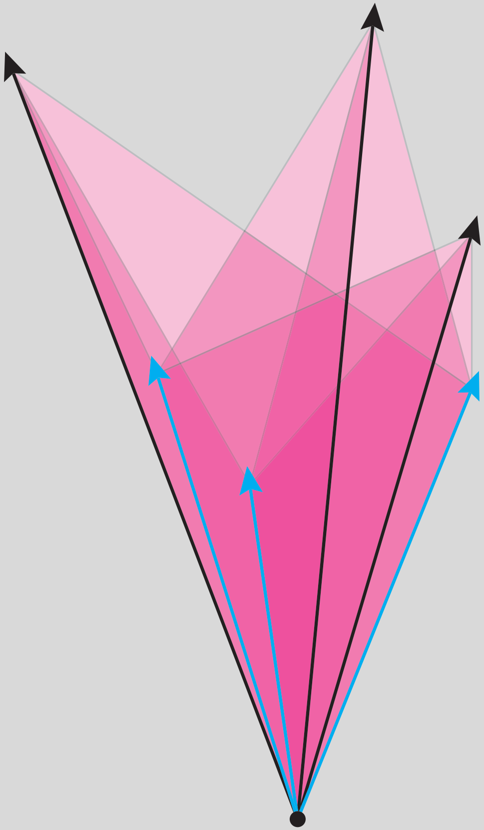
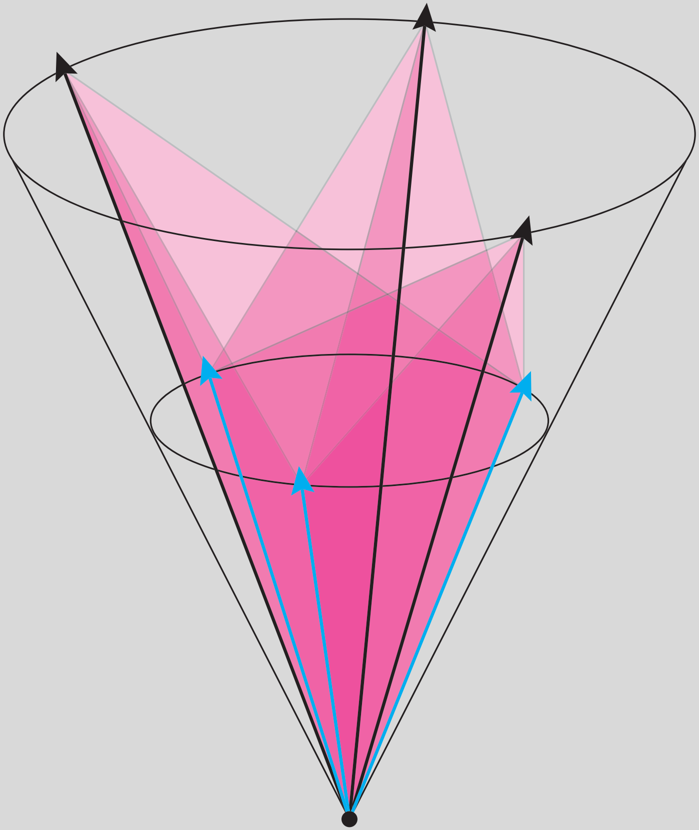
Same count - different arrangement:





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Necessary condition for infinitesimal rigidity

- Each vector has 3 degrees of freedom.
- Each vector contributes a length contraint
- Each edge contributes an “angle” constraint.
- With the origin fixed, there are only 3 trivial modes.

$$|V| + |E| \geq 3|V| - 3$$

$$|E| \geq 2|V| - 3$$

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Invariance under scaling

Each vector can be independently scaled without changing the rank of the matrix.

$$\begin{array}{c|ccccc}
 & & i & & j & \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
(i,j) & \mathbf{0}\cdots\mathbf{0} & \mathbf{r}_j & \mathbf{0}\cdots\mathbf{0} & \mathbf{r}_i & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
i & \mathbf{0}\cdots\mathbf{0} & \mathbf{r}_i & \mathbf{0}\cdots\mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
j & \mathbf{0}\cdots\mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} & \mathbf{r}_j & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}$$

Invariance under scaling

Each vector can be independently scaled without changing the rank of the matrix.

Assume v_1 is incident to $\{v_2, \ldots, v_k\}$

$$\begin{array}{c|cccccc}
 & 1 & 2 & & k & \\
(1,2) & \mathbf{r}_2 & \alpha\mathbf{r}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
(1,3) & \mathbf{r}_3 & \mathbf{0} & \alpha\mathbf{r}_1 & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
(1,k) & \mathbf{r}_k & \mathbf{0} & \ldots & \alpha\mathbf{r}_1 & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
1 & \alpha\mathbf{r}_1 & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
 & \mathbf{0} & \vdots & \vdots & \vdots & \vdots \\
 & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}$$

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Invariance under scaling

Each vector can be independently scaled without changing the rank of the matrix.
Assume v_1 is incident to $\{v_2, \ldots, v_k\}$

$$
\begin{array}{c|ccccc}
 & 1 & 2 & & k & \\
(1,2) & (1/\alpha)\mathbf{r}_2 & \mathbf{r}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
(1,3) & (1/\alpha)\mathbf{r}_3 & \mathbf{0} & \mathbf{r}_1 & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
(1,k) & (1/\alpha)\mathbf{r}_k & \mathbf{0} & \ldots & \mathbf{r}_1 & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
1 & \alpha\mathbf{r}_1 & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
 & \mathbf{0} & \vdots & \vdots & \vdots & \vdots \\
 & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}
$$

Invariance under scaling

Each vector can be independently scaled without changing the rank of the matrix.
Assume v_1 is incident to $\{v_2, \dots, v_k\}$

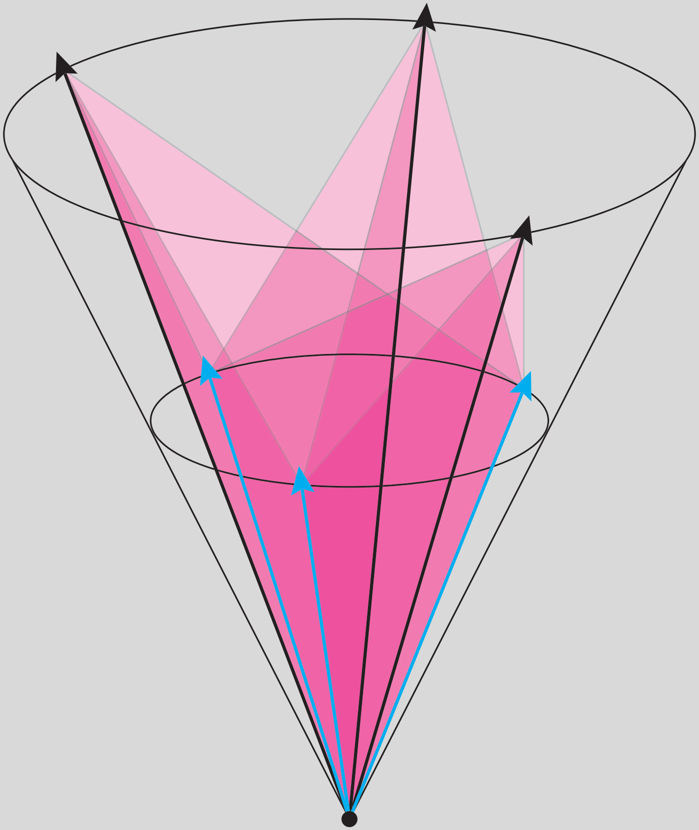
$$
\begin{array}{c|cccccc}
 & 1 & 2 & & k & \\
(1,2) & \mathbf{r}_2 & \mathbf{r}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
(1,3) & \mathbf{r}_3 & \mathbf{0} & \mathbf{r}_1 & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
(1,k) & \mathbf{r}_k & \mathbf{0} & \dots & \mathbf{r}_1 & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
1 & \alpha^2\mathbf{r}_1 & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
 & \mathbf{0} & \vdots & \vdots & \vdots & \vdots \\
 & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}
$$

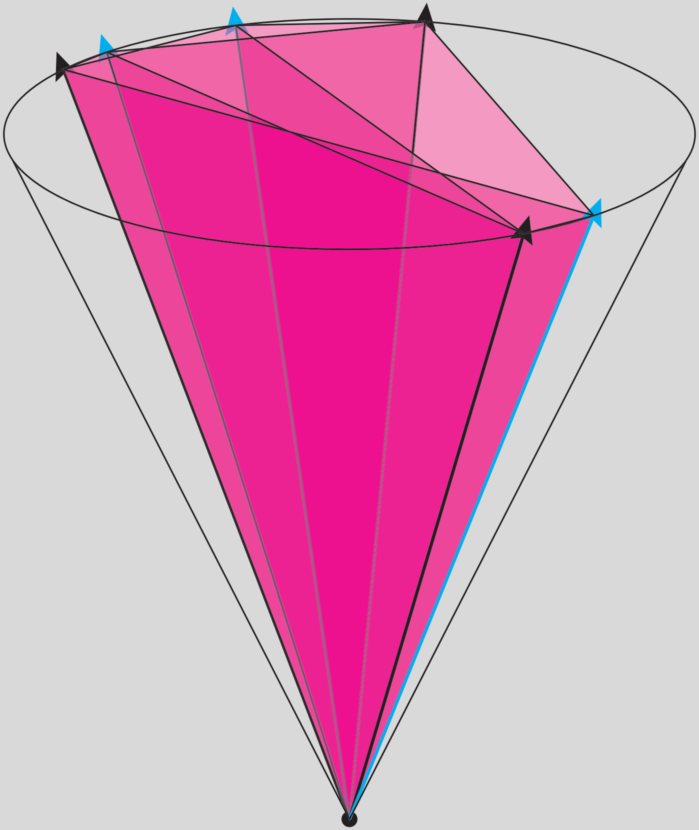
Invariance under scaling

Each vector can be independently scaled without changing the rank of the matrix.

Assume v_1 is incident to $\{v_2, \ldots, v_k\}$

$$\begin{array}{c|ccccc}
 & 1 & 2 & & k & \\
(1,2) & \mathbf{r}_2 & \mathbf{r}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
(1,3) & \mathbf{r}_3 & \mathbf{0} & \mathbf{r}_1 & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
(1,k) & \mathbf{r}_k & \mathbf{0} & \ldots & \mathbf{r}_1 & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
1 & \mathbf{r}_1 & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
 & \mathbf{0} & \vdots & \vdots & \vdots & \vdots \\
 & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}$$





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- Scale each vector so that the third coordinate is 1.

$$\mathbf{r}_i = \mathbf{p}_i + \mathbf{k}$$

$$\begin{array}{c|ccccc}
 & & i & & j & \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
(i,j) & \mathbf{0}\cdots\mathbf{0} & \mathbf{p}_j+\mathbf{k} & \mathbf{0}\cdots\mathbf{0} & \mathbf{p}_i+\mathbf{k} & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
i & \mathbf{0}\cdots\mathbf{0} & \mathbf{p}_i+\mathbf{k} & \mathbf{0}\cdots\mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
j & \mathbf{0}\cdots\mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} & \mathbf{p}_j+\mathbf{k} & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}$$

- Scale each vector so that the third coordinate is 1.

$$\mathbf{r}_i = \mathbf{p}_i + \mathbf{k}$$

- The obvious row operations

$$\begin{array}{c|ccccc}
 & & i & & j & \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
(i,j) & \mathbf{0}\cdots\mathbf{0} & (\mathbf{p}_j - \mathbf{p}_i, 0) & \mathbf{0}\cdots\mathbf{0} & (\mathbf{p}_i - \mathbf{p}_j, 0) & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
i & \mathbf{0}\cdots\mathbf{0} & \mathbf{p}_i + \mathbf{k} & \mathbf{0}\cdots\mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
j & \mathbf{0}\cdots\mathbf{0} & \mathbf{0} & \mathbf{0}\cdots\mathbf{0} & \mathbf{p}_j + \mathbf{k} & \mathbf{0}\cdots\mathbf{0} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}$$

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- Scale each vector so that the third coordinate is 1.

$$\mathbf{r}_i = \mathbf{p}_i + \mathbf{k}$$

- The obvious row operations
- The obvious column operations

$$\begin{array}{cc}
 & \begin{array}{ccccc} \phantom{\mathbf{0}\cdots\mathbf{0}} & i & \phantom{\mathbf{0}\cdots\mathbf{0}} & j & \phantom{\mathbf{0}\cdots\mathbf{0}} \end{array} \\
(i,j) & \left[\begin{array}{ccccc|c}
\vdots & \vdots & \vdots & \vdots & \vdots & \mathbf{0} \\
\mathbf{0}\cdots\mathbf{0} & \mathbf{p}_j - \mathbf{p}_i & \mathbf{0}\cdots\mathbf{0} & \mathbf{p}_i - \mathbf{p}_j & \mathbf{0}\cdots\mathbf{0} & \mathbf{0} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \mathbf{0} \\
\hline
 & \vdots & \mathbf{0} & \vdots & & \mathbf{I}
\end{array}\right]
\end{array}$$

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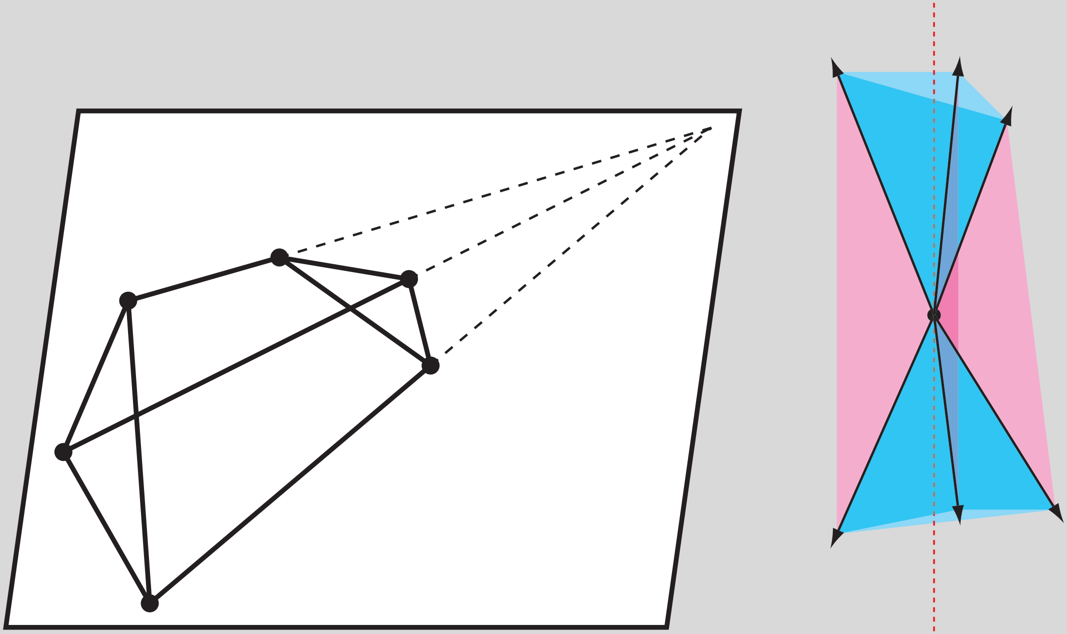
2. Bars

Independence of a configuration 3D position vectors is equivalent to the independence the the configuration as a 2D bar and joint framework.



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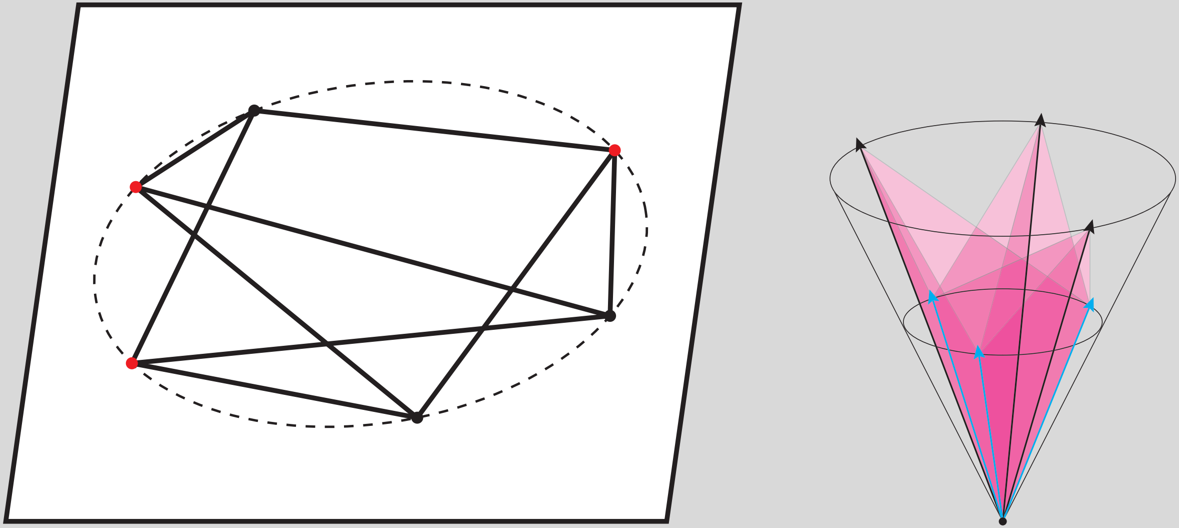
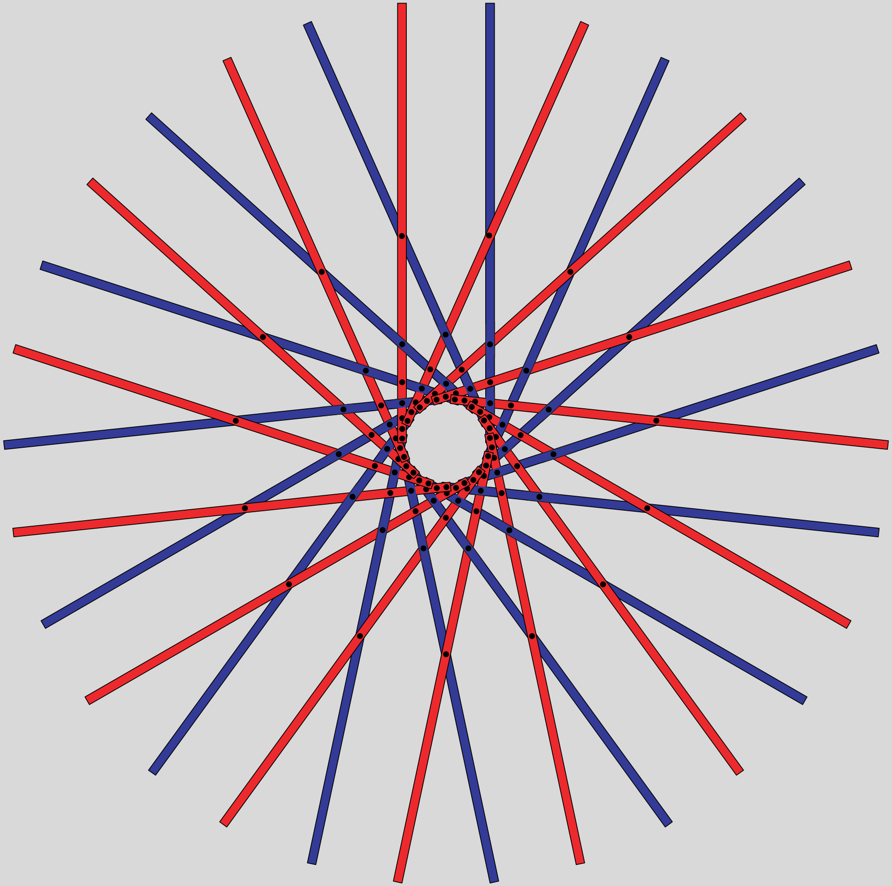
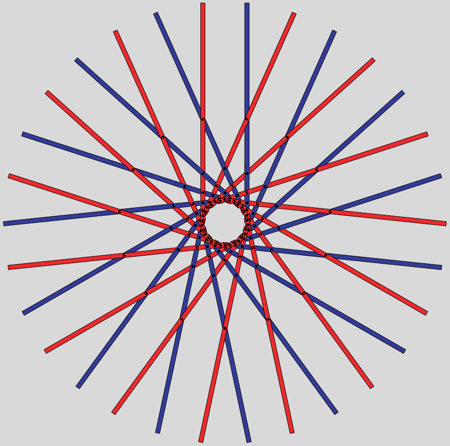


3. Beams, Contacts, and Twists

- Framework of beams, some of which have contact points.

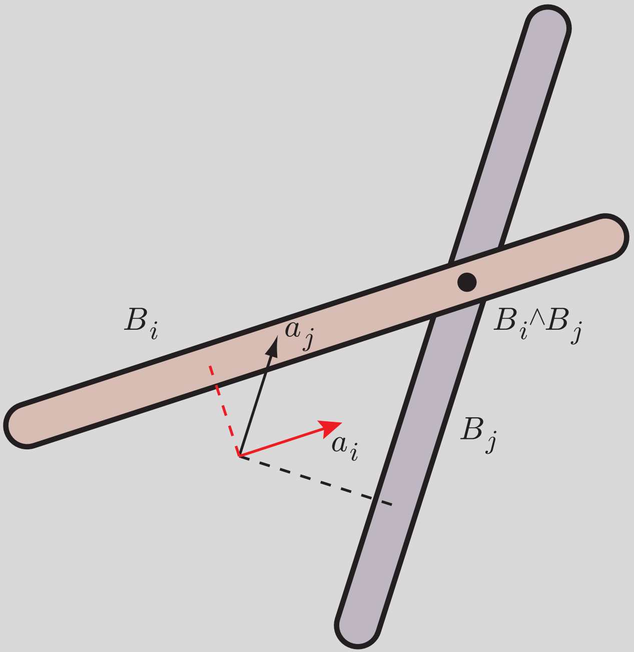


- Framework of beams, some of which have contact points.
- Motion:
  - Twists the beams out of the plane, into space.
  - Preserves the contact points
  - Preserves the projection onto the xy-plane.
- Animation: Infinitesimal Motion:
  - Show - A twist out of square
  - Show - Bird's eye view
  - Show - Edge view



- Infinitesimal Beam twists: $\{\mathbf{a}_1, \ldots, \mathbf{a}_b\}$
- Compatibility Condition

$$\mathbf{a}_i \cdot (B_i \wedge B_j) = \mathbf{a}_i \cdot (B_i \wedge B_j)$$



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Infinitesimal System

$$\begin{array}{cc} & \begin{array}{ccccc} \phantom{\mathbf{0}\cdots\mathbf{0}} & i & \phantom{\mathbf{0}\cdots\mathbf{0}} & j & \phantom{\mathbf{0}\cdots\mathbf{0}} \end{array} \\ (i,j) & \left[\begin{array}{ccccc} \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}\cdots\mathbf{0} & \mathbf{B}_i \wedge \mathbf{B}_j & \mathbf{0}\cdots\mathbf{0} & -\mathbf{B}_j \wedge \mathbf{B}_i & \mathbf{0}\cdots\mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{array}\right] \end{array} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_n \end{bmatrix}$$

$B_i \wedge B_j$

- $\mathbf{p}_i \cdot \mathbf{x} = 1$

$$\begin{array}{cc}
 & \begin{array}{ccccc} \phantom{\mathbf{0}\cdots\mathbf{0}} & i & \phantom{\mathbf{0}\cdots\mathbf{0}} & j & \phantom{\mathbf{0}\cdots\mathbf{0}} \end{array} \\
(i,j) & \left[\begin{array}{ccccc}
\vdots & \vdots & \vdots & \vdots & \vdots \\
\mathbf{0}\cdots\mathbf{0} & \mathbf{B}_i \wedge \mathbf{B}_j & \mathbf{0}\cdots\mathbf{0} & -\mathbf{B}_j \wedge \mathbf{B}_i & \mathbf{0}\cdots\mathbf{0} \\
\vdots & \vdots & \vdots & \vdots & \vdots
\end{array}\right]
\end{array}$$

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$B_i \wedge B_j$

- $\mathbf{p}_i \cdot \mathbf{x} = 1$
- $(\mathbf{p}_i - \mathbf{k}) \cdot (\mathbf{x} + \mathbf{k}) = \mathbf{0}$

$$\begin{array}{cc} & \begin{array}{ccccc} \phantom{\mathbf{0}\cdots\mathbf{0}} & i & \phantom{\mathbf{0}\cdots\mathbf{0}} & j & \phantom{\mathbf{0}\cdots\mathbf{0}} \end{array} \\ (i,j) & \left[\begin{array}{ccccc} \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}\cdots\mathbf{0} & \mathbf{B}_i \wedge \mathbf{B}_j & \mathbf{0}\cdots\mathbf{0} & -\mathbf{B}_j \wedge \mathbf{B}_i & \mathbf{0}\cdots\mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{array}\right] \end{array}$$

$B_i \wedge B_j$

- $\mathbf{p}_i \cdot \mathbf{x} = 1$
- $(\mathbf{p}_i - \mathbf{k}) \cdot (\mathbf{x} + \mathbf{k}) = \mathbf{0}$
- $$\begin{aligned} &(\mathbf{p}_j - \mathbf{k}) \times (\mathbf{p}_i - \mathbf{k}) = \mathbf{p}_j \times \mathbf{p}_i - \mathbf{k} \times \mathbf{p}_i - \mathbf{p}_j \times \mathbf{k} \\ &= \mathbf{p}_j \times \mathbf{p}_i + \mathbf{k} \times (\mathbf{p}_j - \mathbf{p}_i) \\ &\sim \mathbf{k} + \frac{\mathbf{k} \times (\mathbf{p}_j - \mathbf{p}_i)}{(\mathbf{p}_j \times \mathbf{p}_i) \cdot \mathbf{k}} \end{aligned}$$

$$\begin{array}{cccccc} & & i & & j & \\ & \vdots & \vdots & \vdots & \vdots & \vdots \\ (i,j) & \mathbf{0}\cdots\mathbf{0} & \mathbf{B}_i \wedge \mathbf{B}_j & \mathbf{0}\cdots\mathbf{0} & -\mathbf{B}_j \wedge \mathbf{B}_i & \mathbf{0}\cdots\mathbf{0} \\ & \vdots & \vdots & \vdots & \vdots & \vdots \end{array}$$

$B_i \wedge B_j$

- $\mathbf{p}_i \cdot \mathbf{x} = 1$
- $(\mathbf{p}_i - \mathbf{k}) \cdot (\mathbf{x} + \mathbf{k}) = \mathbf{0}$
- $(\mathbf{p}_j - \mathbf{k}) \times (\mathbf{p}_i - \mathbf{k}) = \mathbf{p}_j \times \mathbf{p}_i - \mathbf{k} \times \mathbf{p}_i - \mathbf{p}_j \times \mathbf{k}$
  $= \mathbf{p}_j \times \mathbf{p}_i + \mathbf{k} \times (\mathbf{p}_j - \mathbf{p}_i)$
  $\sim \mathbf{k} + \dfrac{\mathbf{k} \times (\mathbf{p}_j - \mathbf{p}_i)}{(\mathbf{p}_j \times \mathbf{p}_i) \cdot \mathbf{k}}$

$$\begin{array}{cc} & \begin{array}{ccccc} \phantom{\mathbf{0}\cdots\mathbf{0}} & i & \phantom{\mathbf{0}\cdots\mathbf{0}} & j & \phantom{\mathbf{0}\cdots\mathbf{0}} \end{array} \\ (i,j) & \left[\begin{array}{ccccc} \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}\cdots\mathbf{0} & \dfrac{\mathbf{k} \times (\mathbf{p}_j - \mathbf{p}_i)}{(\mathbf{p}_j \times \mathbf{p}_i) \cdot \mathbf{k}} & \mathbf{0}\cdots\mathbf{0} & -\dfrac{\mathbf{k} \times (\mathbf{p}_j - \mathbf{p}_i)}{(\mathbf{p}_j \times \mathbf{p}_i) \cdot \mathbf{k}} & \mathbf{0}\cdots\mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{array}\right] \end{array}$$

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$B_i \wedge B_j$

- $\mathbf{p}_i \cdot \mathbf{x} = 1$
- $(\mathbf{p}_i - \mathbf{k}) \cdot (\mathbf{x} + \mathbf{k}) = 1$
- $(\mathbf{p}_i - \mathbf{k}) \times (\mathbf{p}_j - \mathbf{k}) = \mathbf{p}_i \times \mathbf{p}_j - \mathbf{k} \times \mathbf{p}_j - \mathbf{p}_i \times \mathbf{k}$
  $= \mathbf{p}_i \times \mathbf{p}_j + \mathbf{k} \times (\mathbf{p}_i - \mathbf{p}_j)$
  $\sim \mathbf{k} + \dfrac{\mathbf{k} \times (\mathbf{p}_i - \mathbf{p}_j)}{(\mathbf{p}_i \times \mathbf{p}_j) \cdot \mathbf{k}}$

$$\begin{array}{cc}
 & \begin{array}{ccccc} & i & & j & \end{array} \\
(i,j) & \left[\begin{array}{ccccc}
\vdots & \vdots & \vdots & \vdots & \vdots \\
\mathbf{0}\cdots\mathbf{0} & \mathbf{k} \times (\mathbf{p}_j - \mathbf{p}_i) & \mathbf{0}\cdots\mathbf{0} & -\mathbf{k} \times (\mathbf{p}_j - \mathbf{p}_i) & \mathbf{0}\cdots\mathbf{0} \\
\vdots & \vdots & \vdots & \vdots & \vdots
\end{array}\right]
\end{array}$$

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$B_i \wedge B_j$

- $\mathbf{p}_i \cdot \mathbf{x} = 1$
- $(\mathbf{p}_i - \mathbf{k}) \cdot (\mathbf{x} + \mathbf{k}) = 1$
- $(\mathbf{p}_i - \mathbf{k}) \times (\mathbf{p}_j - \mathbf{k}) = \mathbf{p}_i \times \mathbf{p}_j - \mathbf{k} \times \mathbf{p}_j - \mathbf{p}_i \times \mathbf{k}$
  $= \mathbf{p}_i \times \mathbf{p}_j + \mathbf{k} \times (\mathbf{p}_i - \mathbf{p}_j)$
  $\sim \mathbf{k} + \dfrac{\mathbf{k} \times (\mathbf{p}_i - \mathbf{p}_j)}{(\mathbf{p}_i \times \mathbf{p}_j) \cdot \mathbf{k}}$

$$\begin{array}{cc} & \begin{array}{ccccc} \phantom{\mathbf{0}\cdots\mathbf{0}} & i & \phantom{\mathbf{0}\cdots\mathbf{0}} & j & \phantom{\mathbf{0}\cdots\mathbf{0}} \end{array} \\ (i,j) & \left[\begin{array}{ccccc} \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}\cdots\mathbf{0} & (\mathbf{p}_j - \mathbf{p}_i) & \mathbf{0}\cdots\mathbf{0} & (\mathbf{p}_i - \mathbf{p}_j) & \mathbf{0}\cdots\mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{array}\right] \end{array}$$

Conclusion

Independence for position vector system $\{\mathbf{r}_i\}$ in $\mathbb{R}^3$ is is equivalent to the independence in the corresponding bar and joint framework, or its polar beam and twist framework.

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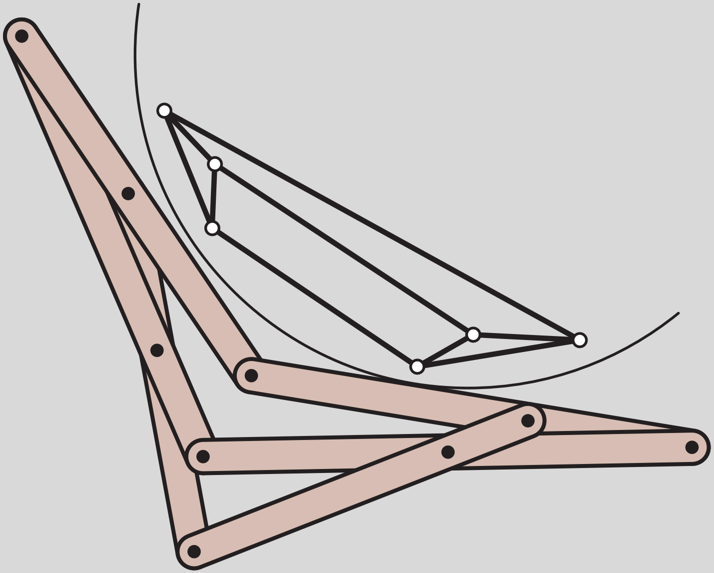

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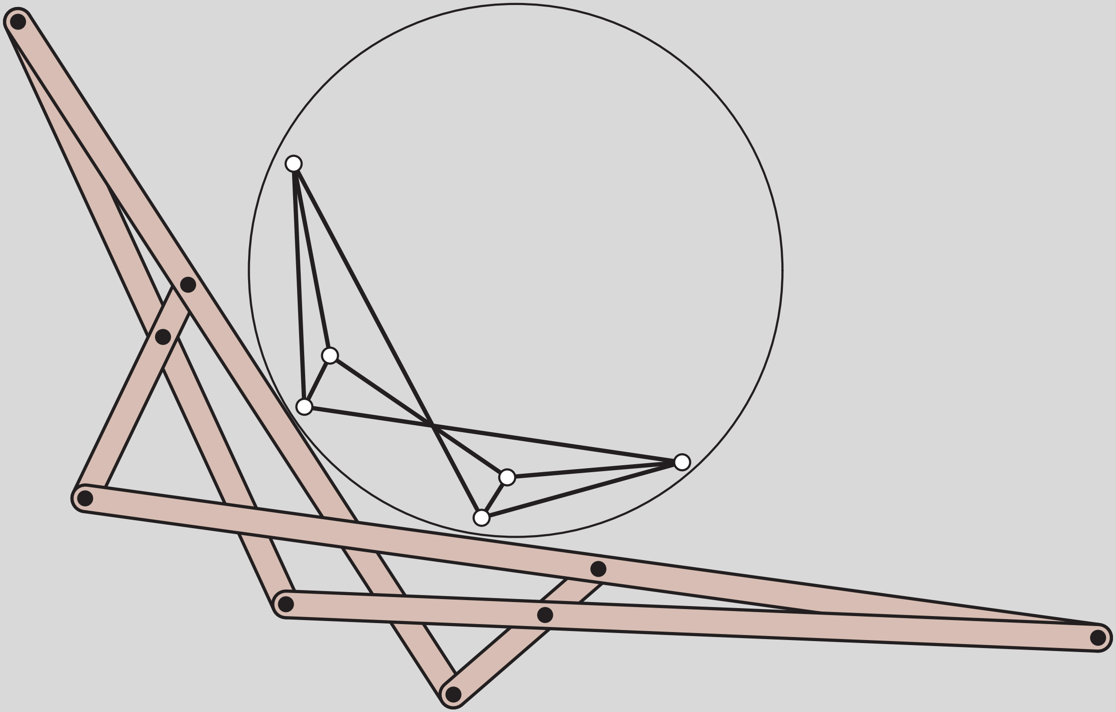

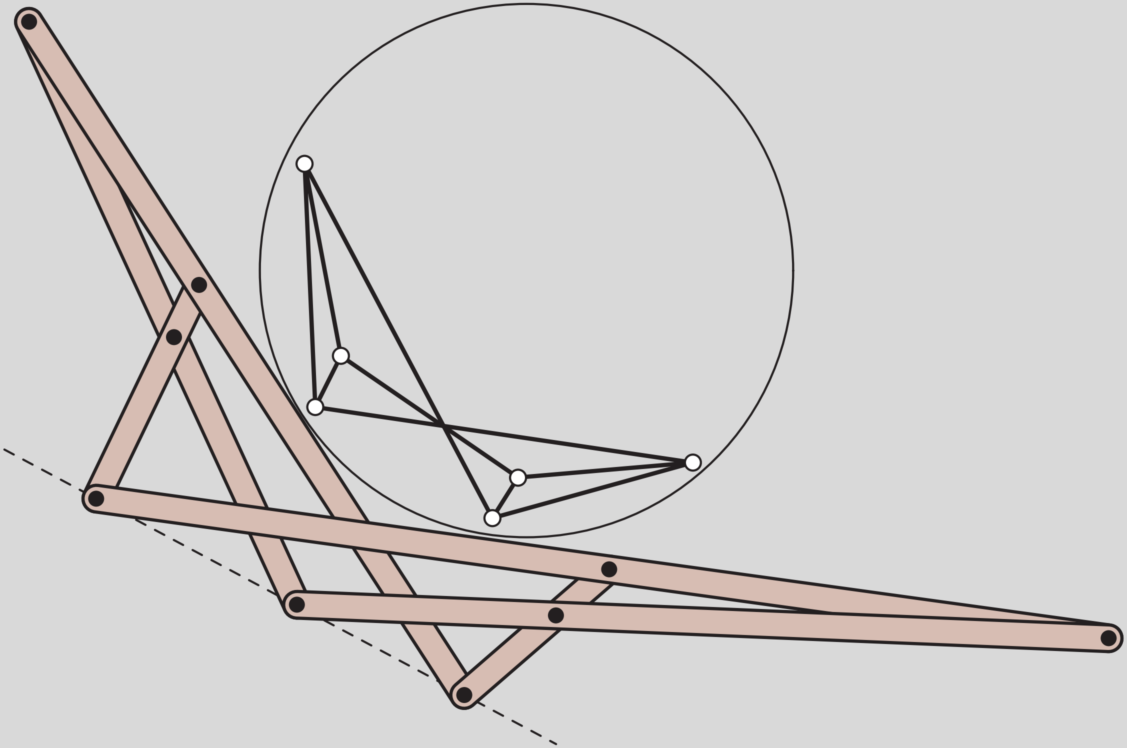

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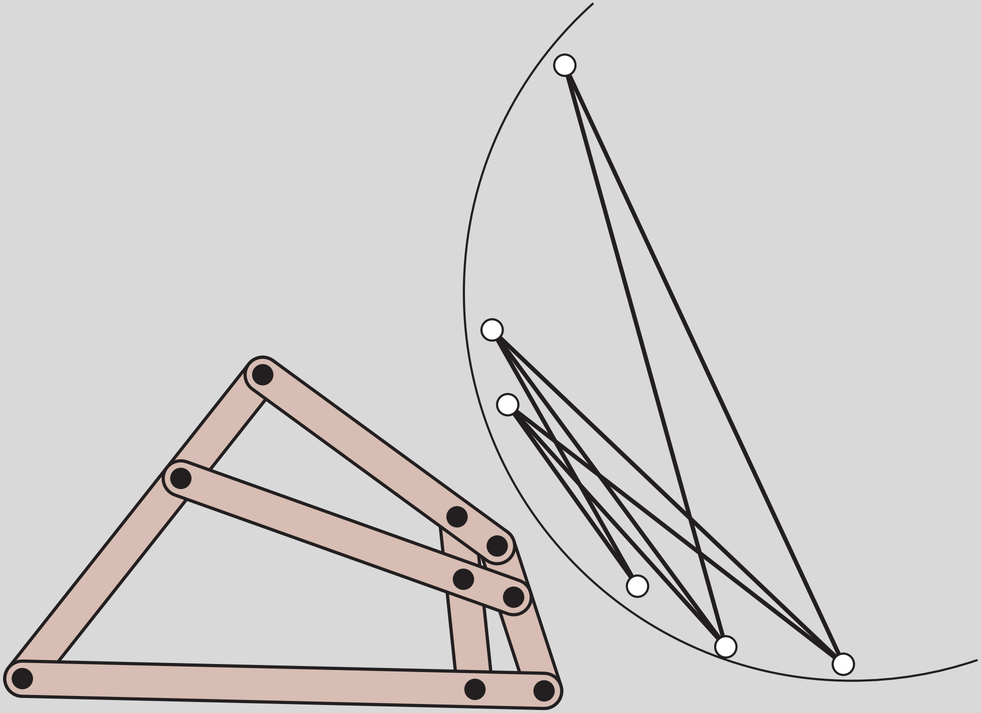

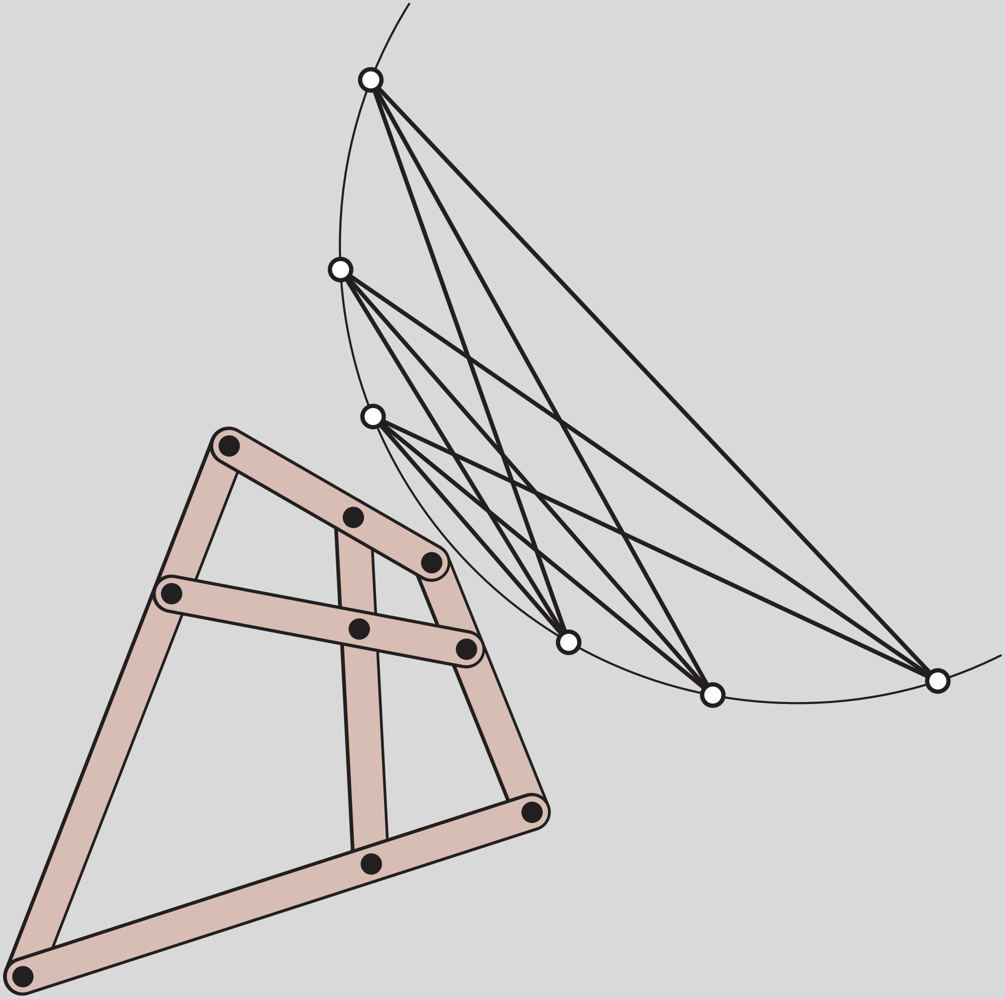

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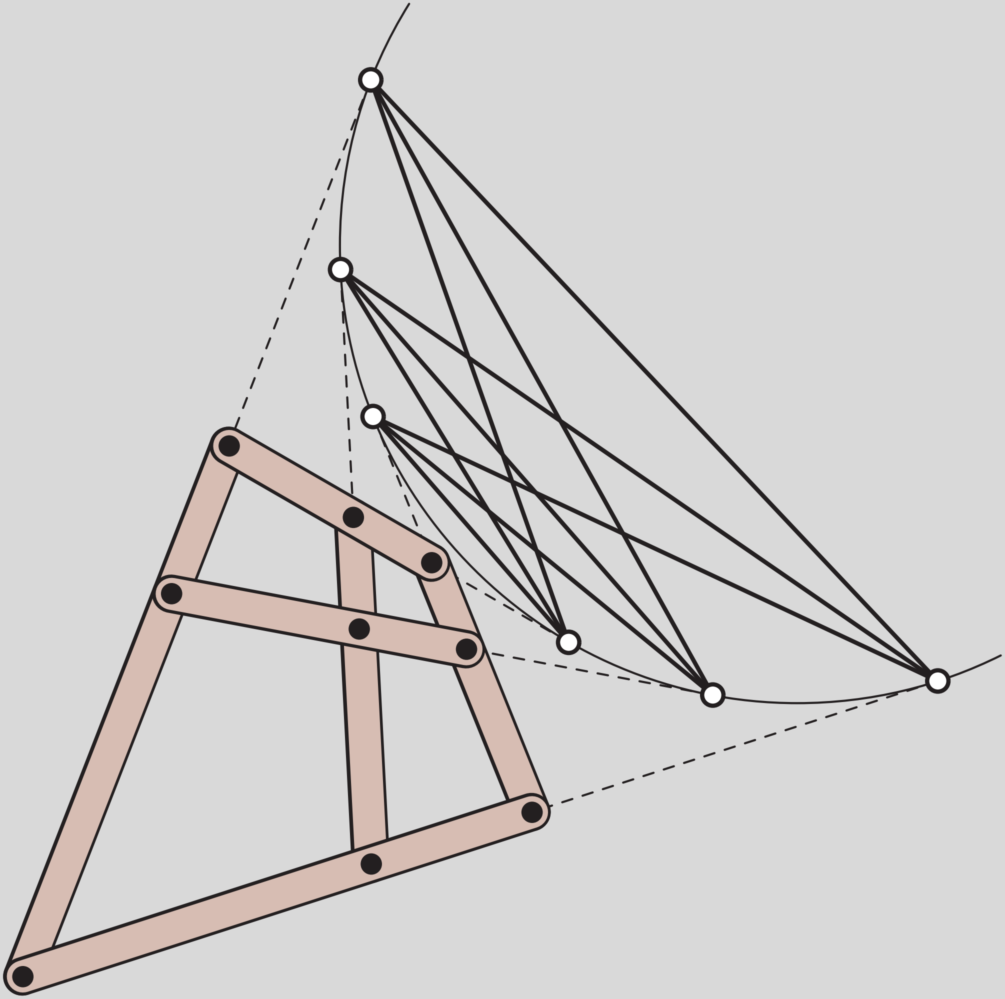

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4. Generic Rigidity

- Laman's Condition
  - $|E| = 2|V(E)| - 3,$
  - $|F| \leq |V(F)| - 3$ for $F \subseteq E$
- Henneberg Moves
- Pebble Game
- Tree Decompositions

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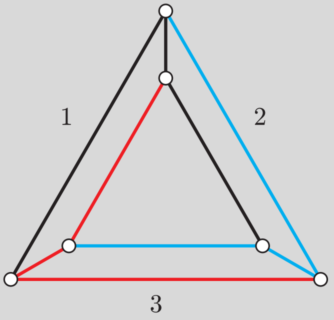
Direct Proof of Sufficiency

Suppose $G = (V, E)$ satisfies Crapo's Proper 3T2 Condition:

- E is the union of 3 trees
- Every vertices belongs to exactly two of them.
- No two subtrees have the same span (proper).

V_{ij} – vertices incident to trees i and j.

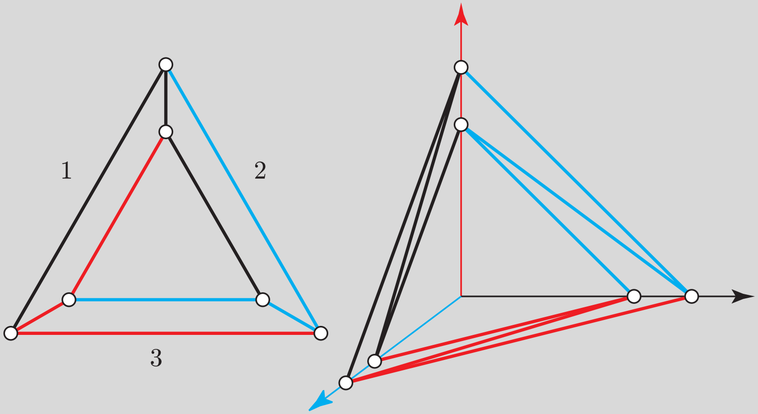


Tripartite Case:

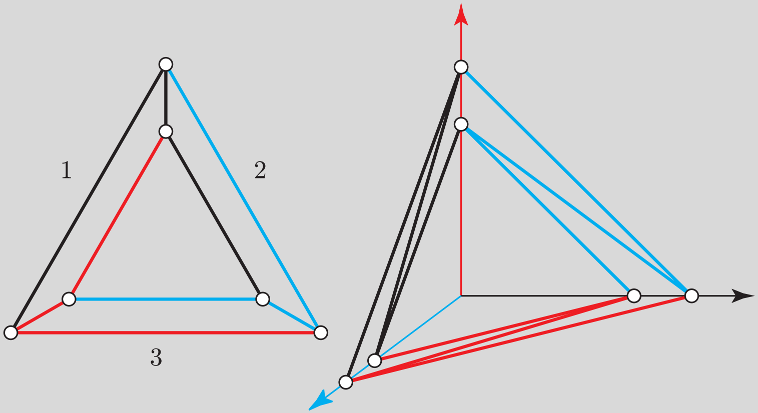
No edges interior to any of the sets V_{ij}.

Map:

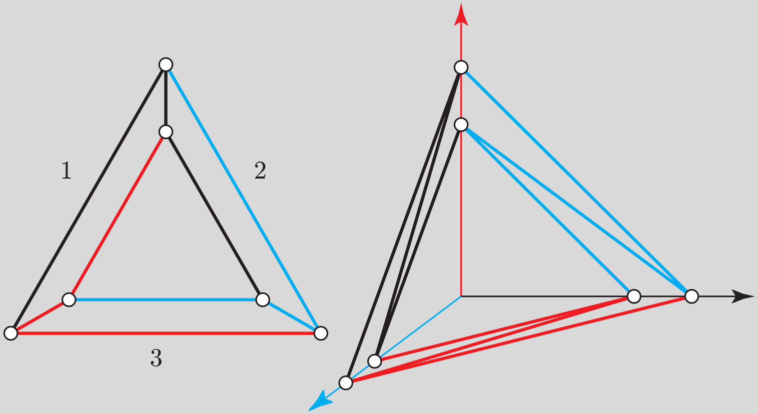
- $V_{23} \longrightarrow$ map to $\mathbf{e}_1$.
- $V_{13} \longrightarrow$ map to $\mathbf{e}_2$.
- $V_{12} \longrightarrow$ map to $\mathbf{e}_3$.



$$\begin{array}{c|ccc}
 & V_{12} & V_{23} & V_{13} \\
T_1 & \mathbf{e}_2 & 0 & \mathbf{e}_3 \\
 & & \vdots & \\
T_2 & \mathbf{e}_1 & \mathbf{e}_3 & 0 \\
 & & \vdots & \\
T_3 & 0 & \mathbf{e}_2 & \mathbf{e}_1 \\
 & & \vdots & \\
\hline
V_{12} & \mathbf{e}_3 & 0 & 0 \\
 & & \vdots & \\
V_{23} & 0 & \mathbf{e}_1 & 0 \\
 & & \vdots & \\
V_{13} & 0 & 0 & \mathbf{e}_2
\end{array}$$



$$\begin{array}{c|cc|cc|cc|ccc|}
 & V_{12,y} & V_{13,z} & V_{12,x} & V_{23,z} & V_{13,x} & V_{23,y} & V_{12,z} & V_{23,x} & V_{13,y} \\
T_1 & \multicolumn{2}{c}{A_1} & \multicolumn{2}{c}{0} & \multicolumn{2}{c}{0} & \multicolumn{3}{c}{0} \\
T_2 & \multicolumn{2}{c}{0} & \multicolumn{2}{c}{A_2} & \multicolumn{2}{c}{0} & \multicolumn{3}{c}{0} \\
T_3 & \multicolumn{2}{c}{0} & \multicolumn{2}{c}{0} & \multicolumn{2}{c}{A_3} & \multicolumn{3}{c}{0} \\
\left\{\begin{matrix} V_{12} \\ V_{23} \\ V_{13} \end{matrix}\right. & \multicolumn{2}{c}{0} & \multicolumn{2}{c}{0} & \multicolumn{2}{c}{0} & \multicolumn{3}{c}{I}
\end{array}$$



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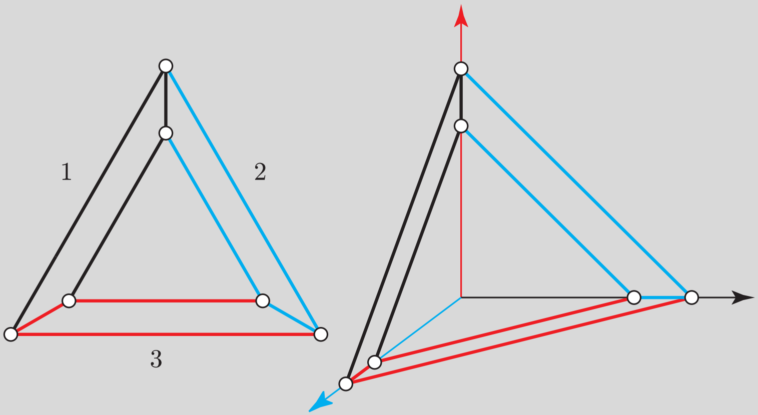
Non-tripartite case

$$\begin{array}{c|ccccccc}
 & & V_{12} & & V_{23} & & V_{13} & \\
T_1 & & \mathbf{e}_2 & & 0 & & \mathbf{e}_3 & \\
(i,j) & \mathbf{e}_3 & & \mathbf{e}_3 & 0 & & & \\
 & & & & 0 & \mathbf{e}_2 & & \mathbf{e}_2 \\
 & & & & \vdots & & & \\
\hline
i & & & \mathbf{e}_3 & 0 & & 0 & \\
j & \mathbf{e}_3 & & & 0 & & 0 & \\
 & & & & \vdots & & & \\
V_{13} & & 0 & & 0 & & \mathbf{e}_2 &
\end{array}$$

- Replace the rows for interior edges with difference vectors



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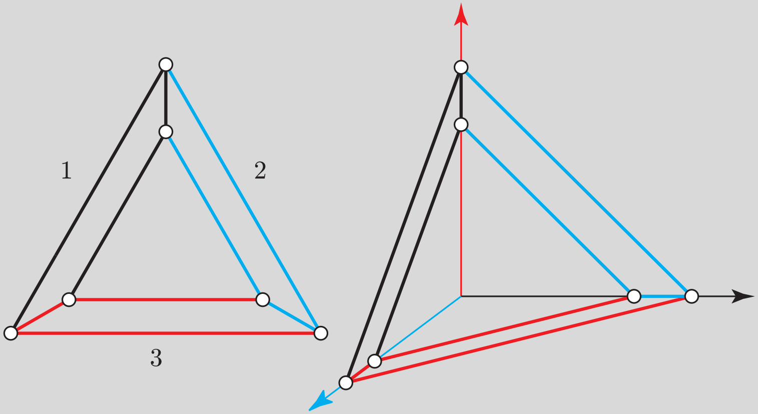
$$\begin{array}{c|ccccccc}
 & & V_{12} & & V_{23} & & V_{13} & \\
T_1 & & \mathbf{e}_2 & & 0 & & \mathbf{e}_3 & \\
(i,j) & \mathbf{e}_3-\mathbf{e}_3 & & \mathbf{e}_3-\mathbf{e}_3 & 0 & & & \\
 & & & & 0 & \mathbf{e}_2 & & \mathbf{e}_2 \\
 & & & & \vdots & & & \\
\hline
i & & & \mathbf{e}_3 & 0 & & 0 & \\
j & \mathbf{e}_3 & & & 0 & & 0 & \\
 & & & & \vdots & & & \\
V_{13} & & 0 & & 0 & & \mathbf{e}_2 &
\end{array}$$

- Replace the rows for interior edges with difference vectors



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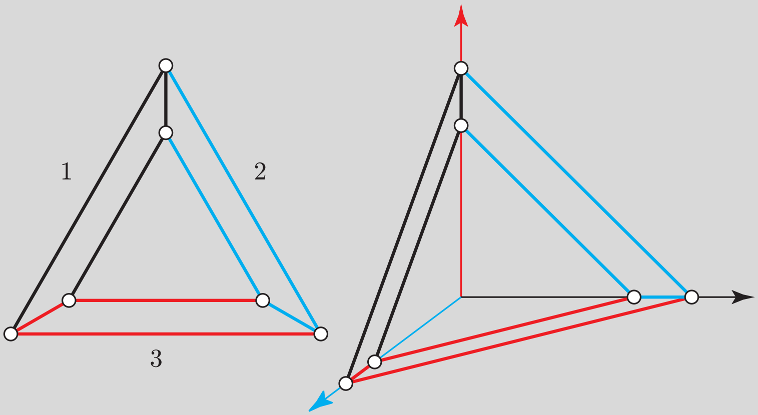
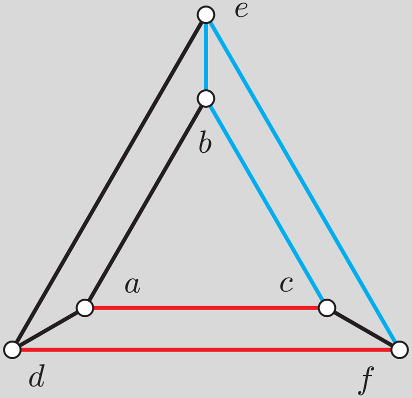
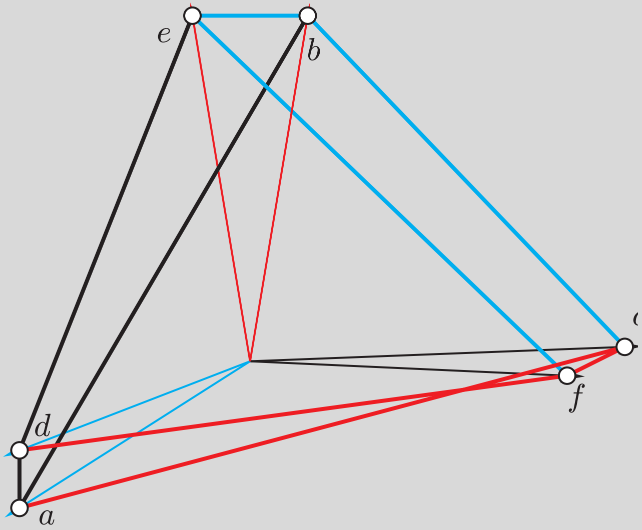
$$\begin{array}{c|cccccccc}
 & & V_{12} & & V_{23} & & V_{13} & \\
T_1 & & \mathbf{e}_2 & & 0 & & \mathbf{e}_3 & \\
(i,j) & -\epsilon\mathbf{e}_2 & & +\epsilon\mathbf{e}_2 & 0 & & & \\
 & & & & 0 & \mathbf{e}_2 & & \mathbf{e}_2 \\
 & & & & \vdots & & & \\
\hline
i & & & \mathbf{e}_3+\epsilon\mathbf{e}_2 & 0 & & 0 & \\
j & \mathbf{e}_3-\epsilon\mathbf{e}_2 & & & 0 & & 0 & \\
 & & & & \vdots & & & \\
V_{13} & & 0 & & 0 & & \mathbf{e}_2 &
\end{array}$$

- Replace the rows for interior edges with difference vectors
- But first, perturb



The exterior edges are now mis-aligned





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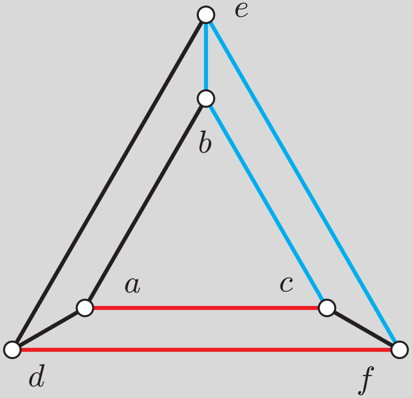
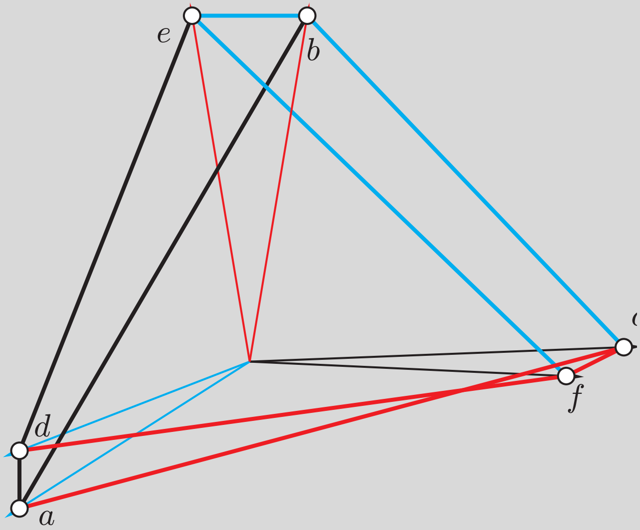
The exterior edges are now mis-aligned

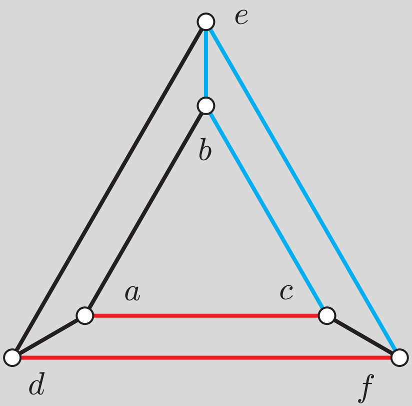
- Normalize the rows corresponding to interior edges.
- Let perturbation go to zero.

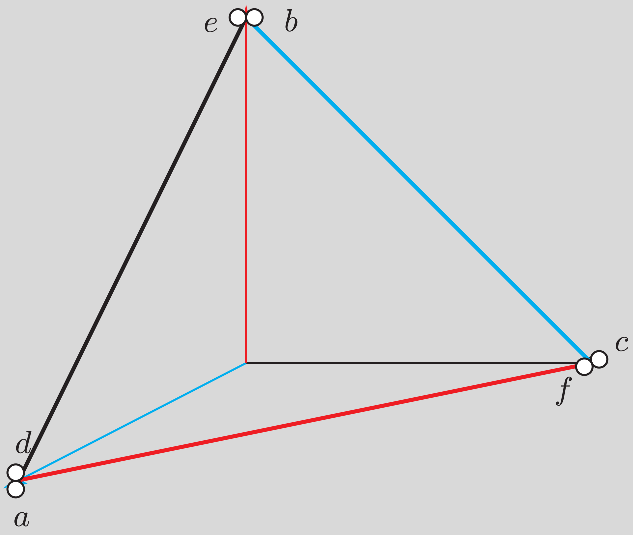




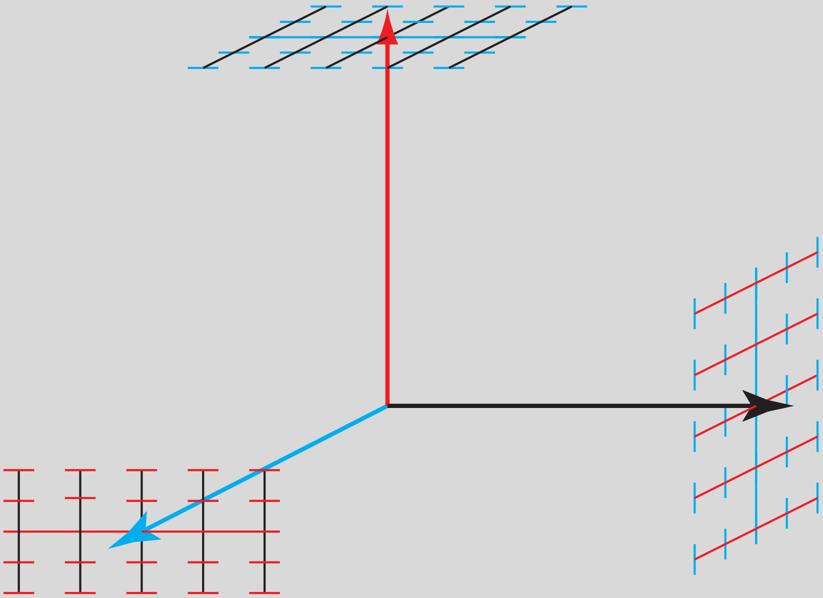
The exterior edges are now mis-aligned

- Normalize the rows corresponding to interior edges.
- Let perturbation go to zero.

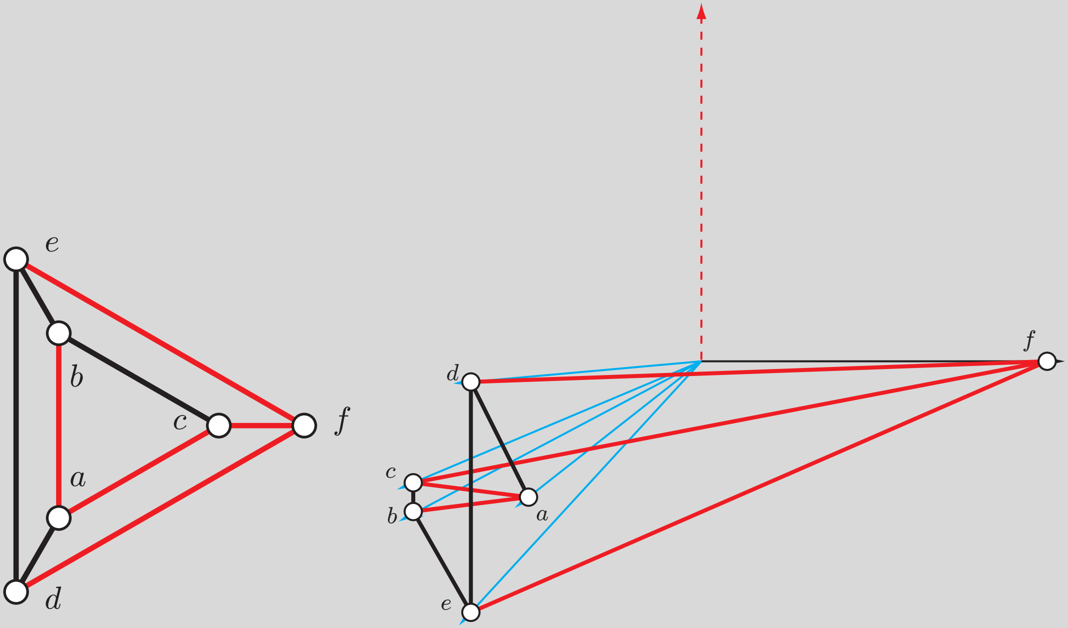




In general, there will be several levels of perturbation

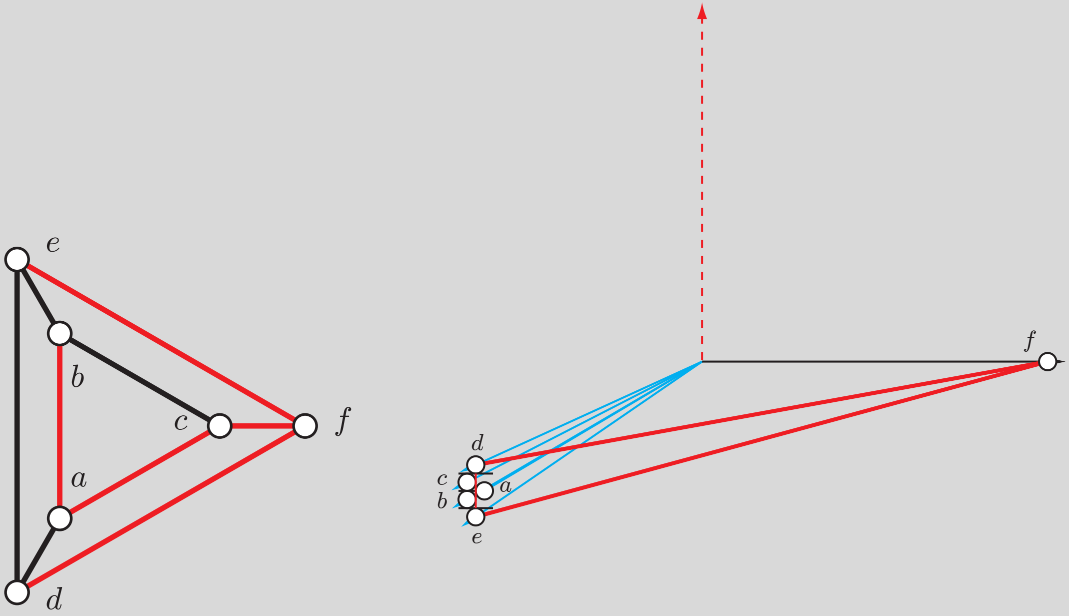


(Note that one of the three sets may be empty:)



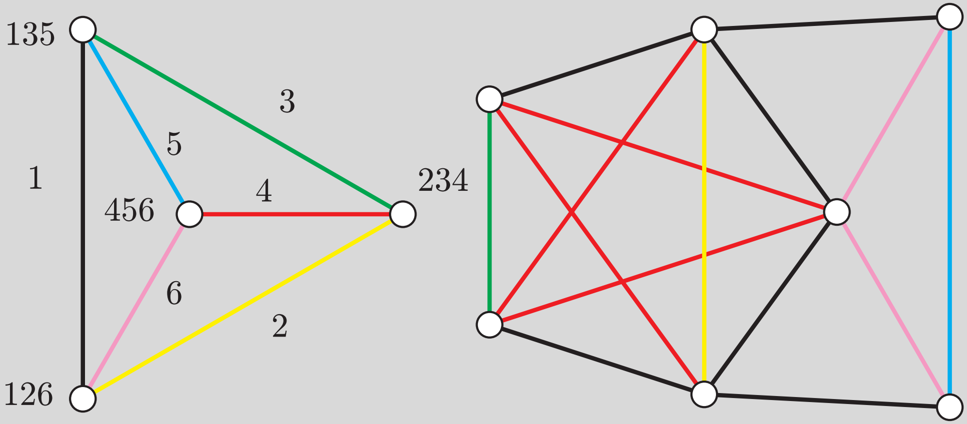
(Note that one of the three sets may be empty:)

The interior edges of a Proper 3T2 decomposition are *combable*.



5. Dimension 3

- $3|V| - 6$
- Laman's Condition is not Sufficient
- Crapo's 6T3:
  - Six trees,
  - Every vertex belongs to 3 of them.
  - The decomposition is "proper".
  - $\binom{6}{3} = 20$ possibilities for vertices.



- $3|V| - 6$
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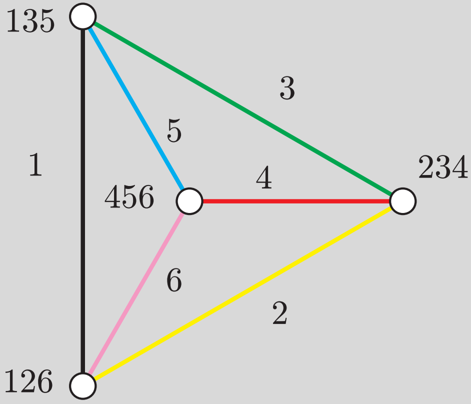

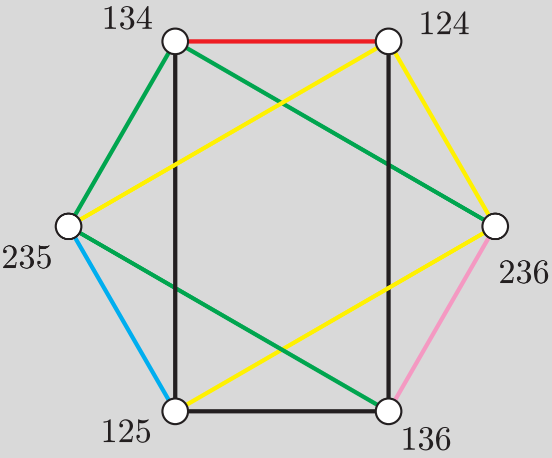

- $3|V| - 6$
- Laman's Condition is not Sufficient
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  - Six trees,
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  - The decomposition is "proper".
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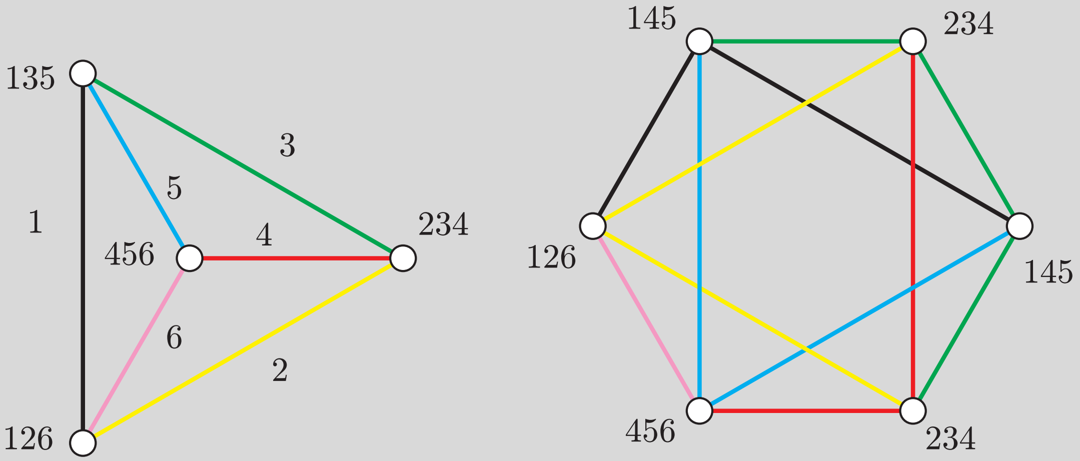

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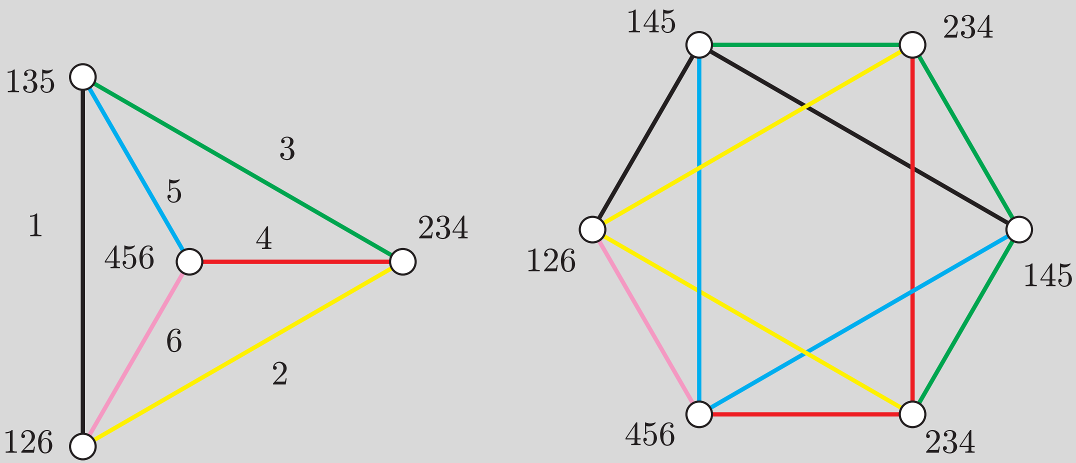
If

- G has a Proper 6T3 decomposition
- G and has only 4 vertex color classes
-
-

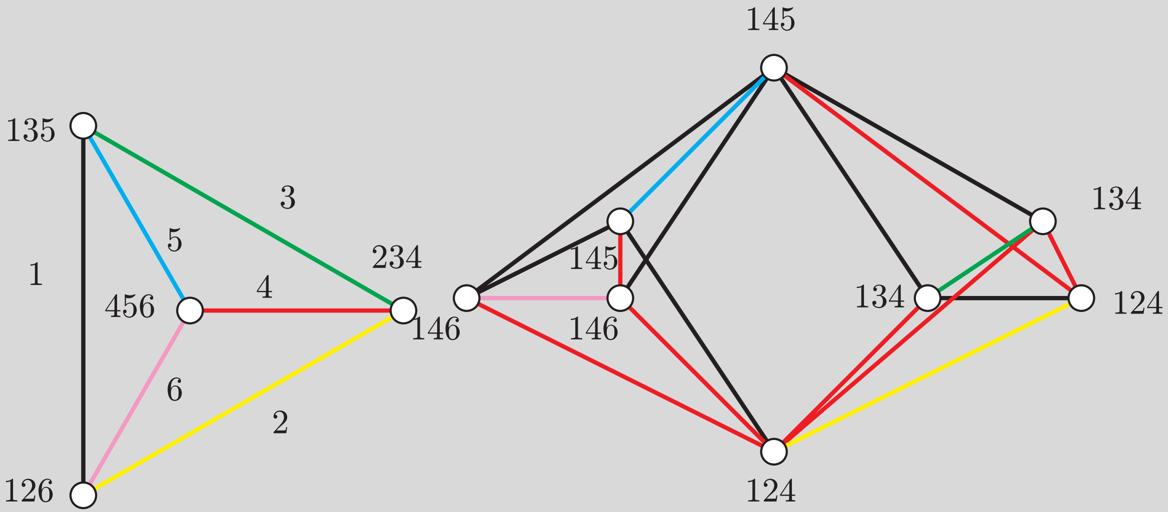
G is generically rigid as a configuration of position vectors in $\mathbb{R}^4$.



If

- G has a Proper 6T3 decomposition
- G and has only 4 vertex color classes
-
-

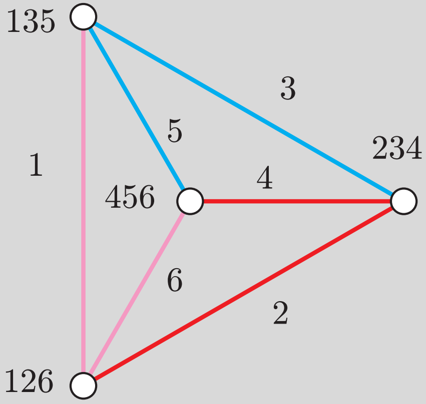
G is generically rigid as a configuration of position vectors in $\mathbb{R}^4$.

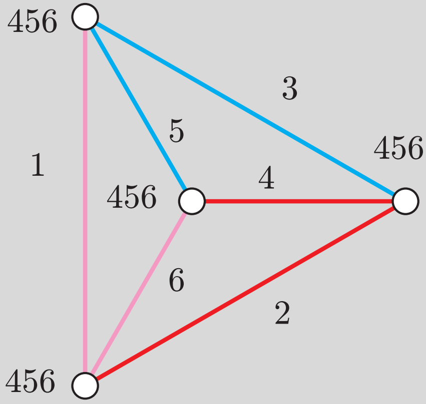


If

- G has a Proper 6T3 decomposition
- G and has only 4 vertex color classes
- The color classes intersect in at most one color
-

G is generically rigid as a configuration of position vectors in $\mathbb{R}^4$.

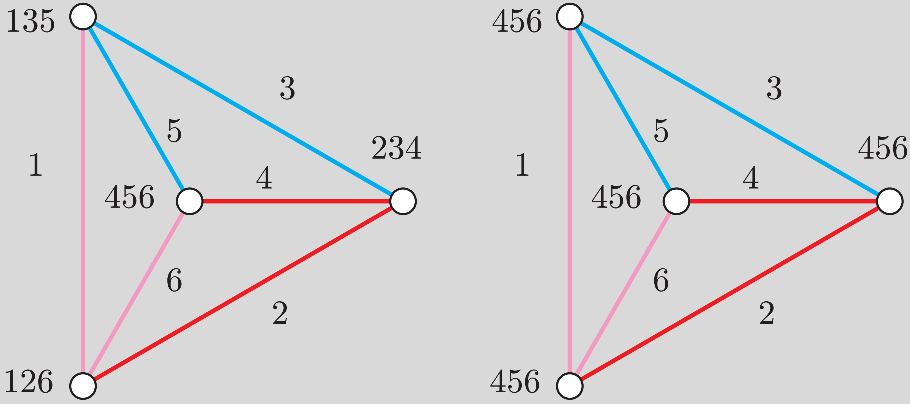




If

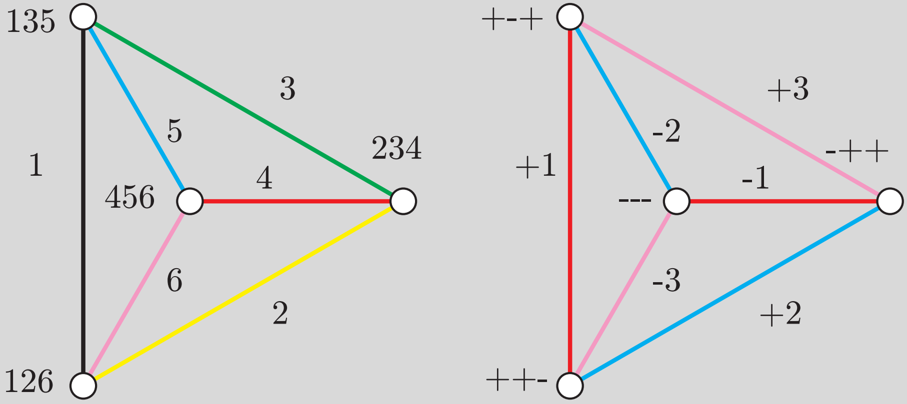
- G has a Proper 6T3 decomposition
- G and has only 4 vertex color classes
- The color classes intersect in at most one color
- The interior edges to the color classes are "combable".

Then G is generically rigid as a configuration of position vectors in $\mathbb{R}^4$.



3BT3

G has a proper 6T3 decomposition if and only if G has a proper decomposition as 3 spanning "broken" trees.



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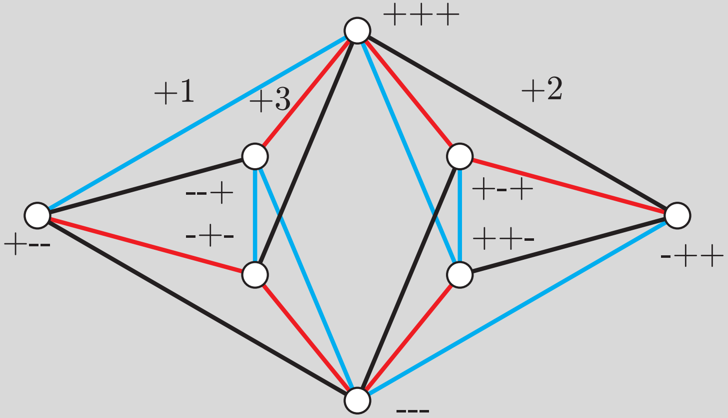
3BT3

G has a proper 6T3 decomposition if and only if G has a proper decomposition as 3 spanning “broken” trees.



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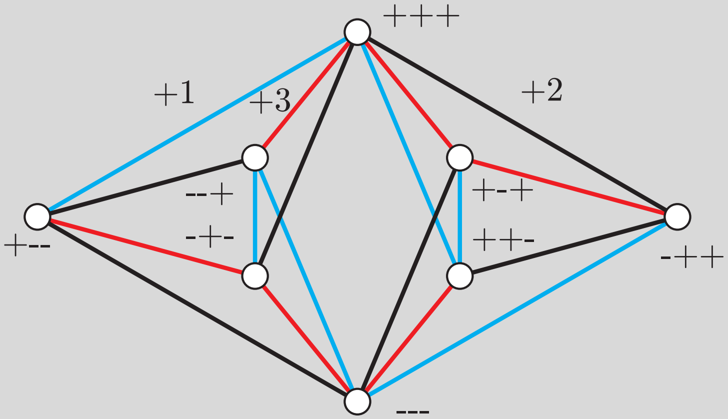
3BT3

G has a proper 6T3 decomposition if and only if G has a proper decomposition as 3 spanning "broken" trees.

- The are at most 8 classes of vertices: $(\pm 1, \pm 1, \pm 1)$.
- No edge can join class (i, j, k) to $(-i, -j, -k)$
- An edge is said to *knit* if its endpoint classes agree in only one coordinate.



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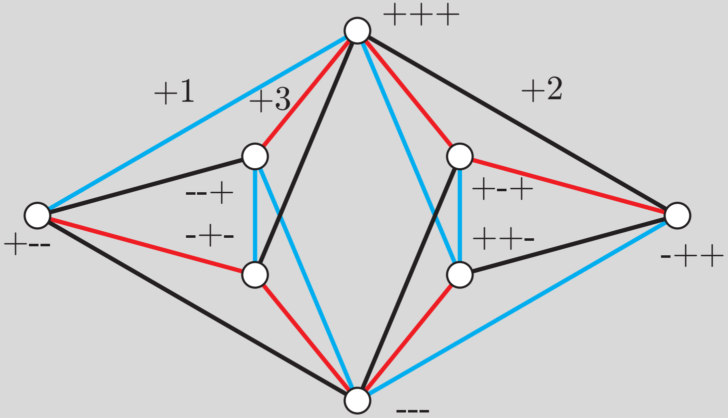
3BT3

If G has a proper decomposition as 3 spanning “broken” trees and

- The interior edges of each class are combable
- the non-interior edges of each class knit

Then G is generically rigid as a configuration of position vectors in $\mathbb{R}^4$.



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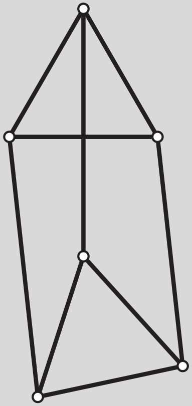
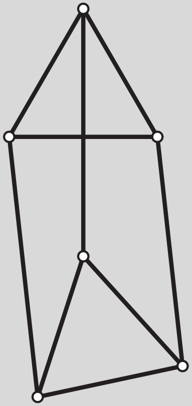
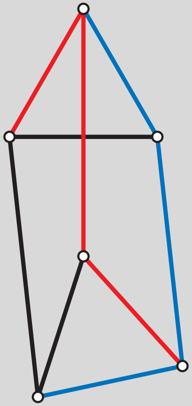
6. Bar/Joint



Framework



Structure Graph



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Objects Joints $\{\mathbf{p}_1, \ldots, \mathbf{p}_n\}$

Constraints Bar Lengths

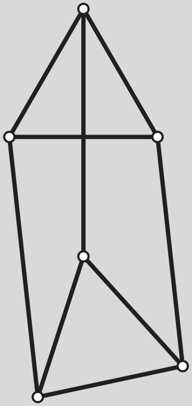
$$(\mathbf{p}_j - \mathbf{p}_i) \cdot (\mathbf{p}_j - \mathbf{p}_i) = \lambda_{ij}^2$$

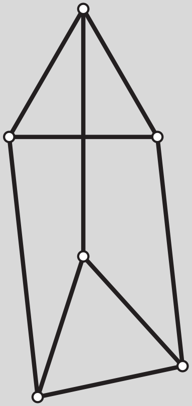
Infinitesimal Constraints Scalar equation

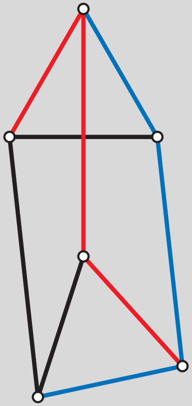
$$(\mathbf{p}_j - \mathbf{p}_i) \cdot (\dot{\mathbf{p}}_j - \dot{\mathbf{p}}_i) = 0$$

Combinatorics Bar Joint Graph.

Tree Decomposition Proper 3T2


Framework


Structure Graph


Tree Decomposition

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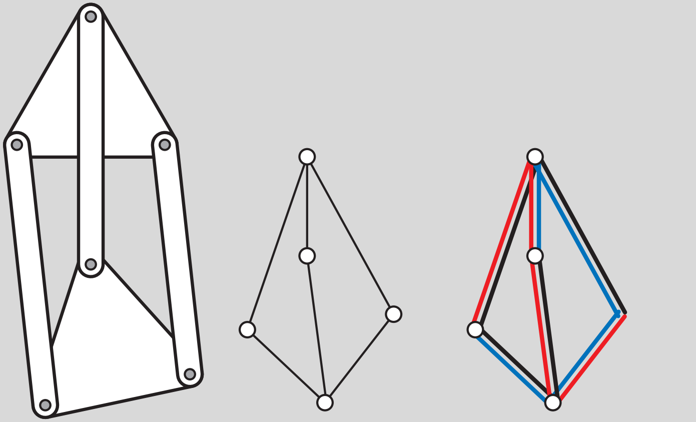
7. Body/Simple Pin



Framework Structure Graph Tree Decomposition

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Objects Body Motions $\mathbf{x} \longrightarrow \mathbf{T}_t + O_t\mathbf{x}$

Constraints Pin Locations $\{\mathbf{p}_{ij}\}$

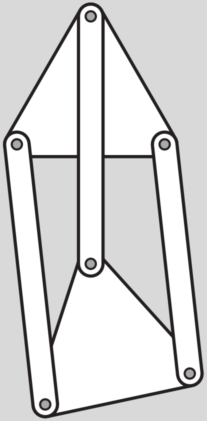
$$\mathbf{T}_i + O_i\mathbf{p}_{ij} = \mathbf{T}_j + O_j\mathbf{p}_{ij}$$

Infinitesimal Constraints Vector Equation (dependent)

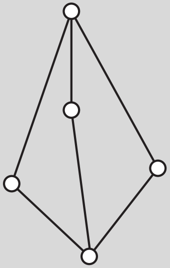
$$A_i + \mathbf{w}_i \times \mathbf{p}_{ij} = A_j + \mathbf{w_j} \times \mathbf{p}_{ij}$$

Combinatorics Vertices: Bodies, Edges: Pins.
$\binom{n+1}{2}|B|$ vs $n|P|$

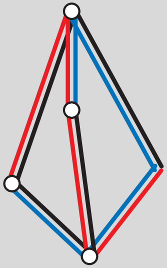
Tree Decomposition n edged Body-Pin graph decomposes as $\binom{n+1}{2}$-trees



Framework



Structure Graph



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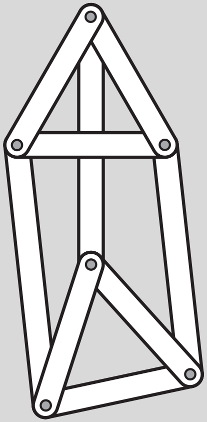
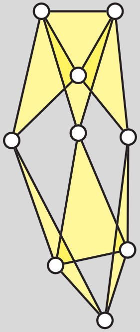
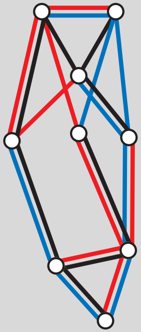
8. Body/Complex Pin



Framework



Structure Graph



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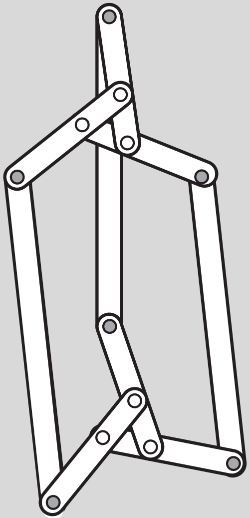
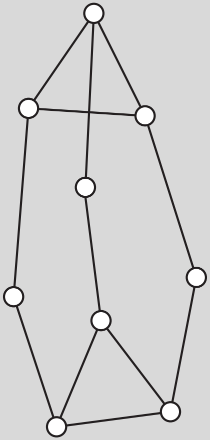
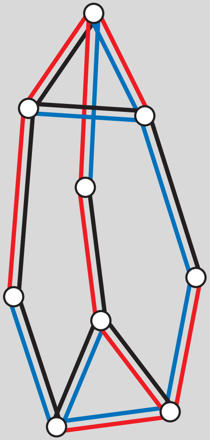


Framework



Structure Graph



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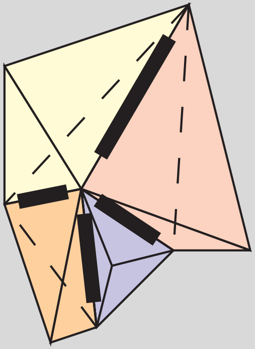
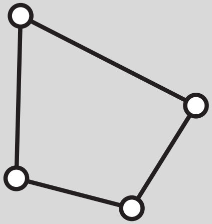
9. Body/hinge

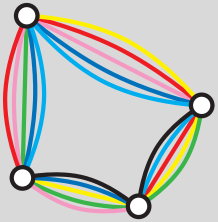
A “hinge” is a co-dimension 2 subspace on which the motions of the bodies agree.
For $\mathbb{R}^2$, Body/Hinge is the same as Body/Pin.
For $\mathbb{R}^3$, a hinge is a line, which is encoded by two points, $\mathbf{p}_{ij}$ and $\mathbf{q}_{ij}$.



Framework



Structure Graph



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Objects Body Motions $\mathbf{x} \longrightarrow \mathbf{T} + \mathrm{O}\mathbf{x}$

Constraints Hinge Locations $\{\mathbf{p}_{ij}\}$ ($\mathbb{R}^2$: same as body/pin)

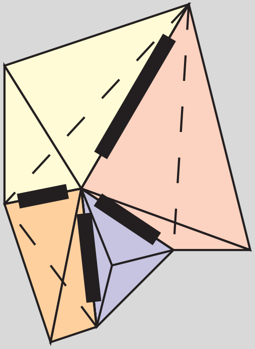
$$\mathbf{T}_i + O_i\mathbf{x} = \mathbf{T}_j + \mathrm{O}_j\mathbf{x}, \qquad \mathbf{x} \in \{\mathbf{p}_{ij}, \mathbf{q}_{ij}\}$$

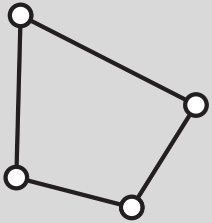
Infinitesimal Constraints Vector Equation (dependent)

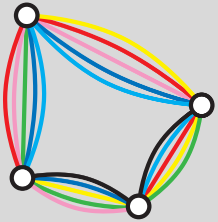
$$\mathbf{a}_i + \mathbf{w}_i \times \mathbf{x} = \mathbf{a}_j + \mathbf{w}_j \times \mathbf{x}, \qquad \mathbf{x} \in \{\mathbf{p}_{ij}, \mathbf{q}_{ij}\}$$

Combinatorics Vertices: Bodies, Edges: Hinges
$\binom{n+1}{2}|B|$ vs $(2n-1)|H|$

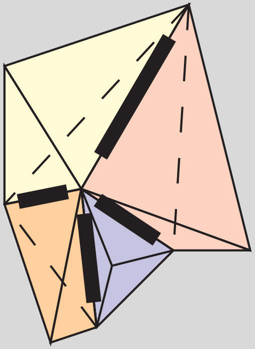
Tree Decomposition n edged Body-Pin graph decomposes as $\binom{n+1}{2}$-trees


Framework

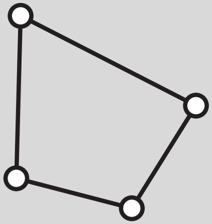

Structure Graph


Tree Decomposition

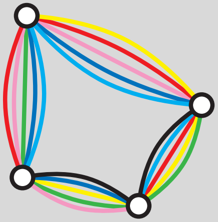
$$\begin{array}{cc} & \begin{array}{ccccccc} & T_i & R_i & & T_j & R_j & \end{array} \\ \begin{array}{c} (i,j)_1 \\ (i,j)_2 \end{array} & \left[\begin{array}{ccccccc} 0\cdots0 & \mathrm{I}_{3\times3} & [(-)\times\mathbf{p}_{ij}] & 0\cdots0 & -\mathrm{I}_{3\times3} & [\mathbf{p}_{ij}\times(-)] & 0\cdots0 \\ 0\cdots0 & \mathrm{I}_{3\times3} & [(-)\times\mathbf{q}_{ij}] & 0\cdots0 & -\mathrm{I}_{3\times3} & [\mathbf{q}_{ij}\times(-)] & 0\cdots0 \\ & & & & & & \end{array}\right] \end{array}$$



Framework



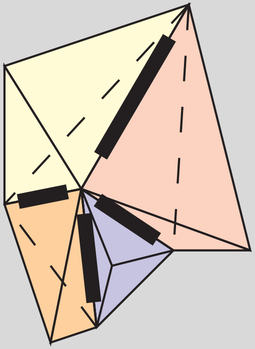
Structure Graph



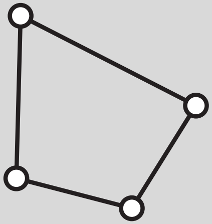
Tree Decomposition

$$\begin{array}{cc} & \begin{array}{cccc} T_i & R_i & T_j & R_j \end{array} \\ \begin{array}{c} (i,j)_1 \\ (i,j)_2 \end{array} & \left[\begin{array}{cccc} I_{3\times 3} & [\times \mathbf{p}_{ij}] & -I_{3\times 3} & [\mathbf{p}_{ij}\times] \\ 0_{3\times 3} & [\times(\mathbf{q}_{ij}-\mathbf{p}_{ij})] & 0_{3\times 3} & [(\mathbf{q}_{ij}-\mathbf{p}_{ij})\times] \end{array}\right] \end{array}$$

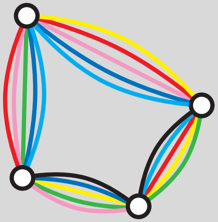
In this form we see the rows $(i,j)_2$ are dependent



Framework



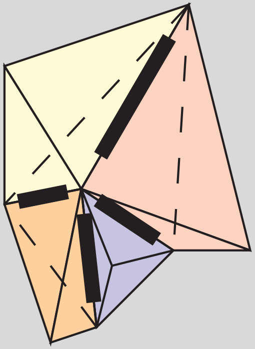
Structure Graph



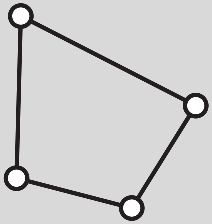
Tree Decomposition

$$\begin{array}{c|cccc|}
 & T_i & R_i & T_j & R_j \\
(i,j)_1 & \mathrm{I}_{3\times3} & [\times\mathbf{p}_{ij}] & -\mathrm{I}_{3\times3} & [\mathbf{p}_{ij}\times] \\
(i,j)_2 & 0_{1\times3} & [\mathbf{q}_{ij}\times\mathbf{p}_{ij}] & 0_{1\times3} & [\mathbf{q}_{ij}\times\mathbf{p}_{ij}] \\
 & 0_{1\times3} & [(\mathbf{q}_{ij}\times\mathbf{p}_{ij})\times\mathbf{p}_{ij}] & 0_{1\times3} & [(\mathbf{q}_{ij}\times\mathbf{p}_{ij})\times\mathbf{p}_{ij}]
\end{array}$$

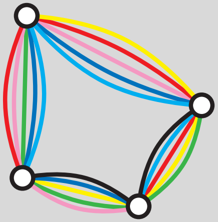
Reducing, we have a somewhat less symmetric $5|E| \times 6|B|$ matrix



Framework



Structure Graph



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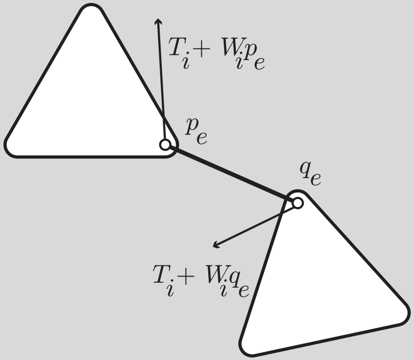
10. Body/Bar

$$[(T_i + W_i\mathbf{p}_e) - (T_j + W_j\mathbf{q}_e)] \cdot [\mathbf{p}_e - \mathbf{q}_e] = 0$$

$$[T_i - T_j] \cdot [\mathbf{p}_e - \mathbf{q}_e] - [W_i\mathbf{p}_e \cdot \mathbf{q}_e + W_j\mathbf{q}_e \cdot \mathbf{p}_e] = 0$$

$$\begin{array}{c c} & \begin{array}{cccc} T_i & R_i & T_j & R_j \end{array} \\ e & \left[\begin{array}{cccc} (\mathbf{q}_e - \mathbf{p}_e) & (\mathbf{q}_e \times \mathbf{p}_e) \cdot \mathbf{k} & (\mathbf{p}_e - \mathbf{q}_e) & (\mathbf{p}_e \times \mathbf{q}_e) \cdot \mathbf{k} \end{array} \right] \end{array}$$



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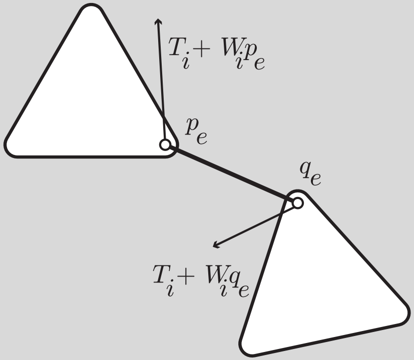
Or in homogeneous coordinates:

$$\begin{array}{c|cc|} & \text{Body } i & \text{Body } j \\ e & (\mathbf{q}_e, 1) \times (\mathbf{p}_e, 1) & (\mathbf{p}_e, 1) \times (\mathbf{q}_e, 1) \end{array}$$



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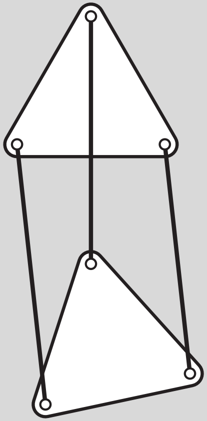
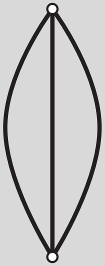


Framework



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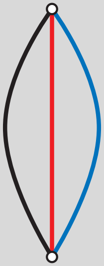

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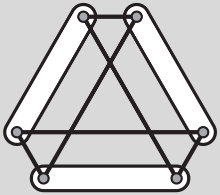
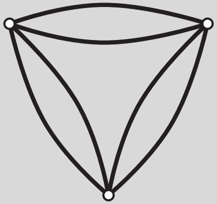
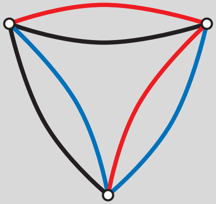


Framework



Structure Graph



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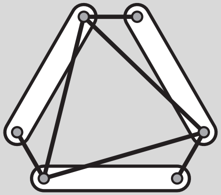
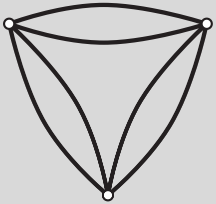
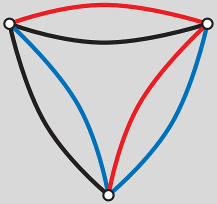


Framework



Structure Graph



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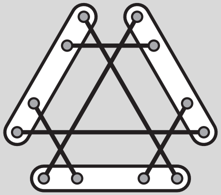
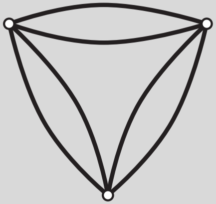
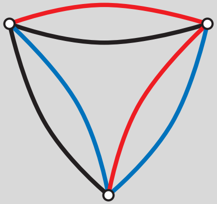


Framework



Structure Graph



Tree Decomposition