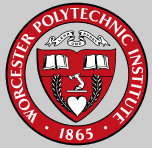
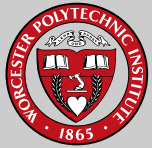


Point Line Configurations and their Realizability

Brigitte Servatius

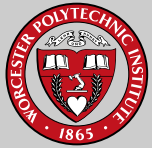


Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 2 of 36
Go Back
Full Screen
Close
Quit

Zur Theorie der Netze und Configurationen von Konrad Zindler in Graz

- Elementary proof of a theorem of Möbius:
Given 4 points in the plane, one can, by ruler alone construct a point in the ϵ-neighborhood of a given 5'th point for any $\epsilon > 0$.

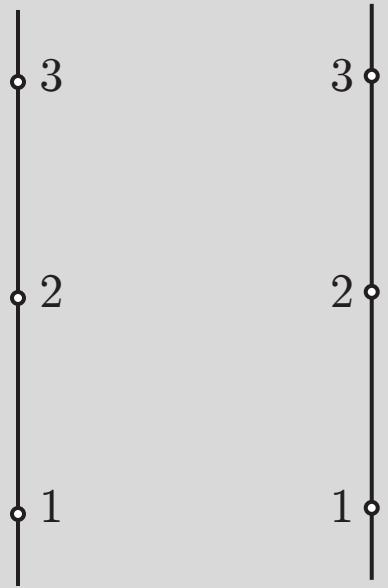
- Generalization of Configuration:
A system of points and lines in the plane such that on every line there are at least 3 points and through every point there are at least 3 lines.

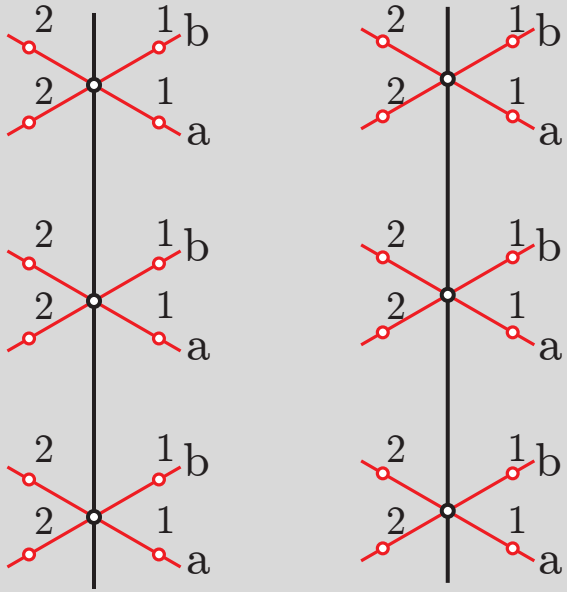


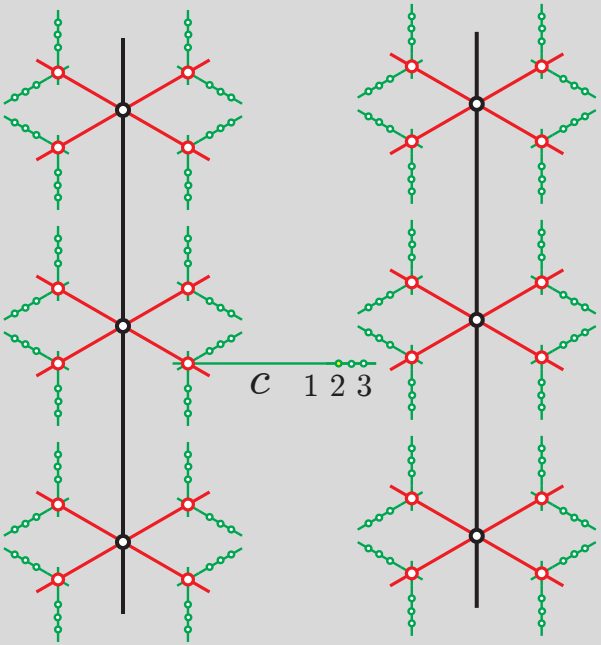
Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 3 of 36
Go Back
Full Screen
Close
Quit

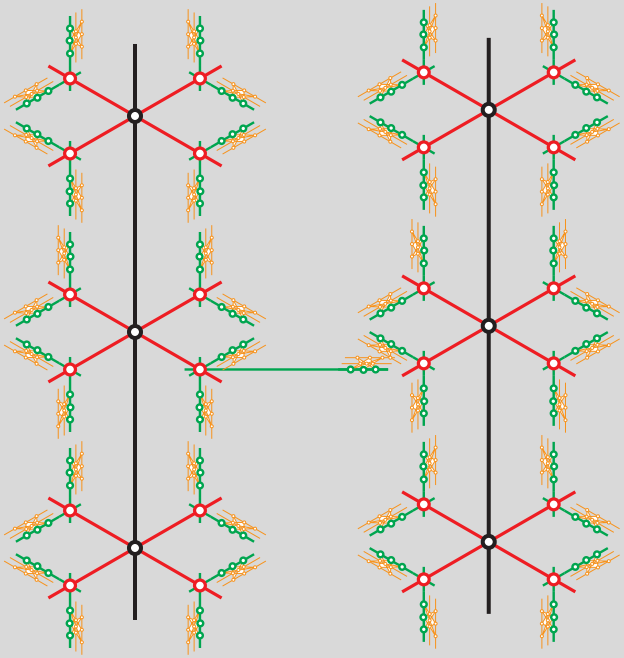
1. Zindler's Construction

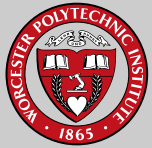
Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 4 of 36
Go Back
Full Screen
Close
Quit



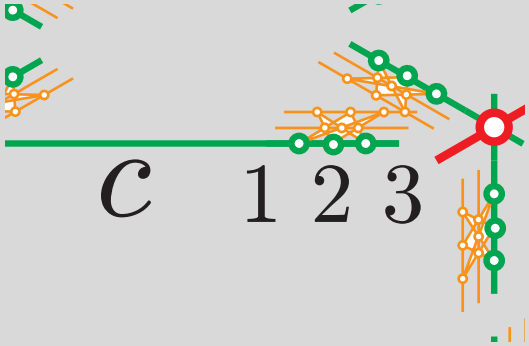

2
1
b
2
1
a
2
1
b
2
1
a
2
1
b
2
1
a
2
1
b
2
1
a
2
1
b
2
1
a

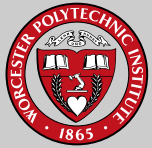

c 1 2 3



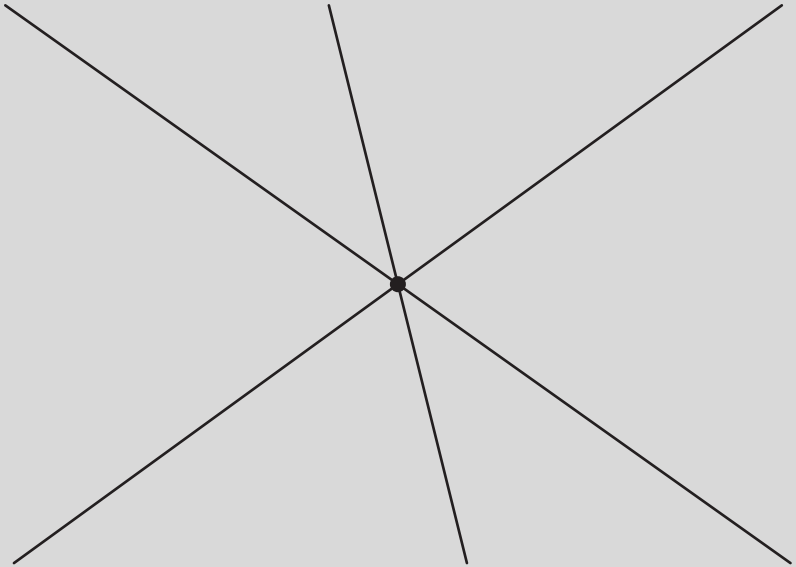


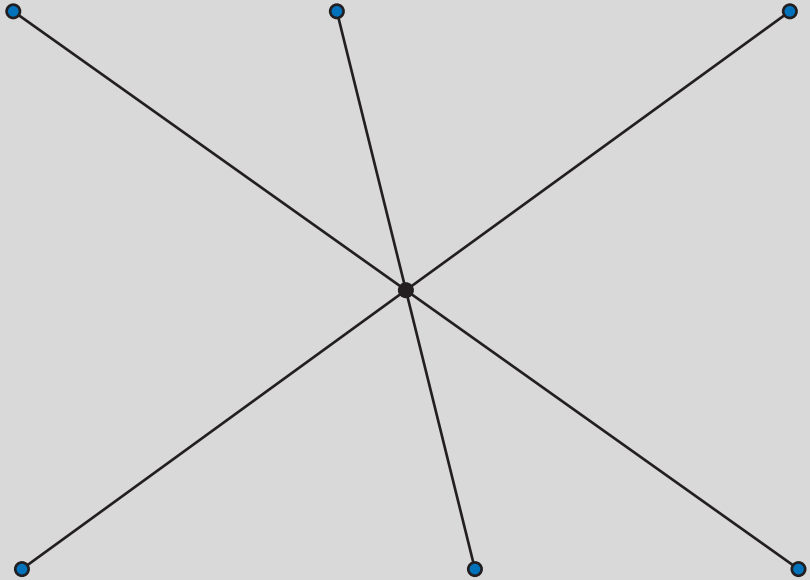
Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 8 of 36
Go Back
Full Screen
Close
Quit

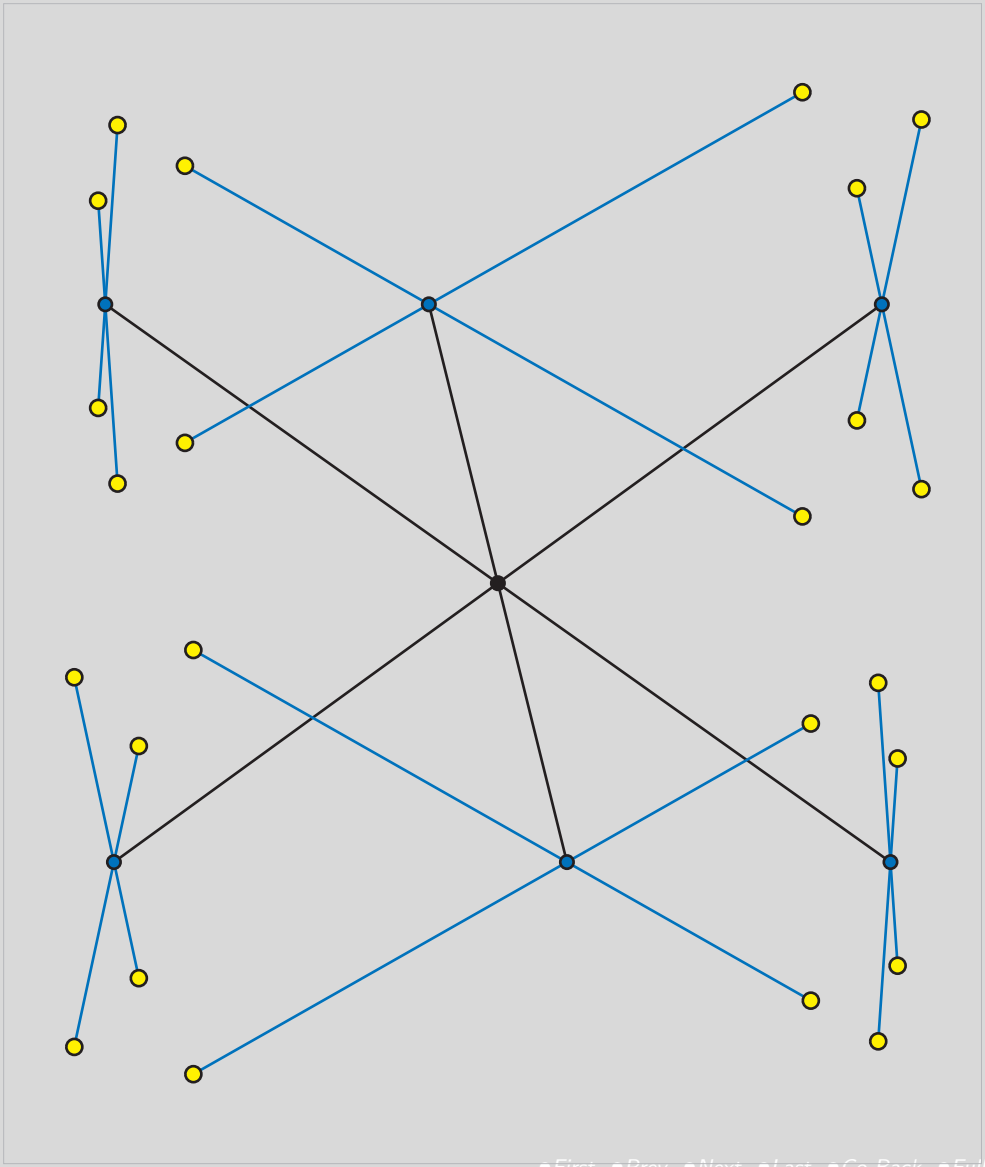


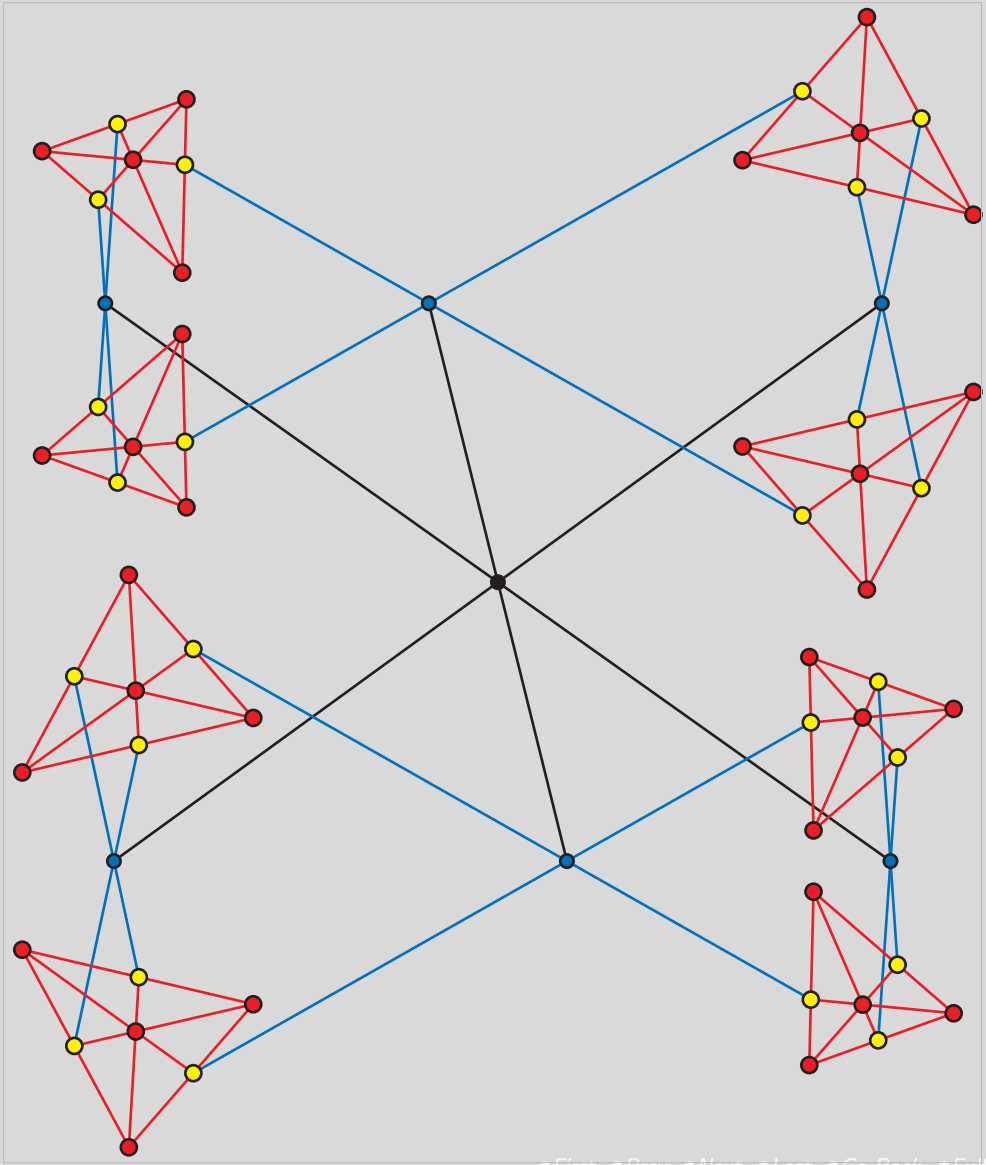


Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 9 of 36
Go Back
Full Screen
Close
Quit

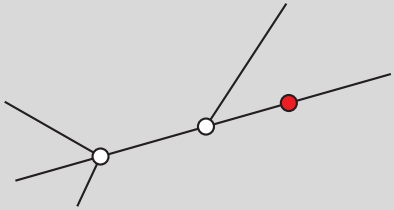




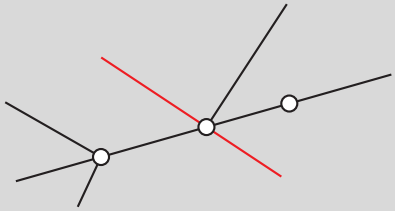




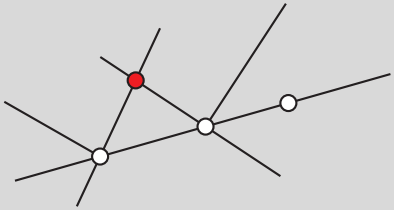
Realizable Moves



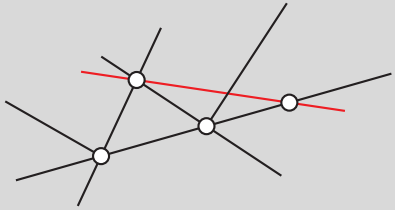
- Put a new point on a line.



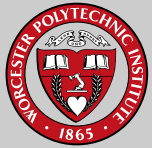
- Put a new line through a point.



- Intersect two lines.

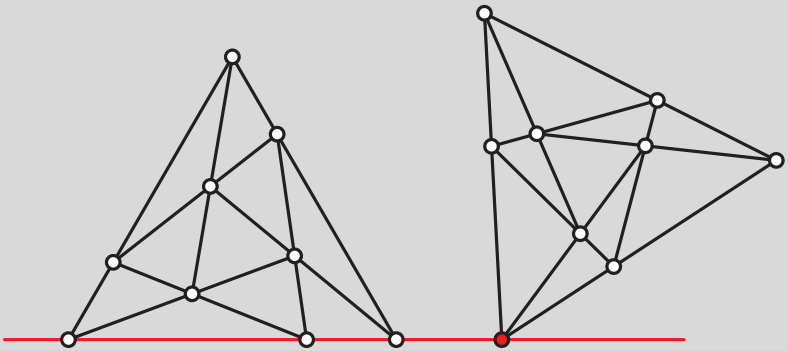


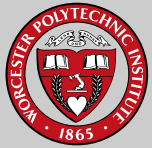
- Draw a line through two points.
- Join two components by putting a point of one component on a line of the other component.



Realizable Moves

- Put a new point on a line.
- Put a new line through a point.
- Intersect two lines.
- Draw a line through two points.
- Join two components by putting a point of one component on a line of the other component.

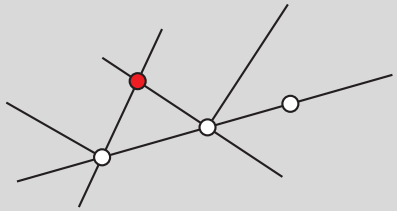




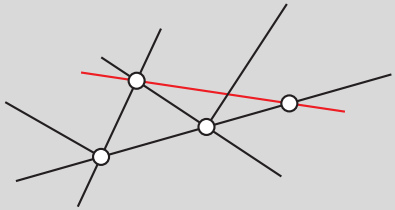
Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 16 of 36
Go Back
Full Screen
Close
Quit

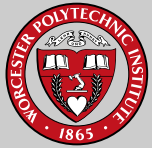
Dangerous Moves

- Intersect two lines.



- Draw a line through two points.

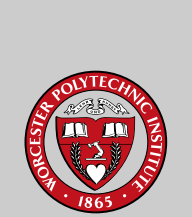




Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 17 of 36
Go Back
Full Screen
Close
Quit

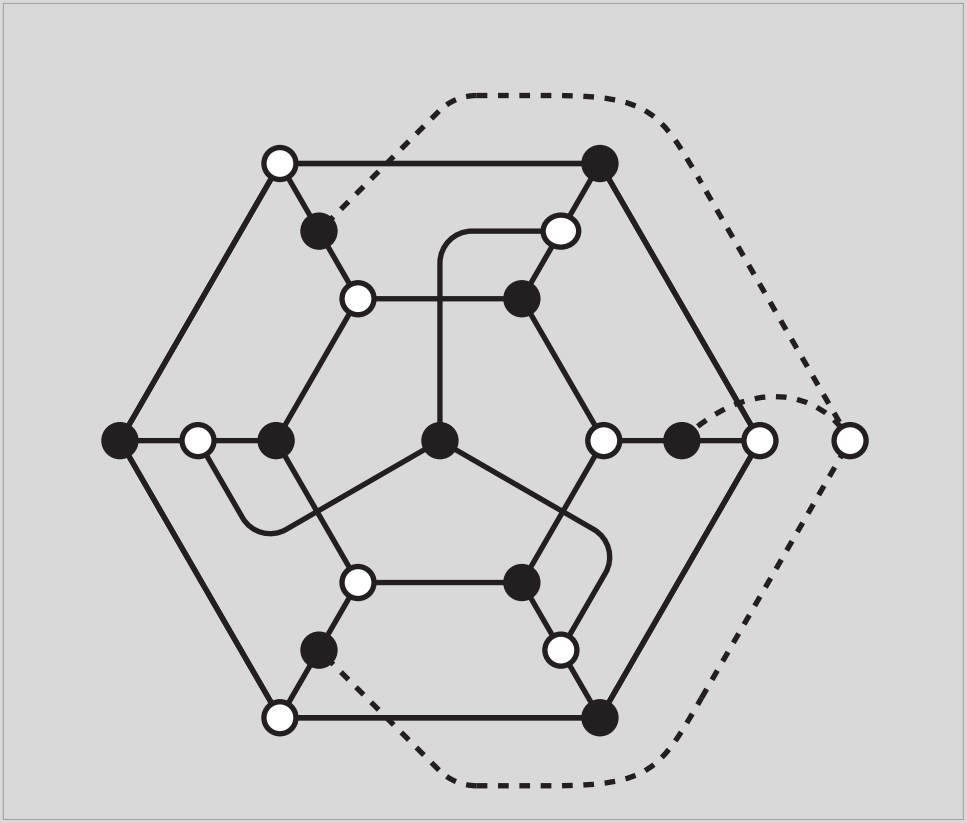
Realizable Moves on the Levi graph

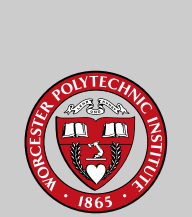
- Add vertices of degree one.
- Add vertices of degree two such that bipartiteness and girth 6 are preserved.
(between points of the same color a distance at least 4 apart.)
- Add edges between connected components (bridges).



Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 18 of 36
Go Back
Full Screen
Close
Quit

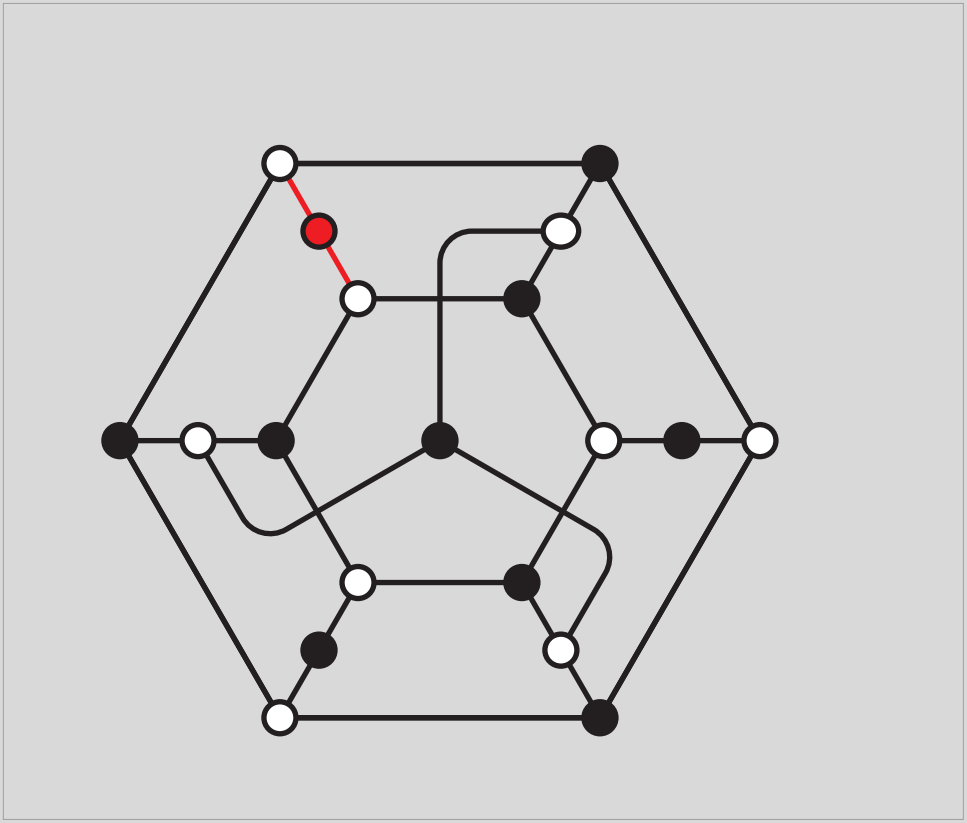
Given a bipartite graph G of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If G is 3-regular, then $G - v$ can be built up from a vertex by allowable moves

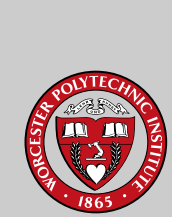




Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 19 of 36
Go Back
Full Screen
Close
Quit

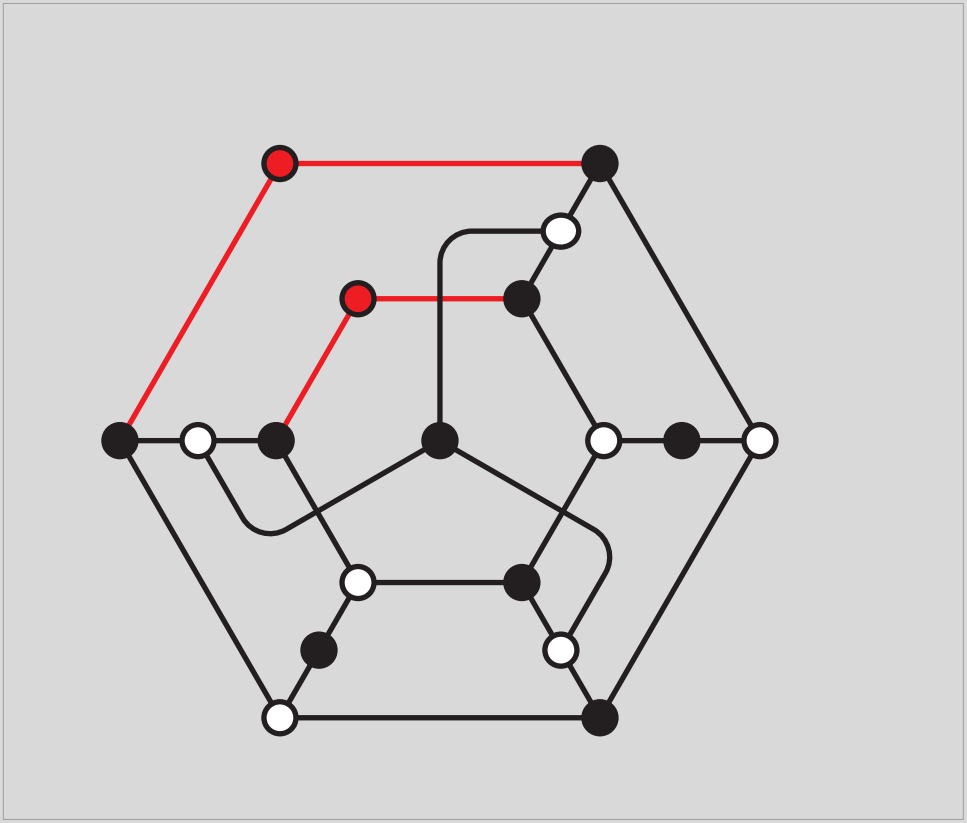
Given a bipartite graph G of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If G is 3-regular, then $G - v$ can be built up from a vertex by allowable moves

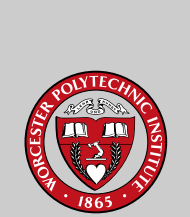




Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 20 of 36
Go Back
Full Screen
Close
Quit

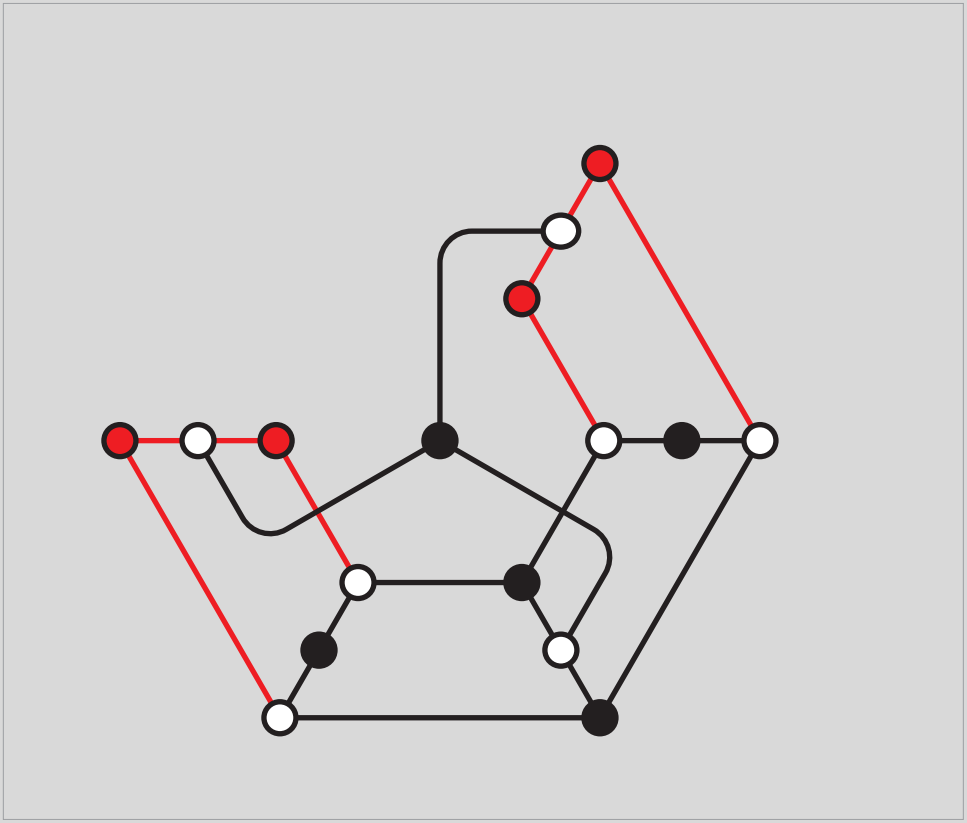
Given a bipartite graph G of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If G is 3-regular, then $G - v$ can be built up from a vertex by allowable moves

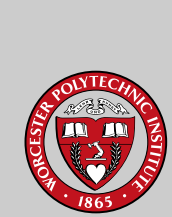




Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 21 of 36
Go Back
Full Screen
Close
Quit

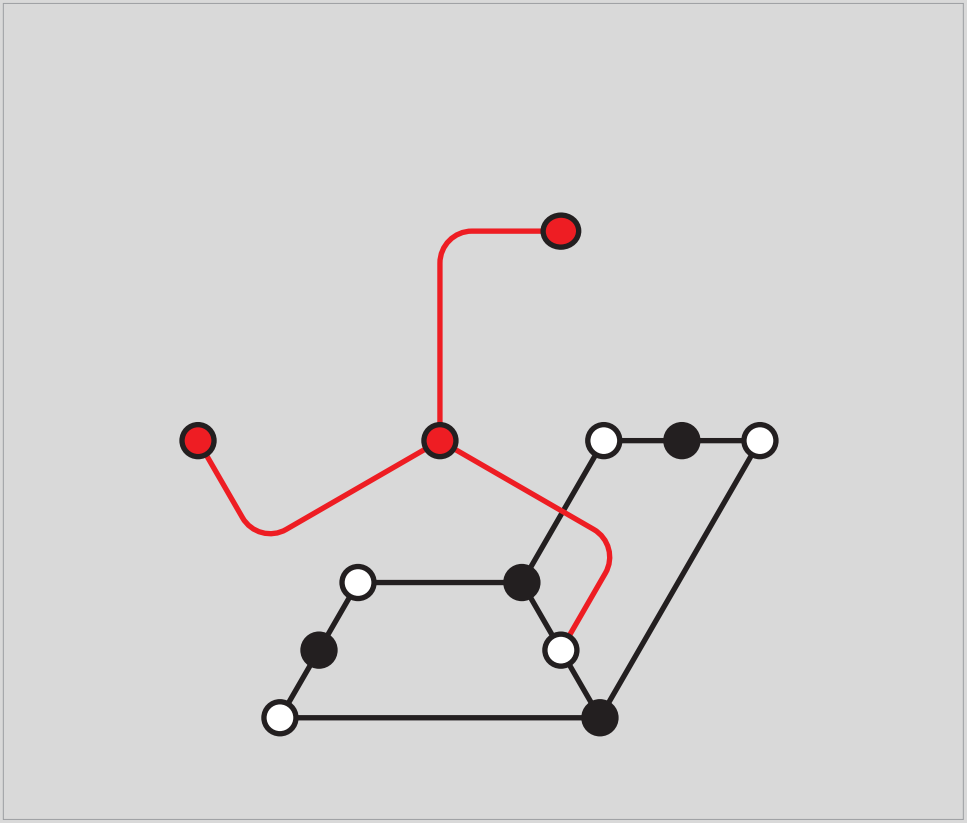
Given a bipartite graph G of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If G is 3-regular, then $G - v$ can be built up from a vertex by allowable moves

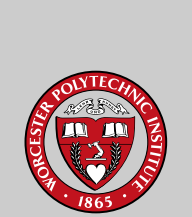




Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 22 of 36
Go Back
Full Screen
Close
Quit

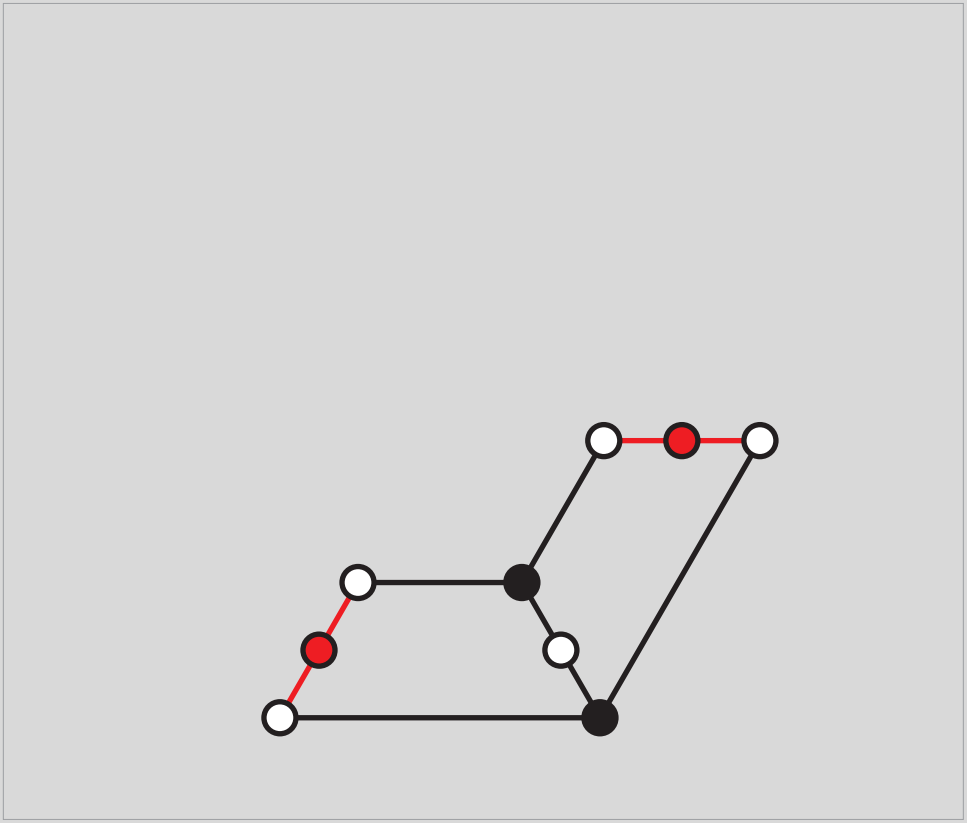
Given a bipartite graph G of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If G is 3-regular, then $G - v$ can be built up from a vertex by allowable moves

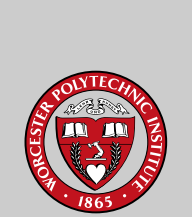




Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 23 of 36
Go Back
Full Screen
Close
Quit

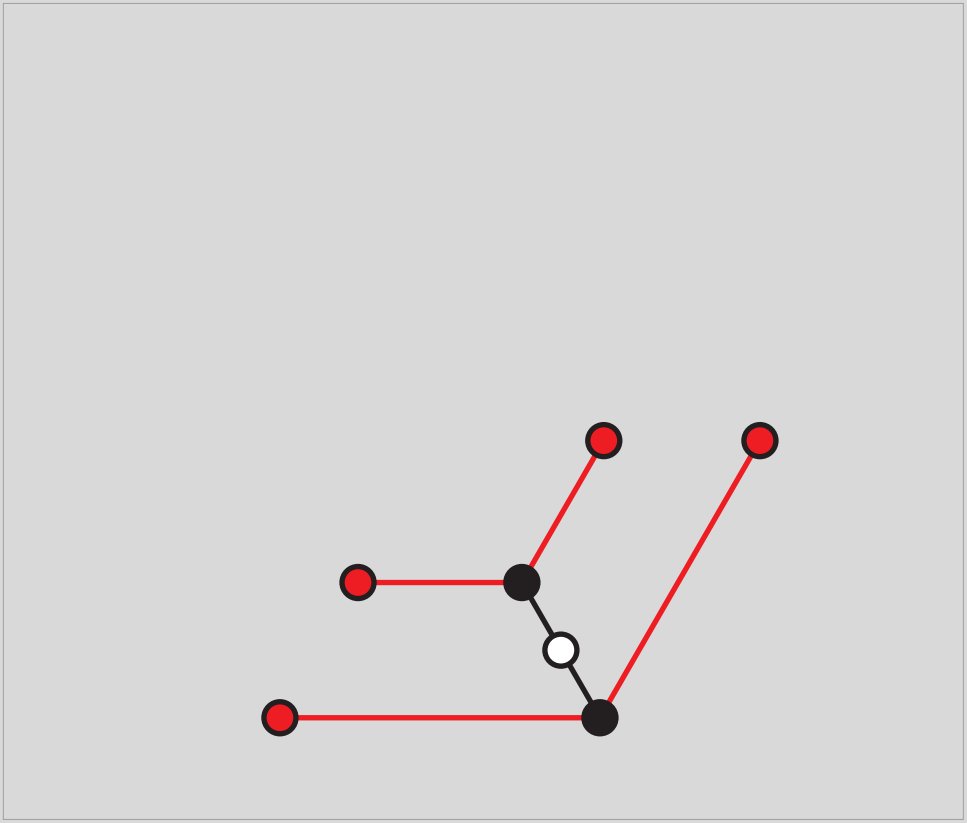
Given a bipartite graph G of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If G is 3-regular, then $G - v$ can be built up from a vertex by allowable moves

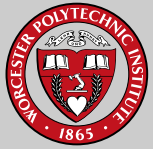




Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 24 of 36
Go Back
Full Screen
Close
Quit

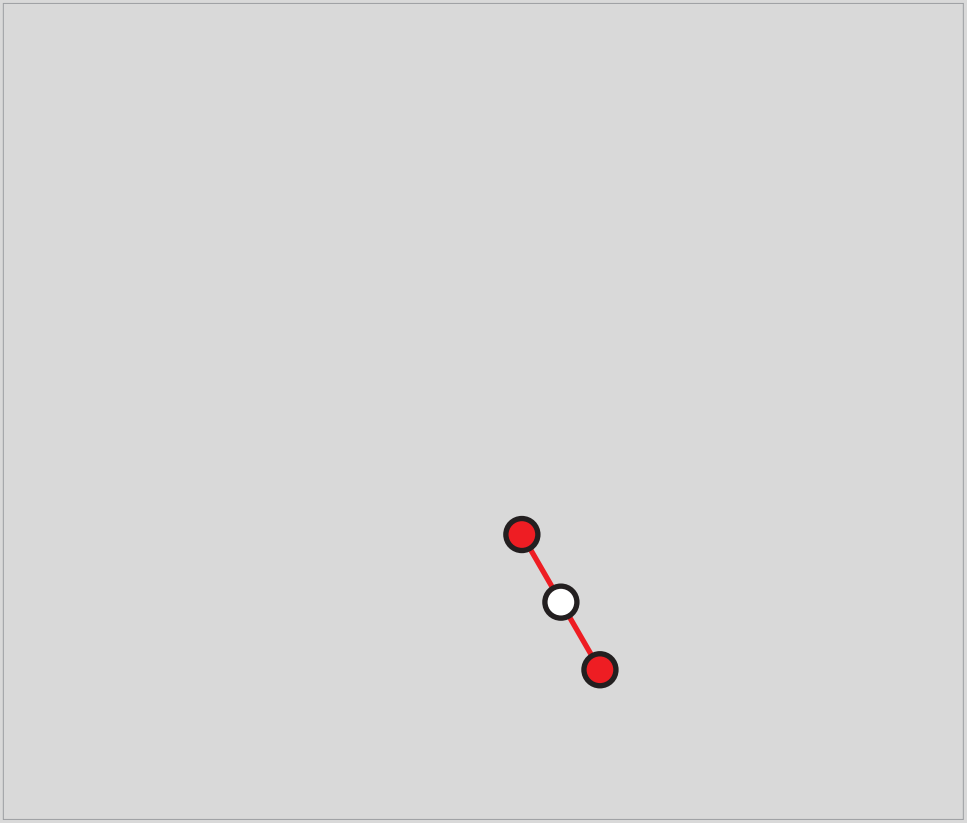
Given a bipartite graph G of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If G is 3-regular, then $G - v$ can be built up from a vertex by allowable moves

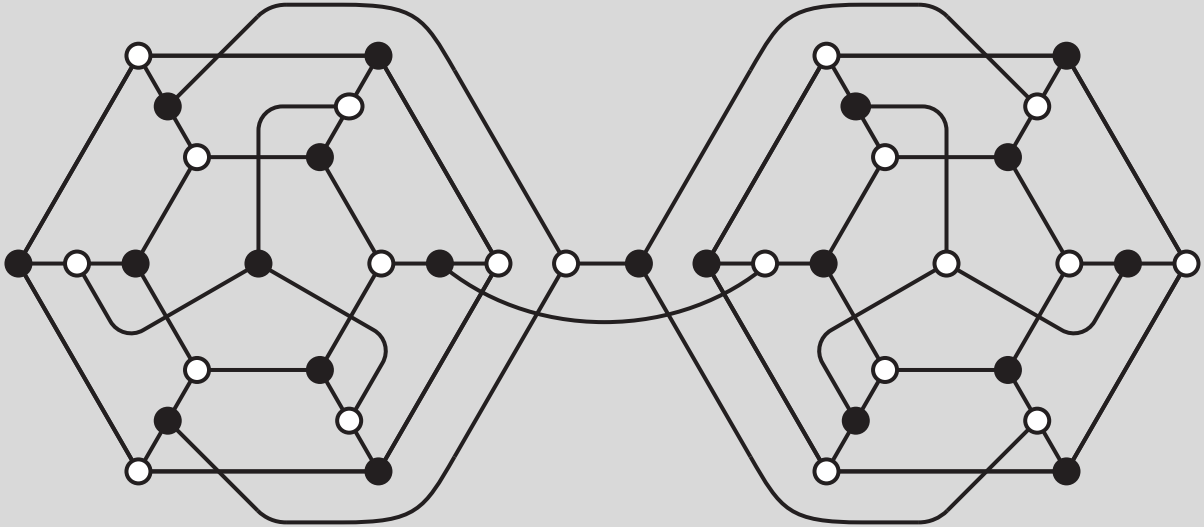


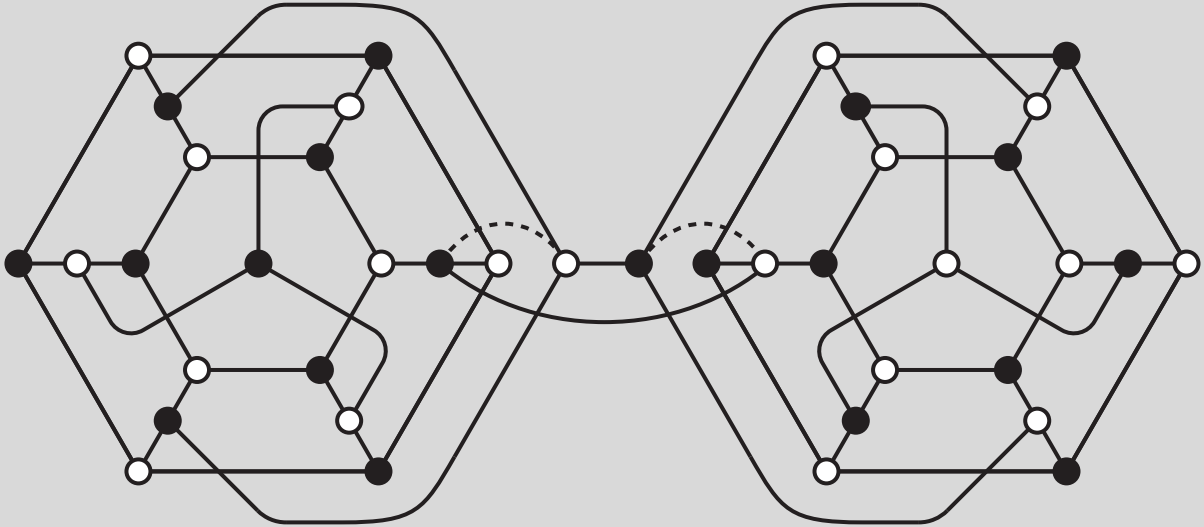


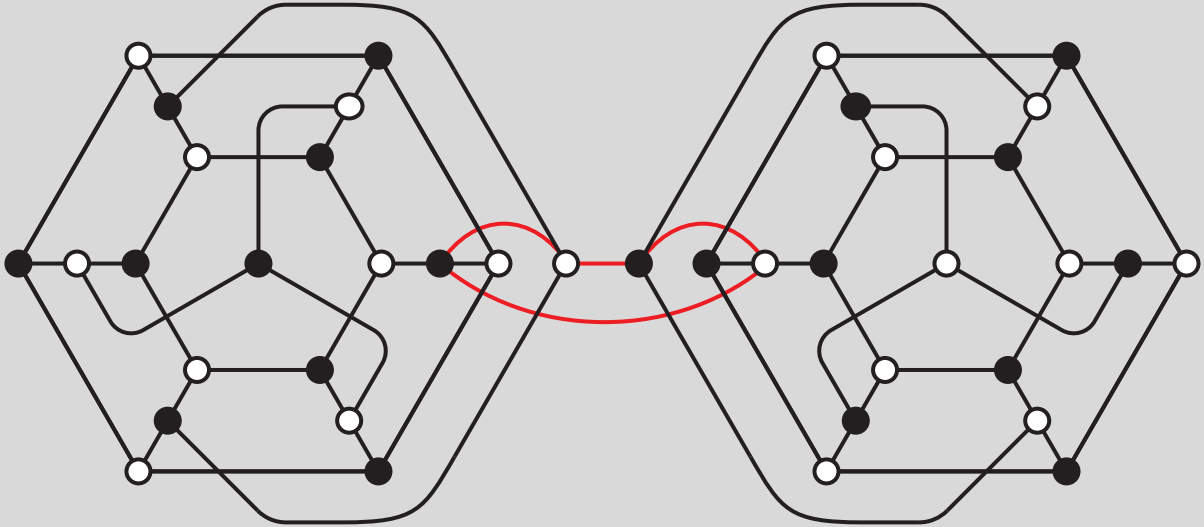
Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 25 of 36
Go Back
Full Screen
Close
Quit

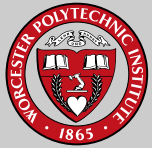
Given a bipartite graph G of girth 6, these moves may be reversed, provided there exists a vertex of degree at most 2. If G is 3-regular, then $G - v$ can be built up from a vertex by allowable moves









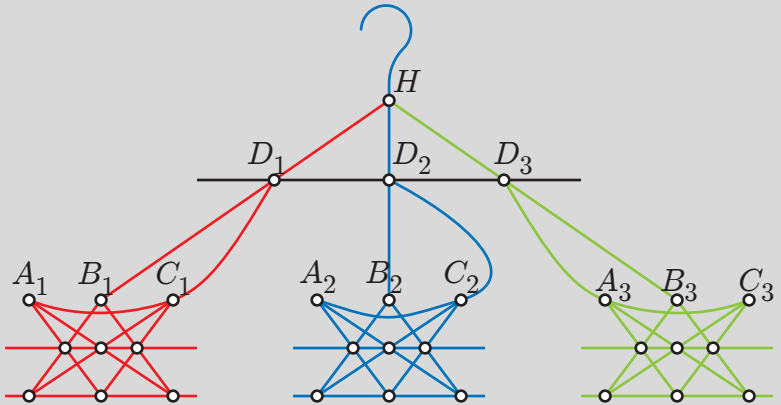


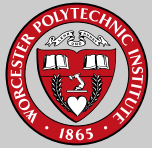
Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 29 of 36
Go Back
Full Screen
Close
Quit

Steinitz's Theorem

Every symmetric v_3 configuration has a realization in the plane with at most one curved line.

Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 30 of 36
Go Back
Full Screen
Close
Quit





Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 31 of 36
Go Back
Full Screen
Close
Quit

Grünbaum's Conjecture

Steinitz's Theorem is true for configurations whose Levi graph is 3-connected.

Theorem

Steinitz's Theorem is true for configurations whose Levi graph is 3-connected. and edge 4-connected.

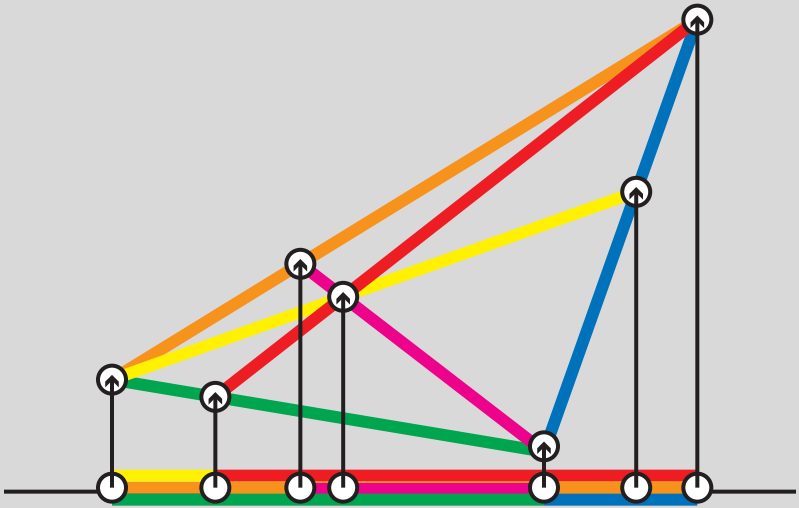
Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 32 of 36
Go Back
Full Screen
Close
Quit

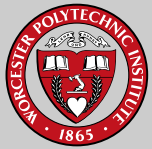
2. Whiteley's Theorem

A generic picture in $k-1$ space of an incidence structure lifts to a sharp scene in k-space if and only if

$$i \leq a + kb - (k+1)$$

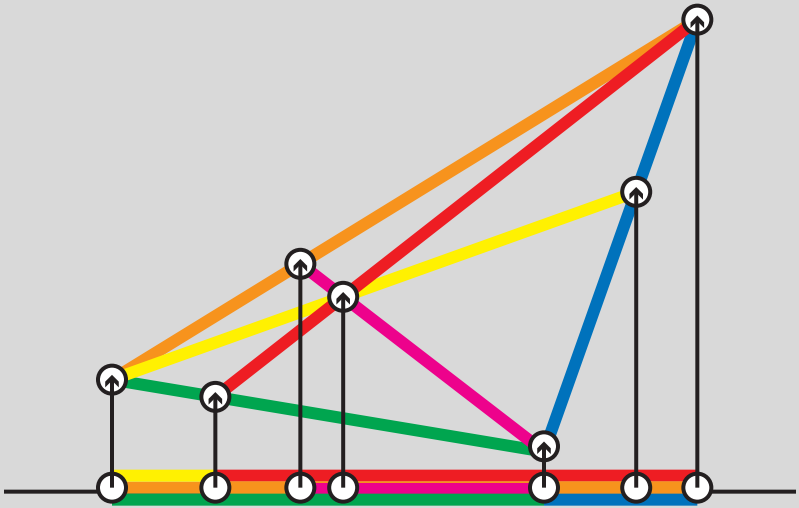
for all sub-incidence structures having at least two blocks.



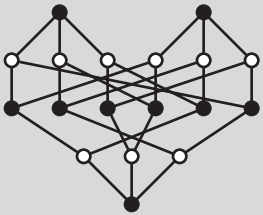


For a 3-regular bipartite graph of girth six Whiteley's count is violated by three.

Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 33 of 36
Go Back
Full Screen
Close
Quit



For a 3-regular bipartite graph of girth six Whiteley's count is violated by three.

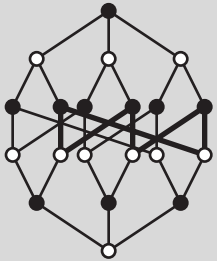


$i = 6 \quad p = 2 \quad l = 6 \quad l + 2p - 2 = 8 \quad 2l + p - 2 = 12$

$i = 18 \quad p = 8 \quad l = 6 \quad l + 2p - 2 = 20 \quad 2l + p - 2 = 18$

$i = 24 \quad p = 8 \quad l = 9 \quad l + 2p - 2 = 23 \quad 2l + p - 2 = 24$

$i = 27 \quad p = 9 \quad l = 9 \quad l + 2p - 2 = 25 \quad 2l + p - 2 = 25$



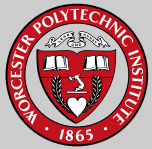
$i = 3 \quad p = 1 \quad l = 6 \quad l + 2p - 2 = 8 \quad 2l + p - 2 = 12$

$i = 9 \quad p = 8 \quad l = 6 \quad l + 2p - 2 = 20 \quad 2l + p - 2 = 18$

$i = 21 \quad p = 8 \quad l = 9 \quad l + 2p - 2 = 23 \quad 2l + p - 2 = 24$

$i = 27 \quad p = 9 \quad l = 9 \quad l + 2p - 2 = 25 \quad 2l + p - 2 = 25$

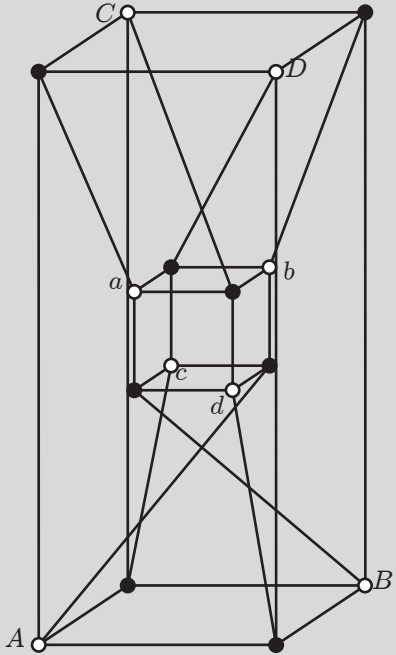
$i = 30 \quad p = 9 \quad l = 9 \quad l + 2p - 2 = 25 \quad 2l + p - 2 = 25$

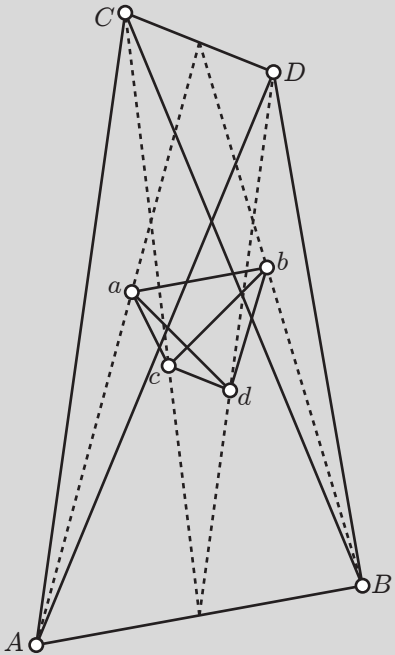


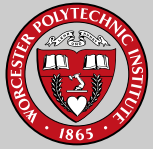
Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 35 of 36
Go Back
Full Screen
Close
Quit

An (8_4) spatial configuration.
$a = 8$, $b = 8$, $i = 32$,

$$a + 3b - 4 = 28$$







Home Page
Print
Title Page
◀◀ ▶▶
◀ ▶
Page 36 of 36
Go Back
Full Screen
Close
Quit

A similar (8_4) spatial configuration.
Levi graph is a hypercube
$a = 8$, $b = 8$, $i = 32$,

$$a + 3b - 4 = 28$$