Discreete Time

Continuous Time

z - transform ⟷ Laplace transform

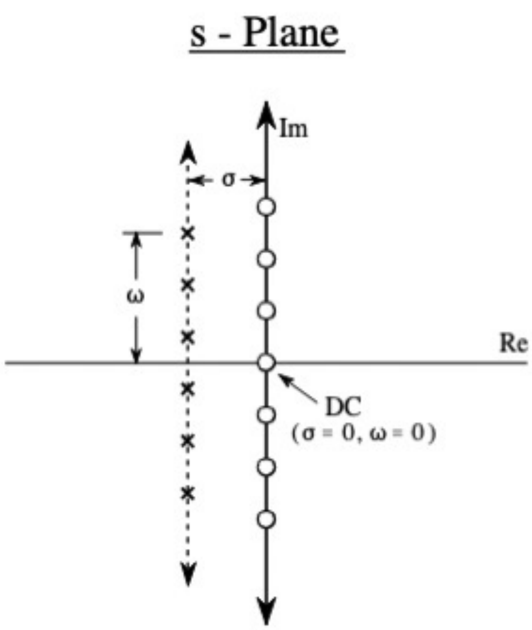
⟵ Fourier Transform ⟶

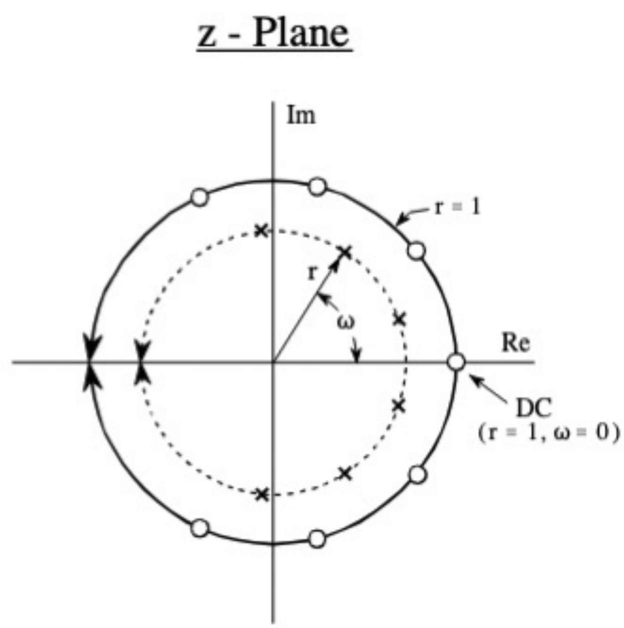
Z - Transform

Z- Transform converts a discrete-time signal, which is a sequence of or numbers, into (z-domain or z-plane) representation.

In Laplace Transformation → - domain's line

In Z-Transformation → in z domain.





- z – Transform is defined by –

for z

$X(z)$

$= r(\cos\omega + j\sin\omega)$

$x[n] \Longleftrightarrow X(z)$

With z – Transform, the s-plane is represented as a set of complex signals.

For any given LTI system, some of these signals may cause of the system to , while others may cause the output to

The set of signals that cause the system's output to lies in the

$X(z)$ is a power series → must specify the ROC
values of z s.t.

power series. $\sum_{n=0}^{\infty} a_n (x-c)^n = \sum_{n=0}^{\infty} a_0 + a_1 (x-c) + a_2 (x-c)^2 + \cdots$

z - Transform of Impulse function. $\delta[n]$

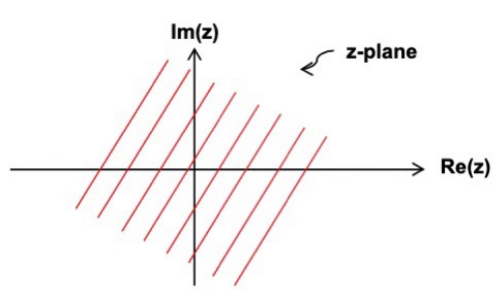
If $x[n] = \delta[n]$

$X(z)$

For $n<0 \rightarrow \delta[n]$

For $n>0$, $\delta[n] = 0$

ROC



z - Transform of a unit step

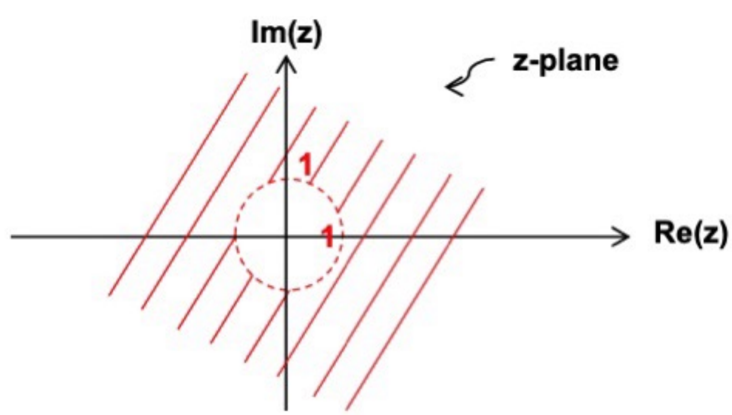
$x[n] = \mu[n]$

$X(z)$

$\frac{z}{z-1}$

not converging

ROC

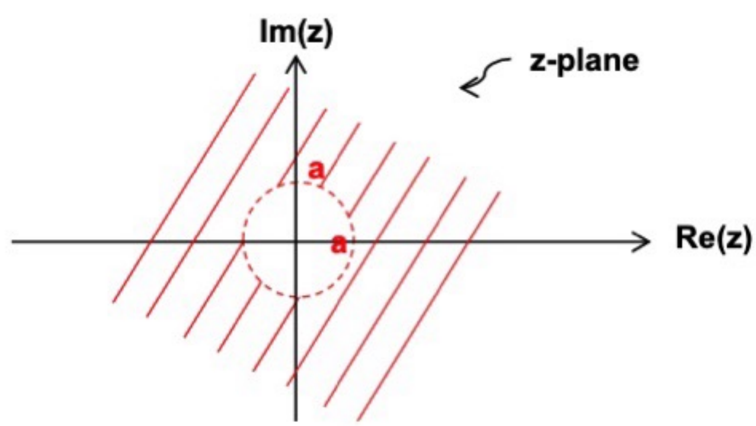


Z-Transform of a Power Series.

$x[n] = a^n u[n]$, $a > 0$ & real

$X(z)$

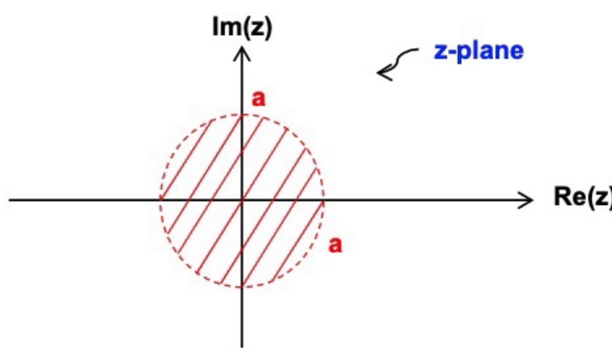
ROC



Z Transform of Anti Causal Power Series.

$$x[n] = -a^n \mu[-n-1] \quad , a > 0 \ \text{\& real}$$

ROC



z Transform of a finite duration sequence.

If $x[n] = \{ \underset{\substack{\uparrow \\ 0}}{4}, 1, -7 \}$

$X(z) =$

ROC

If $x[n] = \{8, 3, \underset{\uparrow}{2}, 1\}$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

ROC $\Rightarrow$

$x[n] = 4\mu[n] + 19\,\delta[n]$

ROC

ROC Properties

sequence is	Finite Duration	Infinite Duration

For some radii, r_a r_b

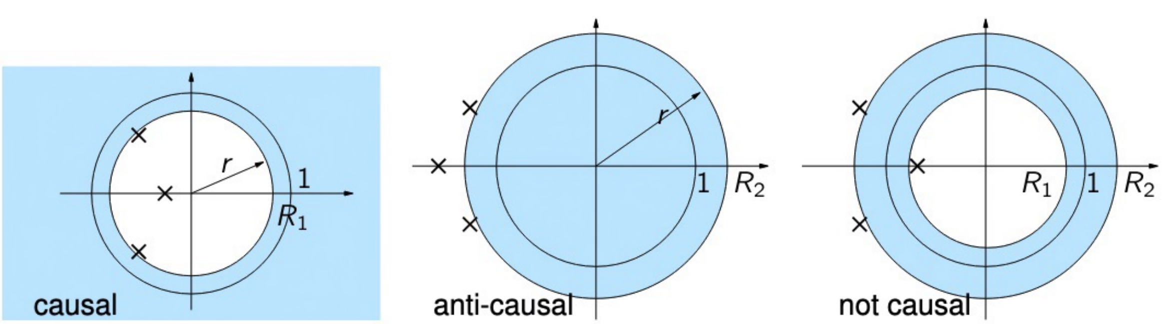


TABLE 3.3 SOME COMMON Z-TRANSFORM PAIRS

	Signal, $x(n)$	z-Transform, $X(z)$	ROC
1	$\delta(n)$	1	All z
2	$u(n)$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3	$a^n u(n)$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
4	$na^n u(n)$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
5	$-a^n u(-n-1)$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
6	$-na^n u(-n-1)$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
7	$(\cos\omega_0 n)u(n)$	$\dfrac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
8	$(\sin\omega_0 n)u(n)$	$\dfrac{z^{-1}\sin\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
9	$(a^n\cos\omega_0 n)u(n)$	$\dfrac{1-az^{-1}\cos\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
10	$(a^n\sin\omega_0 n)u(n)$	$\dfrac{az^{-1}\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

From Proakis and Manolakis

Z-Transform Properties

• Linearity

$$x[n] = a \cdot x_1[n] + b \cdot x_2[n]$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n}$$

ROC is intersection of

$ROC\, X_1$ and $ROC\, X_2$

example

$x_1[n] = \mu[n]$ $\quad$ $x_2[n] = \delta[n]$

Time Shifting

$$y[n] = x[n-m]$$

$$Y(z) = \sum_{n=-\infty}^{\infty} x[n-m]\, z^{-n}$$

Let, $l = n-m \rightarrow n = l+m$

$$Y(z) = \sum_{l+m=-\infty}^{\infty} x[l]\, z^{-(l+m)}$$

$$= \sum_{l=-\infty}^{\infty} x[l] \cdot z^{-l} \cdot z^{-m}$$

$$= z^{-m} \left[\sum_{l=-\infty}^{\infty} x[l] \cdot z^{-l} \right]$$

$$Y(z) = \begin{cases} \text{If } m > 0, \text{ remove } z=0 \text{ from ROC} \\ \text{If } m < 0, \text{ remove } |z| = \infty \text{ from ROC} \end{cases}$$

Time Reversal

$y[n] = x[-n]$

$Y(z) =$

$$\sum_{-m=-\infty}^{\infty} x[m]\, z^{m}$$

$$= \sum_{m=-\infty}^{\infty} x[m]\, [z^{-1}]^{-m}$$

$$\boxed{}$$

If $ROC_x : r_a < |z| < r_b$

Then

Convolution in Time

Let $x[n] = x_1[n] * x_2[n]$

$$= \sum_{m=-\infty}^{\infty} x_1[m]\, x_2[n-m]$$

$$X(z) = \sum_{n=-\infty}^{\infty} \left[\sum_{m=-\infty}^{\infty} x_1[m]\, x_2[n-m] \right] z^{-n}$$

Initial value Theorem

If $x[n] \rightarrow$ causal sequence

then $x[0] = \lim_{z \to \infty} X(z)$

Proof

for $x[n]$ causal sequenc.