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Analyzing the reliability of degradable networks

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ABSTRACT

The reachability of a strongly connected network may be destroyed after link damage. Since many networks have directed links with the potential for reversal, the reachability may be restored by reversing the direction of links. In this paper, the reliability of a network that allows reversal of links is discussed.

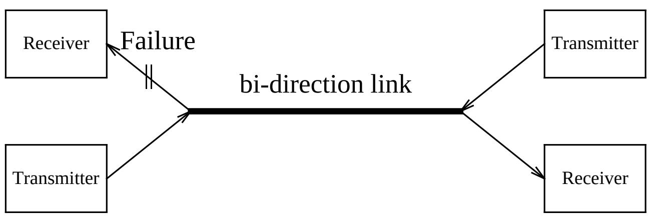
Index Terms: Network reliability, degradable network, network reconfiguration, fault tolerant system, fault tolerant network, strongly connected network.

1. Introduction

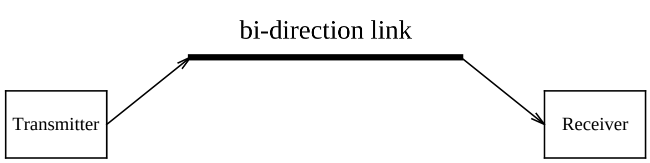
Some networks are vulnerable to failure. A fault in any node or cable segment breaks the network and may bring network operations to an immediate halt. To improve the reliability of networks, the failed network may be reconfigured, thereby obtaining protection from failures while still retaining overall network connectivity. To protect networks from failure, the designers have made great strides and a number of reconfiguration techniques that deal with network recovery have been discussed in the literature [1, 2, 3, 4, 5, 6, 7, 8].

Many systems with potentially reversible directed links are represented by directed networks or mixed networks. For instance, fiber-optic networks are the preferred choice for local area networks, because of the lower cost of uni-directional fiber- optic links. At present, the majority of fiber-optic networks use uni-directional fiber-optic links which can be reversed by interchanging transmitters and receivers.

Although actual links are bi-directional in some networks, in fact, each physical bi-directional link consists of two dedicated uni-directional links. Even though the physical link between two nodes is bi-directional, the direction of the signal, which propagates along the link, is restricted by the transmitter and receiver. A transmitter or receiver failure may destroy signal propagation in only one direction. As shown in Figure 1.1, a bi- directional link becomes a uni-directional link, shown in Figure 1.1(b), and the undirected network becomes a *mixed network* which includes both directed and undirected links. Therefore, to model undirected networks with failure, two uni-directional links in opposing directions should be used instead of a bi-directional link between two nodes. This is also true for frequency-division multiplexing (FDM) networks because the signals in different directions are transferred in different frequency bands along a link and the direction is determined by the transmitters and receivers of nodes. The communication path is equivalent to a number of uni-directional links between the two nodes which correspond to different frequency bands.



(a) Bi-directional link with one receiver failure



(a) A uni-directional link results due to a receiver failure

Figure 1.1

In general, a network is more suitably describe d by a directed network in orderto analyze its reliability. Therefore it is reasonable to study reconfiguration networks by using directed networks and assuming their directed links are reversible.

1.1. Previous results

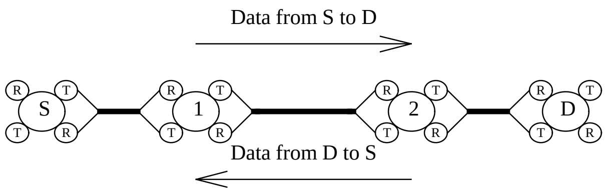
Based on directed networks, extending the results of Robbins [9] and Boesch-Tindell [10], W. Shi [11] developed a reconfiguration approach by using link reversal to reconnect a non-strongly connected network. This non-strongly connected network was created from a strongly connected network through link or node failures. A linear time algorithm which, when reachability has been destroyed by the removal of a single link, optimally restores reachability through the reversal of selected links was developed by [11]. Multi-link failure reconnectability was discussed and an algorithm with polynomial complexity is given which provides a near optimum solution to reconnect the network. Shi [11] shows that allowing link reversals improves the reliability of the network by at least a factor of two. For networks in which reconnection cannot be established through link reversal, reachability is maximized.

1.2. Reliability of links

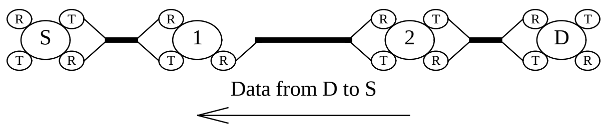
In an undirected network, the unit of measurement of reliability is based on an undirected link. In order to study a degradable network, we represent an undirected link by two directed links in opposite directions. Each one of these directed links will be called *unit directional link*, and its reliability is denoted by $R_{1/2}$. In the following sections, the study of the reliability of degradable networks is based upon the reliablility of the unit directional links.

2. Degradation of failed networks

When an undirected communication network is operating normally, messages or data are transmitted back and forth between two nodes, say S and D, along an undirected path (or channel) in opposite directions, shown in Figure 2.1(a), where S, 1, 2 and D indicate nodes, R and T indicate receiver and transmitter respectively. A transmitter or receiver failure of intermediate nodes may destroy the channel in only one direction. Figure 2.1(b) shows that a transmitter of node 1 has failed. If there is another undirected path between these two nodes, the transmitted message may be sent via the second channel. The interchange of messages or data between S and D can still be carried on as normal, that is, messages can be transmitted along this second undirected path in opposite directions. We call such an operation a *zero order degraded operation.* If bi-directional (i.e. undirected) paths exist between any pair of nodes in a network after failures occur, the network is referred to as a *zero order degraded network.*



(a) Transferring data in opposite directions along the same channel



(b) A transmitter of node 1 is faulty

Figure 2.1

When transmitter/receiver failures of an intermediate node on the second path occur, the second channel may also be destroyed in only one direction. If no more bi-directional paths between D and S exist in the network, then there are only two uni- directional paths between nodes D and S. Furthermore, if these two uni-directional paths are in opposite directions, messages or data can still be transmitted between the two nodes along these two opposite uni-directional links. The operation under this condition is different from normal operation and is called *first order degraded operation,* and such networks are referred to as *first order degraded networks.* In the first order degraded network, the operations contain both zero order degraded operation, if two nodes are connected by a bi-directional path, and first order degraded operation, if two nodes are connected only by two uni-directional paths in opposite directions. Figure 2.2 shows that a receiver of node 2 and a transmitter of node 3 are faulty. The messages from S to D can be transmitted through path $S \rightarrow 1 \rightarrow 2 \rightarrow D$, and the messages from D to S through path $D \rightarrow 4 \rightarrow 3 \rightarrow S$.

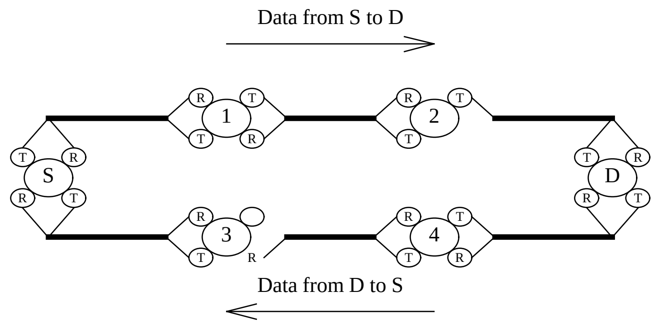
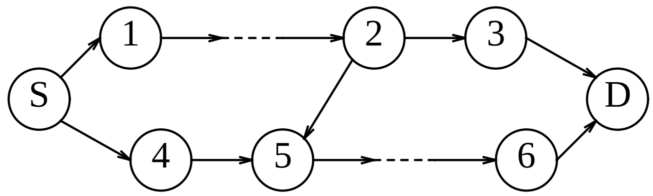
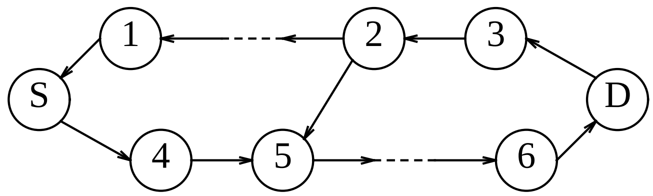


Figure 2.2 First order degraded network

First order degraded operation requires that at least two uni- directional paths exist in opposite directions between two nodes. In some cases two uni-directional paths exist between two nodes, but are not in opposite directions, as shown in Figure 2.3(a). Therefore, the network is not strongly connected. In order to reconnect the network, the direction of links may be reversed to create two directed paths in opposite directions. A reconnected network can be established by reversing the links as shown in Figure 2.3(b). Reversing links can be achieved by interchanging the transmitters and receivers, as shown in Figure 2.4. Figure 2.4(a) shows that a receiver of node S, a receiver of node 1 and a receiver of node 2 on the path are failed respectively. The bi-directional path becomes uni-directional (the direction is from S to D). Figure 2.4(b) shows that switching the connection from transmitters to receivers at node S, node 1 and node 2, respectively, can redirect the path.

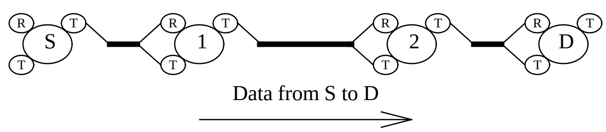


(a) Nodes S and D are not strongly connected

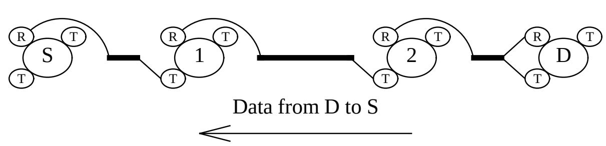


(b) Restored first order degraded network

Figure 2.3



(a) Uni-directed path from S to D



(b) Redirecting the links by interchanging transmitters and receivers

Figure 2.4

In a directed network, whose underlying graph contains bridges, strong connectivity cannot be achieved through link reversals. At this stage, the messages can only be transmitted among nodes within the same strongly connected sub-network. Such a network is called a *second order degraded network.* In a second order degraded network, messages among nodes within a component are transmitted in accordance with either zero order degraded operation or first order degraded operation. However, to exchange messages among components, temporary reversals of bridges may be required. This is called *second order degraded operation.* A special case of a second order degraded network is a tree structured network, each component of the network consists of a signal node only and the underlying network is connected.

A network is referred to as a *third order degraded network* if the network is disconnected. In other words, a third order degraded network consists of a number of separated sub-networks without link connections among them. These sub-networks are either zero order, or first order, or second order degraded sub- networks. In a third order degraded network, messages can be transmitted within a sub-network in accordance with either zero , or first order, or second order degraded operation.

3. State probability of degradable networks

Assume that in a given network N, l unit directional links fail. We denote the resulting network by N'. According to the above description N' may be a zero, first, second, or third order degraded network. Let $P_i(l)$ be the probability that N' is an ith order degraded network. To investigate the state probability $P_i(l)$, two important concepts have to be defined.

Given an undirected network, two nodes n_i and n_j are K *link connected*, if there are K undirected paths without common links between n_i and n_j. A network is called *minimum K link connected* if there does not exist a pair of nodes less than K link connected in the network.

An undirected network is called *maximum U link removable* if there exist U links that can be removed without disconnecting the network, but any removal of $U+1$ links creates disconnection.

These two concepts are easily adapted to directed networks: A directed network is called *maximum D unit directional link removable* if D unit directional links can be removed without generating bridges or disconnecting the network, but any removals of $D+1$ unit directional links creates bridges or disconnection.

Suppose an undirected network N has L undirected links and N is maximum U link removable. If N' is created by removing one unit directional link from each undirected link in N, and N is connected and has no bridges,a total of L unit directional links can be removed without creating bridges or disconnection. Furthermore, $U-1$ unit directional links can be removed from N' without creating bridges or disconnection, since N is maximum U links removable. Therefore, the undirected network N, viewed as a directed network by replacing each of its links by unit directional links in opposite directions, is maximum $L+U-1$ unit directional link removable.

Based on the two parameters defined above, we can investigate state probability and reliability of degradable networks.

Suppose a network N is a minimum K link connected and maximum U link removable. Let V and L denote the number of nodes and links in the network, respectively. Let $C_i(l)$ denote the number of ways that a network under consideration can operate at ith order degraded network state N_i when l failed links occur in a network. Let $C(l)$ denote the total number of ways that l links fail in the network. We have:

$$C(l) = \sum_{i=0}^{3} C_i(l) = \begin{pmatrix} L \\ l \end{pmatrix} = \frac{L!}{(L-l)!l!}$$

If the number of failed unit directional links is less than K (that is $0 \le l \le K-1$), N' remains in state N_0 (zero order degraded), since N is assumed to be minimum K link connected. Therefore, the state probability $P_0(l) = 1$ for $0 \le l \le K-1$. As soon as more than $2U$ failures of unit directional links occur, N' cannot remain in state N_0, since N is assumed to be maximum U link removable and $P_0(l) = 0$ in this case. When the number of failed unit directional links is more than $K-1$ and less than $2U$ ($K \le l \le 2U$), N' could operate either in state N_0 or state N_1 or state N_2 or state N_3 depending on the topology of the network, and we obtain the following formula for $P_0(l)$:

$$P_0(l) = \begin{cases} 1 & 0 \le l \le K-1 \\ \dfrac{C_0(l)}{C(l)} & K \le l \le 2U \\ 0 & 2U+1 \le l \le 2L \end{cases} \qquad 3.1$$

Similarly, we calculate $P_i(l)$ for $i=1,2,3$:

$$P_1(l) = \begin{cases} 0 & 0 \le l \le K-1 \\ \dfrac{C_1(l)}{C(l)} & K \le l \le L+U-1 \\ 0 & L+U \le l \le 2L \end{cases} \qquad 3.2$$

$$P_2(l) = \begin{cases} 0 & 0 \le l \le 2(K-1) \\ \dfrac{C_2(l)}{C(l)} & 2K-1 \le l \le L+U \\ 0 & L+U+1 \le l \le 2L \end{cases} \qquad 3.3$$

$$P_3(l) = \begin{cases} 0 & 0 \le l \le 2K-1 \\ \dfrac{C_3(l)}{C(l)} & 2K \le l \le 2L-V+1 \\ 1 & 2L-V+2 \le l \le 2L \end{cases} \qquad 3.4$$

State probabilities $P_0(l)$, $P_1(l)$, $P_2(l)$ and $P_3(l)$ of network N is plotted below:

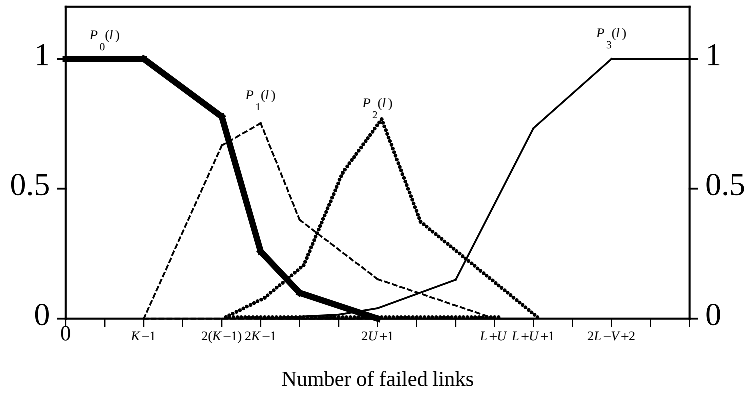


Figure 3.1.
State probability $P_0(l)$, $P_1(l)$, $P_2(l)$ and $P_3(l)$

As an example, the network in Figure 3.2 is minimum 2 undirected link connected, and maximum 2 undirected link removable. $L = 5$ and $V = 4$, respectively.

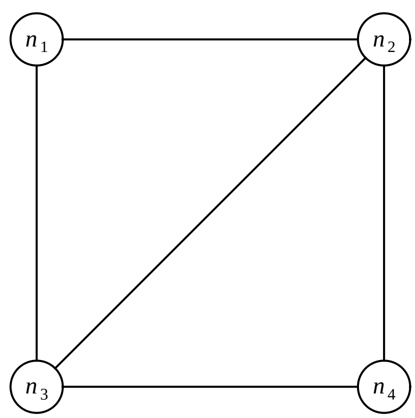


Figure 3.2

The number of ways that the network in Figure 3.2 can operate as ith order degraded network, $C_0(l)$, $C_1(l)$, $C_2(l)$ and $C_3(l)$, is calculated in Table 3.1.

Table 3.1. Number of ways Fig.3.2 operates as i th order degraded network after failure of l links

State	l=0	l=1	l=2	l=3	l=4	l=5	l=6	l=7	l=8	l=9	l=10
$C_0(l)$	1	10	35	31	8	0	0	0	0	0	0
$C_1(l)$	0	0	10	80	158	96	32	0	0	0	0
$C_2(l)$	0	0	0	9	42	150	160	32	0	0	0
$C_3(l)$	0	0	0	0	2	6	18	88	45	10	1
$C(l)$	1	10	45	120	210	252	210	120	45	10	1

The state probabilities of this network were computed from Equations 3.1, 3.2, 3.3 and 3.4 and are shown in Table 3.2:

Table 3.2. State probability $P_i(l)$

State	l=0	l=1	l=2	l=3	l=4	l=5	l=6	l=7	l=8	l=9	l=10
$P_0(l)$	1	1	0.777	0.258	0.038	0.000	0.000	0.000	0	0	0
$P_1(l)$	0	0	0.222	0.666	0.752	0.38	0.152	0.000	0	0	0
$P_2(l)$	0	0	0.000	0.075	0.2	0.586	0.762	0.267	0	0	0
$P_3(l)$	0	0	0.000	0.000	0.0095	0.0238	0.0857	0.733	1	1	1

The state probabilities $P_0(l)$, $P_1(l)$, $P_2(l)$ and $P_3(l)$ of the network in Figure 3.2 are plotted in Figure 3.3.

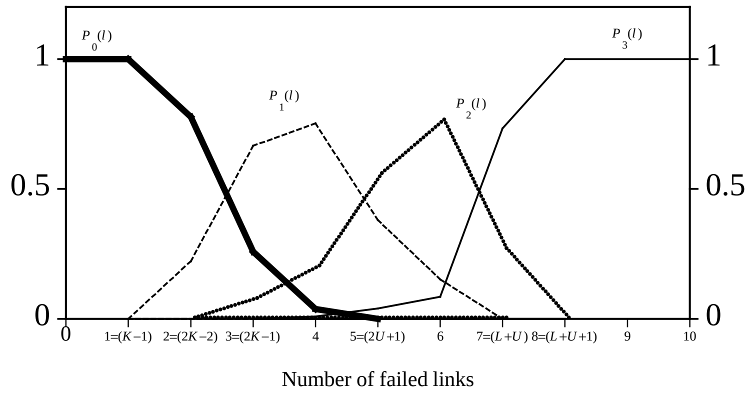


Figure 3.3. State probabilities of Figure 3.2

In a network with reversal strategy, the probability of proper operation is $P_0(l) + P_1(l) + P_2(l)$, since up to second order degraded operation every node can communicate with every other node and proper network operation comes to a halt only when third order degradation is reached. If link reversals are not allowed, the probability for proper operation of the network is only $P_0(l)$, and we see that a degradable network is much more reliable. For our example in Figure 3.2 we have:

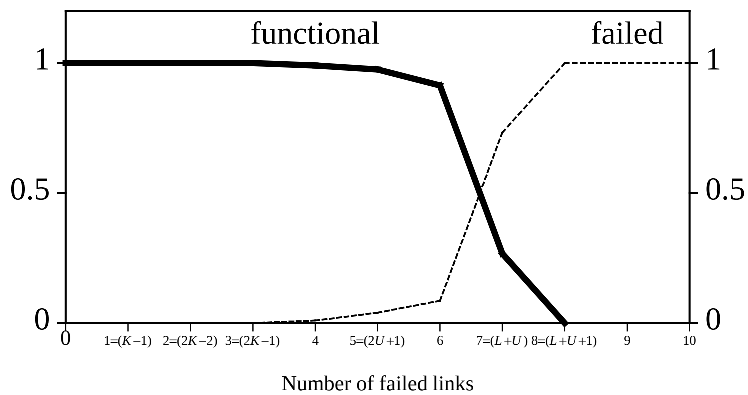


Figure 3.4. State probability of Figure 3.2 as a degradable network

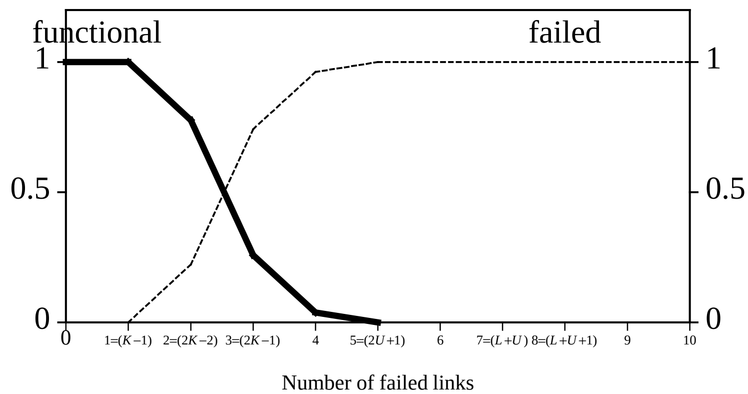


Figure 3.5. State probability of Figure 3.2 as network without reconfiguration

4. The reliability of degradable networks

Given the reliability, $R_{1/2}$, of a unit directional link, we can now easily compute the state probability $P_i(l)$, which is defined to be the probability that the network is i th order degraded. The probability that l links are failed in network N is $R_{1/2}^{L-l}(1-R_{1/2})^l$.

>From equations 3.1 to 3.4 we obtain:

$$R_0 = \sum_{l=0}^{K-1} [C(l)\, R_{1/2}^{L-l}(1-R_{1/2})^l] + \sum_{l=K}^{2U} [C_0(l)\, R_{1/2}^{L-l}(1-R_{1/2})^l] \qquad 4.1$$

$$R_1 = \sum_{l=K}^{L+U-1} [C_1(l)\, R_{1/2}^{L-l}(1-R_{1/2})^l] \qquad 4.2$$

$$R_2 = \sum_{l=2K-1}^{L+U} [C_2(l)\, R_{1/2}^{L-l}(1-R_{1/2})^l] \qquad 4.3$$

$$R_3 = \sum_{l=2K}^{2L-V+1} [C_3(l)\, R_{1/2}^{L-l}(1-R_{1/2})^l] + \sum_{l=2L-V+2}^{2L} [C(l)\, R_{1/2}^{L-l}(1-R_{1/2})^l] \qquad 4.4$$

We define that network N is functional, if N remains in zero, first or second order degraded state. when l failed links occur. Therefore, the reliability of a degraded network is $R = R_0 + R_1 + R_2 = 1 - R_3$, that is:

$$R = \sum_{l=0}^{2K-1} [C(l)\, R_{1/2}^{L-l}(1-R_{1/2})^l] +$$

$$\sum_{l=2K}^{2U} [C_0(l)+C_1(l)+C_2(l)]\, R_{1/2}^{L-l}\,(1-R_{1/2})^l \;+$$

$$\sum_{l=2U+1}^{L+U-1} [C_1(l)+C_2(l)]\, R_{1/2}^{L-l}\,(1-R_{1/2})^l + [C_2(L+U)]\, R_{1/2}^{L-l}\,(1-R_{1/2})^l$$

As an example, consider again the network in Figure 3.2 . Suppose the reliability of a unit directional link is $R_{1/2} = \frac{1}{2}$. Therefore, $R_{1/2}^{L-l}(1-R_{1/2})^l = \left(\frac{1}{2}\right)^L = \left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}}$. A table listed below shows $C_i(l)R_{1/2}^{L}(1-R_{1/2})^l$ and R_i:

State	$l=0$	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$	$l=6$	$l=7$	$l=8$	$l=9$	$l=10$	R_i
$C_0(l)/2^{10}$	0.001	0.01	0.034	0.03	0.008	0	0	0	0	0	0	0.083
$C_1(l)/2^{10}$	0	0	0.01	0.078	0.154	0.094	0.031	0	0	0	0	0.367
$C_2(l)/2^{10}$	0	0	0	0.009	0.041	0.146	0.156	0.031	0	0	0	0.384
$C_3(l)/2^{10}$	0	0	0	0	0.002	0.006	0.018	0.086	0.044	0.01	0.001	0.166

For the reliability of the network in Figure 3.2 we get: $R = \frac{85+376+393}{1024} = \frac{854}{1024} = 0.83$. This is the reliability of the network with degradation.

Now to evaluate the improvement of degradable networks, over networks that do not allow link reversals, we compute the reliability of this network considered as undirected network. To this end we first have to link the reliability of the undirected without degradation. Assuming the reliability of receiver, transmitter and bi- directional link are $r = r_l = (R_{1/2})^{\frac{1}{3}} = 0.79$ (see Figure 1.1), the reliability of undirected link is $R_l = r^4 r_l = r^5 = \left(\frac{1}{2}\right)^{\frac{5}{3}} = 0.314$. The reliability of the undirected network becomes:

$$R = 1 - \sum_{l=0}^{5} C(l) R_l^{5-l}(1-R_l)^l$$

where $C(l)$ denotes the number of ways that the network under consideration can be retained functional when l links are failed. Substituting our numerical values we obtain: $R = R_l^5 + 5R_l^4(1 - R_l) + 8R_l^3(1 - R_l)^2 = 0.154$

5. Degradable networks with repair

In the above discussion we do not consider that the failed links can be repaired. To study the reliability of a network with repair, queuing theory can be adapted.

5.1. Networks with single repair man

First, consider a degradable network with single repair man, containing L links. Suppose the unit directional link failure rate is $\lambda = 1 - R_{1/2}$ and the repair rate is μ for a single repair man. The failure-repair state diagram is shown in Figure 5.1.1

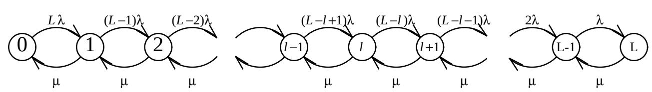


Figure 5.1.1 m repair men failure-repair state diagram

The probability q_l that exactly l links are failed, can be derived from above diagram:

$$L\lambda q_0 = \mu q_1$$

$$(L-1)\lambda q_1 = \mu q_2$$

$$(L-2)\lambda q_2 = \mu q_3$$

$$\ldots\ldots$$

$$(L-l)\lambda q_l = \mu q_{l+1}$$

$$\ldots\ldots$$

$$(L-1)\lambda q_L - 1 = \mu q_L$$

>From above expressions we have:

$$q_l = \frac{L!}{(L-l)!}\left(\frac{\lambda}{\mu}\right)^l q_0 = \frac{L!}{(L-l)!}\rho^l q_0 \qquad 5.1.1$$

Where $\rho = \dfrac{\lambda}{\mu} < 1$. The normalization condition summed over all links gives:

$$\sum_{l=0}^{L} q_l = 1 \qquad 5.1.2$$

Substitute Equation 5.1.1 into Equation 5.1.2

$$q_0 \Big[\sum_{i=0}^{L} \frac{L!}{(L-i)!}\rho^i\Big] = 1$$

$$q_0 = 1 \Big/ L!\Big[\sum_{i=0}^{L} \frac{\rho^i}{(L-i)!}\Big] \qquad 5.1.3$$

Therefore, the probability of exactly l failed links in a reparable network is

$$q_l = \frac{L!\rho^l}{(L-l)!\Big[\sum_{i=0}^{L} \frac{L!}{(L-i)!}\rho^i\Big]} = \frac{\rho^l}{(L-l)!\Big[\sum_{i=0}^{L} \frac{\rho^i}{(L-i)!}\Big]} \qquad 5.1.4$$

5.2. Networks with m repair men

Let L, λ and μ be as above. For a network with m repair men we have the following failure-repair state diagram (Figure 5.2.1.).

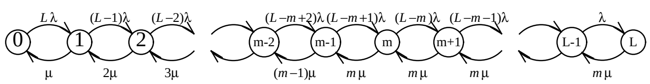


Figure 5.2.1. m repair men failure-repair state diagram

q_l, as derived from above diagram:

$$\begin{cases} (L-l)\lambda q_l = (l+1)\mu q_{l+1} & 0 \le l \le m \\ (L-l)\lambda q_l = m\mu q_{l+1} & m \le l \le L \end{cases}$$

$$\begin{cases} q_l = \dfrac{L!}{(L-l)!\,l!}\rho^l q_0 = C_L^l \rho^l q_0 & 0 \le l \le m \\ q_l = \dfrac{l!}{\cdots^{\,l-m}} C_L^l \rho^l q_0 & m \le l \le L \end{cases} \qquad 5.2.1$$

Where $\rho = \dfrac{\lambda}{\mu} < 1$. The normalization condition summed over all links gives:

$$\sum_{l=0}^{L} q_l = 1 \qquad 5.2.2$$

Substituting Equation 5.1.4 into Equation 5.2.1 we obtain:

$$q_0\Big[\sum_{i=0}^{m} C_L^i \rho^i + \sum_{i=m+1}^{L} \frac{i\,!}{m\,!m^{i-m}} C_L^i \rho^i\Big] = 1$$

$$q_0 = \Big[\sum_{i=0}^{m} C_L^i \rho^i + \sum_{i=m+1}^{L} \frac{i\,!}{m\,!m^{i-m}} C_L^i \rho^i\Big]^{-1} \qquad 5.2.3$$

5.3. The reliability of reparable degradable networks

Once we have the probability, q_l, of l failed links in a network, we can substitute $R_{1/2}^{L-l}(1 - R_{1/2})^l$ for q_l in Equation 4.1, 4.2, 4.3 and 4.4, respectively, where q_l is given by Equation 5.1.3 for single repair man, or given by Equation 5.2.2 for m repair men, to compute the reliability of a degradable network with repair.

>From Equation 4.1, the reliability R_0 of a zero order degraded network with repair is:

$$R_0 = \sum_{l=0}^{2U}\Big[\frac{C_0(l)}{c(l)}\, q_l\Big] = \sum_{l=0}^{K-1}[q_l] + \sum_{l=K-1}^{2U}\Big[\frac{C_0(l)}{C(l)}\, q_l\Big]$$

>From Equation 4.2, the reliability R_1 of a first order degraded network with repair is:

$$R_1 = \sum_{l=K}^{L+U-1}\Big[\frac{C_1(l)}{C(l)}\, q_l\Big]$$

>From Equation 4.3, the reliability R_2 of a second order degraded network with repair is:

$$R_2 = \sum_{l=2K-1}^{L+U}\Big[\frac{C_2(l)}{C(l)}\, q_l\Big]$$

>From Equation 4.4, the reliability R_3 of a third order degraded network with repair is:

$$R_3 = \sum_{l=2K}^{2L} [\frac{C_3(l)}{C(l)} q_l] = \sum_{l=2K}^{2L-V+1} [\frac{C_3(l)}{C(l)} q_l] + \sum_{l=2L-V+2}^{2L} [C(l)\, q_l]$$

6. Conlusions

Based on results of [11], The concept of degradable networks was proposed. In our model, directed networks are used, and redirection of directed links is achieved through exchange of transmitters and receivers in the network. As links are failing in the network, operation may remain normal, (zero order degradation), require redirection of links in order to maintain strong connectivity of the network, (first order degradation), require temporary redirection of bridges, (second order degradation), or come to a halt (third order degradation). Given the reliability of a directed link, the reliability of degradable networks is computed, by computing the probabilities for each of the possible states of the network. Numerical examples are discussed, which show the improvement obtained by the reversal strategy.

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