Abstract

The **tensegrity modules for cable-strut systems** here presented achieve a good compromise between the **structural efficiency** of cable-strut systems and the **deploying capability** and **controllability** of tensegrity systems.

The modules are obtained by **'expansions' of an octahedron**, and assembled by means of **strut-to-strut connections**; **additional cables** make the overall stiffness larger. After the first assembling phase, the structure can be either folded back or further deployed; after the second phase, the structure can support service loads.

1. Tensegrity Systems

Definition. Tensegrity systems are spatial trusses composed by struts and cables, such that:
(1) the collection of cables appears as a connected set (**tens**ile-int**egrity**); **(2)** the struts are never connected to each other (floating compression);
(3) there are infinitesimal mechanisms, stabilized by a self-stress state.
✶ Certain authors regard both properties (1) and (2) as essential, others insist only on (1); others do not include anyone of the two into their definitions.

Form-Finding Property. Given a *n*-element tensegrity system, if the lenghts of *(n-1)* elements are fixed, then a stable equilibrium configuration (a **tensegrity configuration**) obtains when the last cable (strut) has minimal (maximal) lenght.

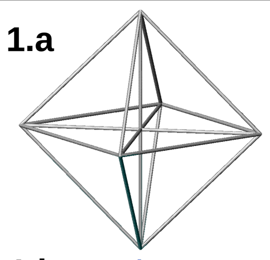
Form-Finding Problem. Find the set of all possible tensegrity configurations for a system of fixed topology.
✶ The solution of this problem constitutes the **form-finding condition** for the existence of a tensegrity system.

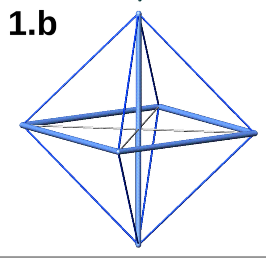
2. Expanding Octahedra

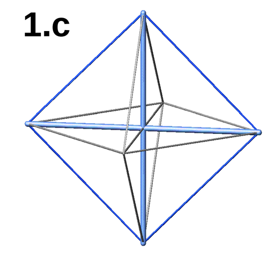
- A strut-and-cable truss having the octahedron topology is modified by adding three elemento, each connecting a couple of opposed vertices (Fig. **1.a**).
- The additional elements induce three independent self-stress states in the system (Figs. **1.b** and **1.c** show the two main types of states).
- From an initial self-stressed configuration, the force vectors concurrent in a node are so decomposed as to justify one or more of the following operations: (1) stressing/unstressing of elements; (2) creation and extension of elements; (3) doubling of nodes; (4) doubling of elements.
✶ Note that (4) implies (3) and (3) implies (2). The resulting system is expanded both in **dimensions** and **number of components**; the **number of mechanisms** increases and the system is in a **tensegrity configuration.**

The Extended Maxwell's Rule: 3n - 6 - e = m - s

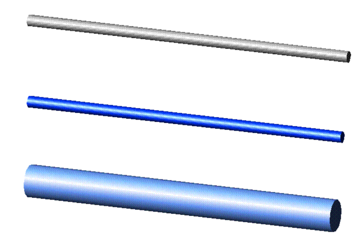
This rule relates the number of **nodes** (**n**) and **elements**, stressed or not (**e**) of the system with the number of **mechanisms** (**m**) and **self-stress states** (**s**); six is the number of rigidmotions in 3D.

1.a 

1.b 

1.c 

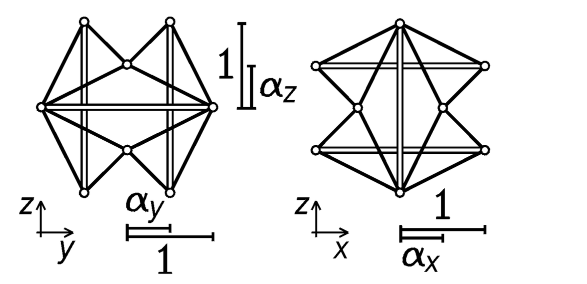
1.a - Modified Octahedron
1.b & **1.c** - Self-stress states induced by added elements connecting pairs of opposed vertices.



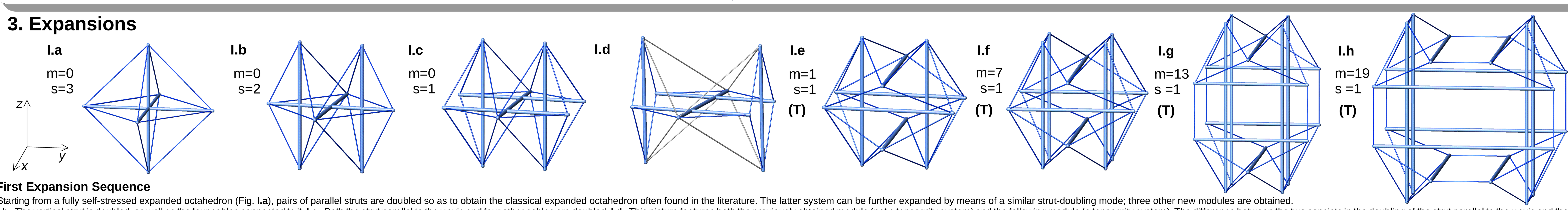
Classical Expanded Octahedron

Form-finding condition

$$\alpha_x \alpha_y \alpha_z = (1-\alpha_x)(1-\alpha_y)(1-\alpha_z)$$
$$\alpha_x, \alpha_y, \alpha_z \leqslant 1$$

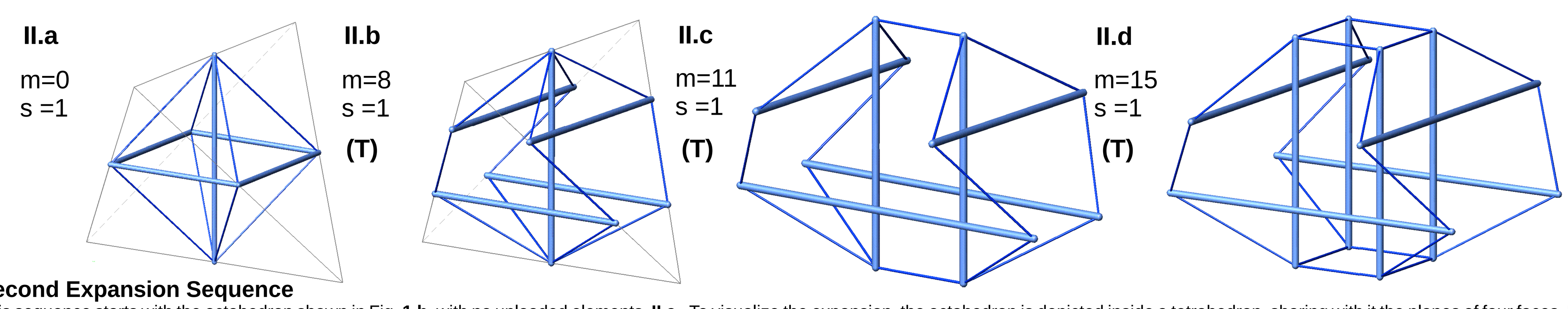
2 

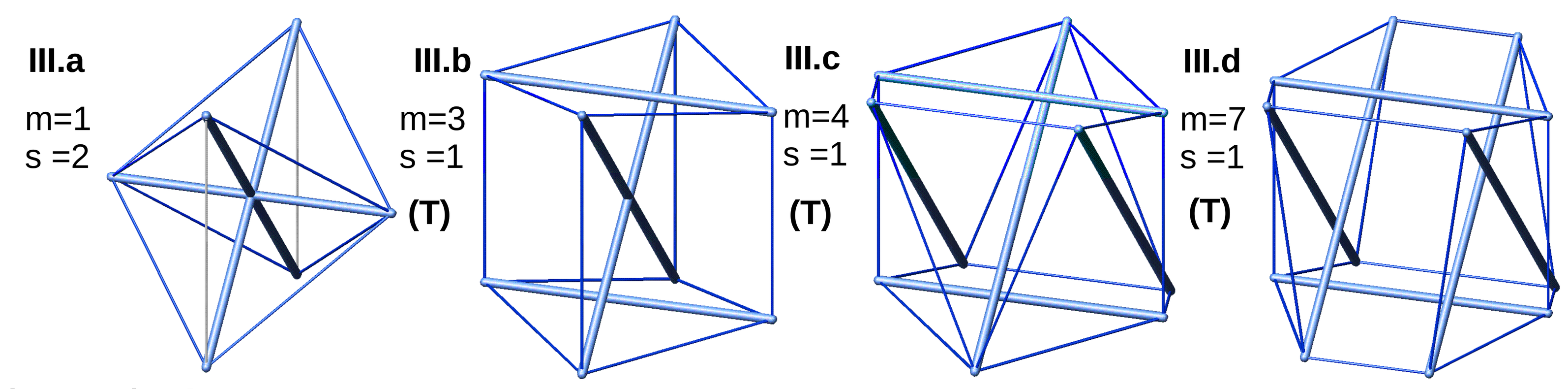
3. Expansions



First Expansion Sequence

Starting from a fully self-stressed expanded octahedron (Fig. **I.a**), pairs of parallel struts are doubled so as to obtain the classical expanded octahedron often found in the literature. The latter system can be further expanded by means of a similar strut-doubling mode; three other new modules are obtained.
I.b - The vertical strut is doubled, as well as the four cables connected to it. **I.c** - Both the strut parallel to the *y* axis and four other cables are doubled. **I.d -** This picture features both the previously obtained module (not a tensegrity system) and the following module (a tensegrity system). The difference between the two consists in the doubling of the strut parallel to the *x* axis and the cables connected to it. In this special configuration, 8-16 cables have null stress. While doubling the strut parallel to the x axis, it is necessary to pass through this configuration to satisfy the form-finding condition of the tensegrity system. **I.e** - This module is the classical expanded octahedron. **I.f** - First re-expansion: vertical struts are doubled; between their doubled end nodes, four cables are created, parallel to the *x* axis. **I.g** - Second re-expansion: struts parallel to the *y* axis are doubled; four other cables parallel to the *z* axis are created.**I.h** -Third re-expansion: struts parallel to the *x* axis are doubled; four other cables parallel to the y axis are created.
✶ **Form-finding condition.** This condition is independent of the distance between pairs of doubling struts, but it does depend on the parameters indicated in Fig. 2 in Sections 2.





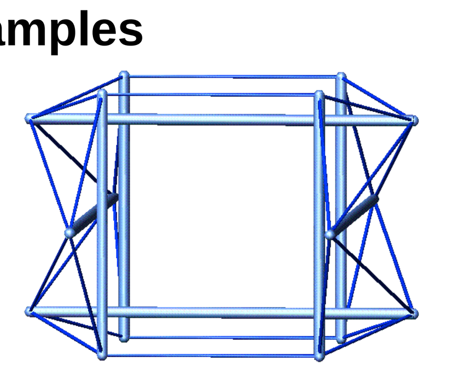
Second Expansion Sequence

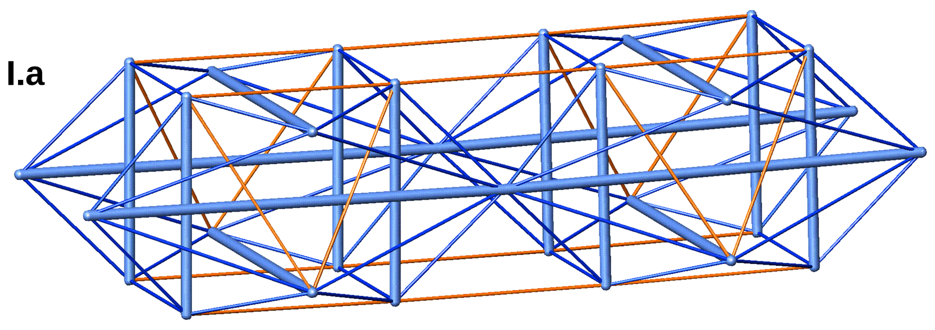
This sequence starts with the octahedron shown in Fig. **1.b**, with no unloaded elements. **II.a** - To visualize the expansion, the octahedron is depicted inside a tetrahedron, sharing with it the planes of four faces. **II.b** - The nodes of the compressed horizontal square are doubled, horizontal struts are separated and four new cables are created between their end nodes (to obtain a tensegrity systems, these elements must lay on the edges of the tetrahedron). **II.c -** Themodule is further re-expanded by doubling the vertical strut and creating horizontal cables between the doubled nodes. **II.d** - Themodule in this figure obtains just as before.
✶ **Form-finding condition.** This condition requires that there the aforementioned tetrahedron exists.

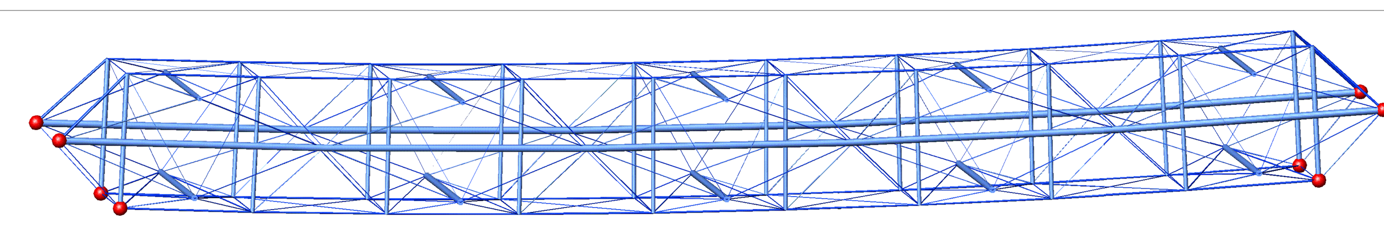
Third Expansion Sequence

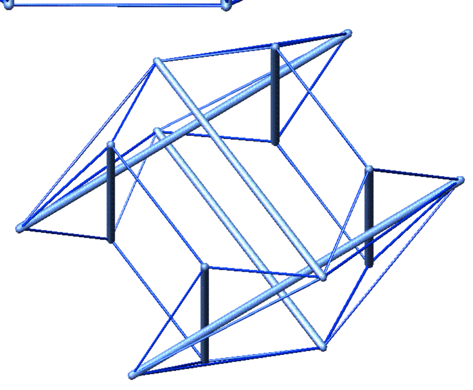
The starting octahedron (Fig. **III.a**) has two self-stress states of the type shown in Fig. **1.c**. Two struts are represented in a diagonal configuration; two of the four unloaded elements are vertical, the remaining two are removed. **III.b** - The horizontal strut is doubled, two new vertical cables are created; the other vertical elements are tensioned. **III.c** - One diagonal strut is doubled, as well as the four cables connected to it; two horizontal cables are created between doubled nodes. **III.d** - The other diagonal element is doubled and two horizontal cables are created.
✶ **Form-finding condition.** In this special case this condition is always satisfied.

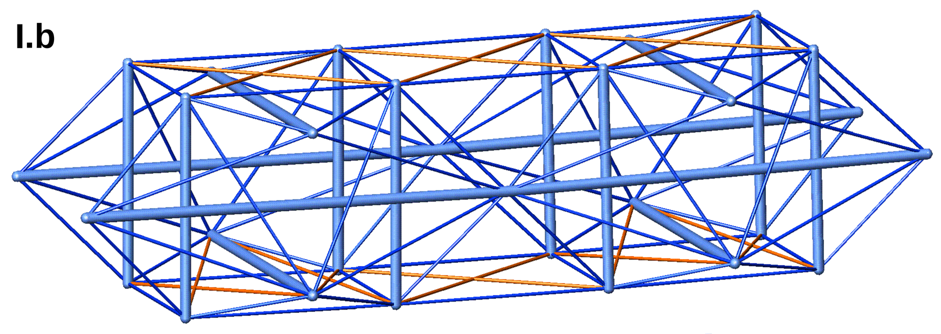
4. Examples

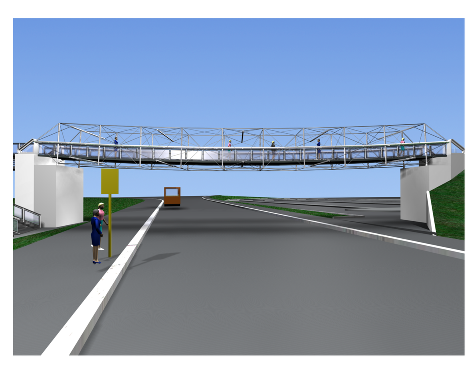


I.a 





I.b 

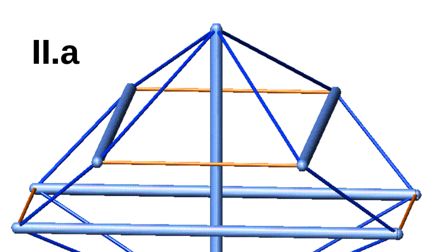


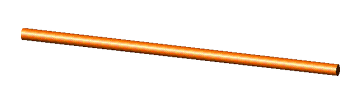
The TVF

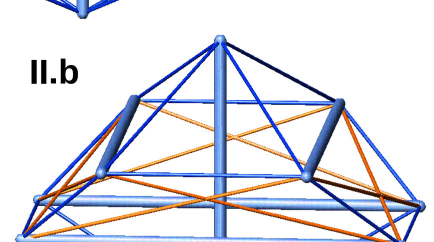
The **TorVergata Footbridge** is to be built near the School of Engineering of the University of RomeTorVergata, to allow safe pedestrian crossing of a motor way bordering the Campus. Our choice of a tensegrity concept has been influenced by the sense of transparency and lightness emanating from the large tensegrity sculp/struc-tures realized by K. Snelson, which are looked at not only from a distance but also from their inside: we have chosen a structure people become familiar with as they walk through it.
We chose the module in Fig. **I.f** in Section 3 because it enjoys a wide `cross-sectional' space and the smallest number of mechanisms. The module is 3 m wide and 2.6 m high, enough to accomodate a deck that can be walked on confortably. The footbridge consists of 5 modules and has a span of 32 m in total; it has a 'banana shape', to have a higher geometric stiffness at the expenses of heavier foundations. Modules are to be assembled in the plant. For ease of transportation, some elements are detached from the module allowing for folding into a compact bundle (Figs. **I.a** and **I.b** in Section 5). On location, the modules are assembled together and supplementary elements are added. Finally, the deck is installed, and the whole footbridge is lifted up and placed on its supports. Figs. **I.a** and **I.b** show two sets of stiffening elements: the first set resists vertical actions, the second horizontal and torsional actions (this type of structure is especially sensitive to torsional actions).

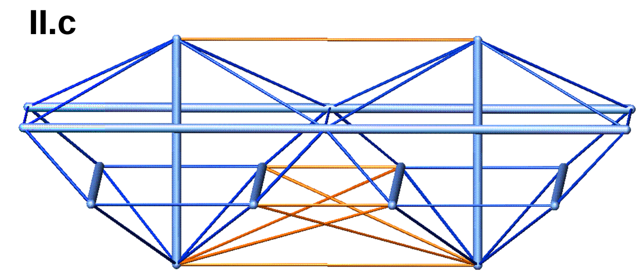
A Plate-like Assembly

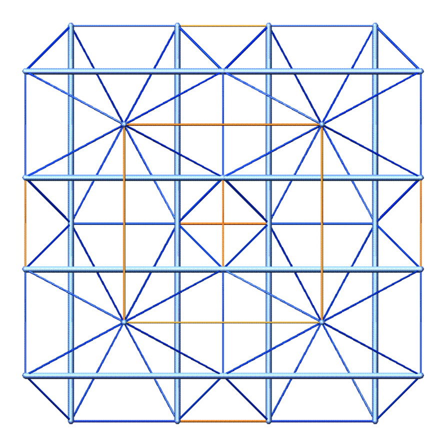
Modules of the second type are well-suited to realize `two-dimensional' cable-strut structures. Bracing elements can be employed both to remove global mechanisms and to stiffen the structure in correspondence of concentrated actions.
Fig. **II.a** shows the module of Fig. **II.b** in Section 3 with a first set of four additional cables; the result is a tensegrity system with less mechanisms; a second set of bracing elements (Fig. **II.b**) removes allmechanisms and gives the module a higher stiffness.
Fig. **II.c** shows a 2-module assembly realizing straight compressive and tensile paths; additional bracings are also shown.
Fig.s **II.d** and **II.e** show grids with 2 by 2 and 5 by 5 modules, respectively, without bracing elements.

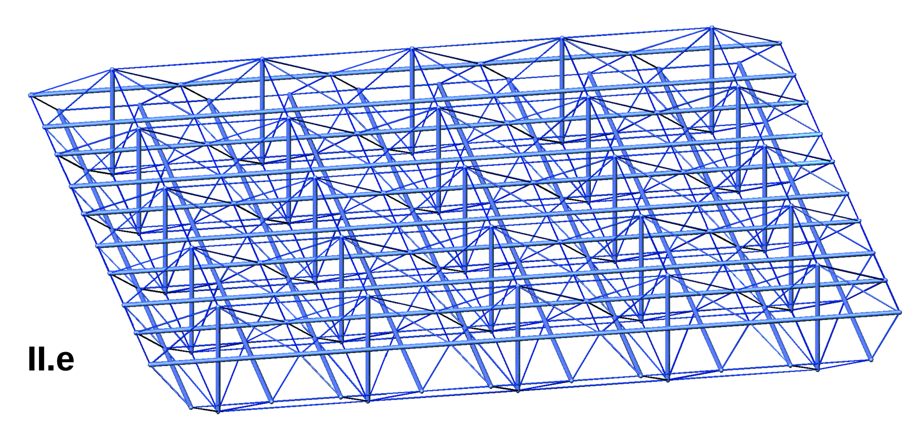
II.a 



II.b 

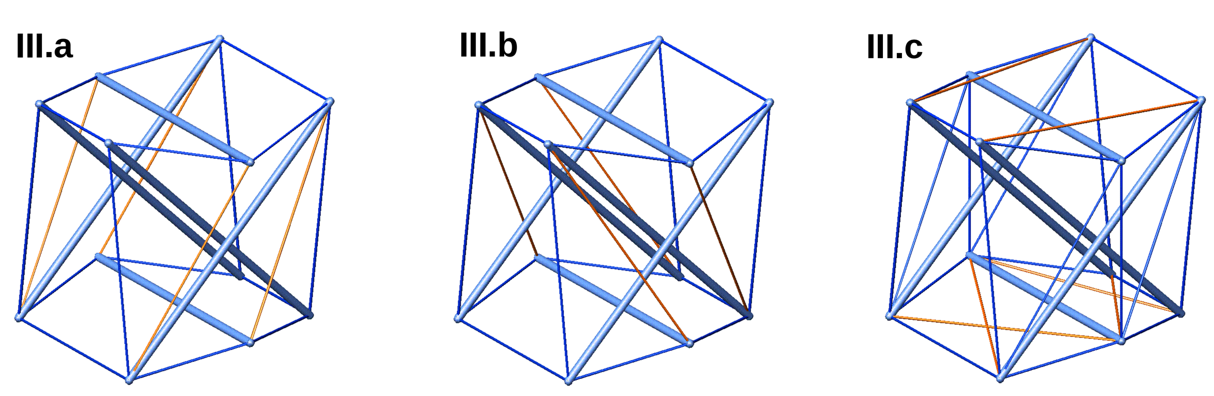
II.c 

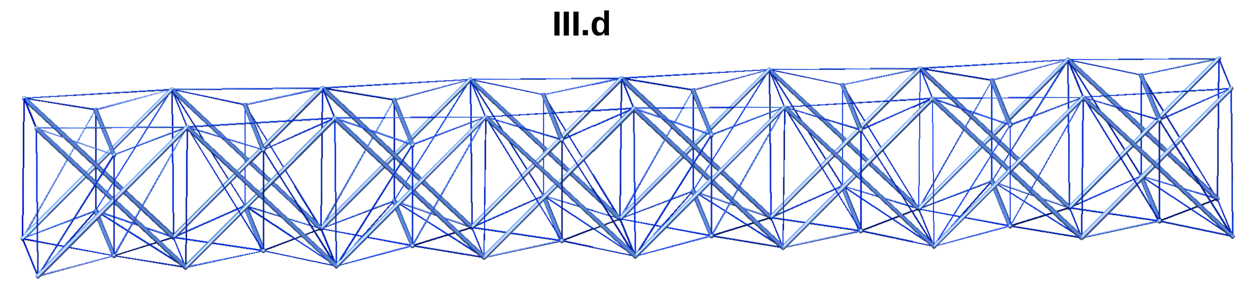
II.d 

II.e 

A Beam-like Assembly

Modules of the third type need no intermodule elements to be assembled; the intrinsic module stiffeness suffices.
Both Fig. **III.a** and Fig. **III.b** show the module in Fig. **III.d** in Section 3 with four additional cables replacing the vertical ones; in both cases the result is another type of tensegrity system.
Fig. **III.c** shows certain admissible placements for bracing and horizontal elements (note that horizontal elements are not anymore necessary).
Fig. **III.d** shows an 8-module beam. As for the **TVF**, this structure suffers torsional actions, a problem that can be eliminated by employing the stiffer module in Fig. **III.c** in Section 3.



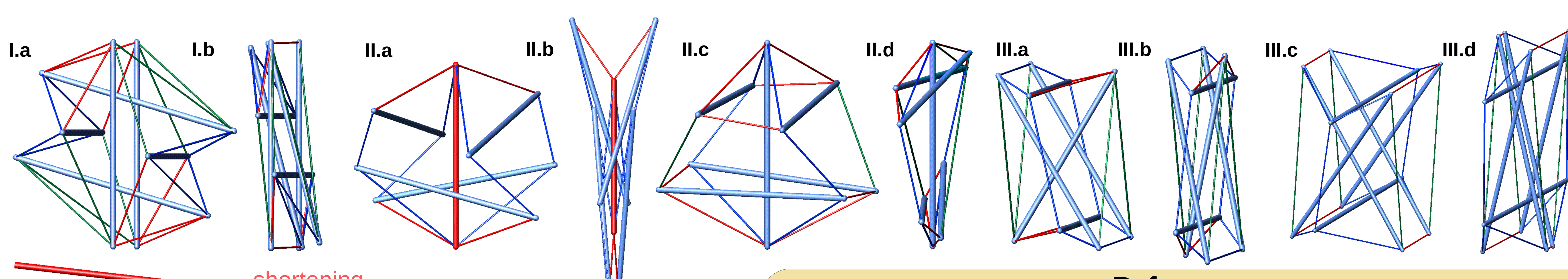
III.d 

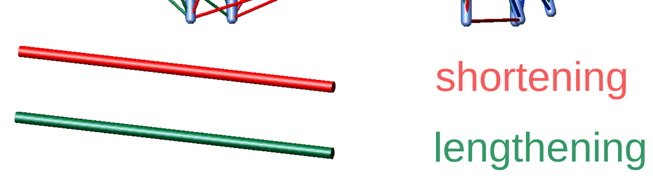
5. Folding Strategies

Tensegrity systems change their shape through a **form-finding** process; they pass from one tensegrity placement to another by changing the length of two (or more) elements at the same time. Prof. **W.O. Williams** (at Carnegie Mellon University, Pittsburgh, PA) has developed the analytic framework we here use to study such shape changes as **folding processes**, according to which the system follows a continuous path in the space of tensegrity placements.

Folding Tips

Active and passive modes. A system can be folded either by means of embedded **actuators** changing the length of some elements (**active mode**) or with the aid of **external forces**, e.g., by human action (**passive mode**). In the former mode, the prestress state is continuously controlled and the transformation choices are reduced. In the latter mode, it is sufficient to disconnect some cables to fold the system.Williams' analytic approach allows to design both.
- **Strut and cable modes.** If both struts and cables change their length (**strut mode**) then more space is saved than when only cables do (**cable mode**), because struts usually are the longest elements; on the other hand, struts usually are the most stressed elements as well, hence the strut mode requires higher actuating energy.
- **Number of mechanisms.** Many mechanisms mean an easy-to-fold systems, possibly exhibiting equilibrium bifurcations or snap-through phenomena.
- **Kinematic compatibility between adjacent modules.** This condition affects the nodes shared by adjacent modules and the intermodule cables.





Figures

I.a,I.b - Cable Mode, Passive (a similar strategy works for the module of the TorVergata Footbridge).
II.a, II.b - Strut Mode, Passive (the kinematic compatibility condition between adjacent modules of a grid would not be satisfied were this foldingmodeactive).
II.c, II.d - CableMode, Active.
III.a, III.b - Cable Mode, Active or Passive.
III.c, III.d - Cable Mode (all elements involved are shared with the adjacent modules).

References

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