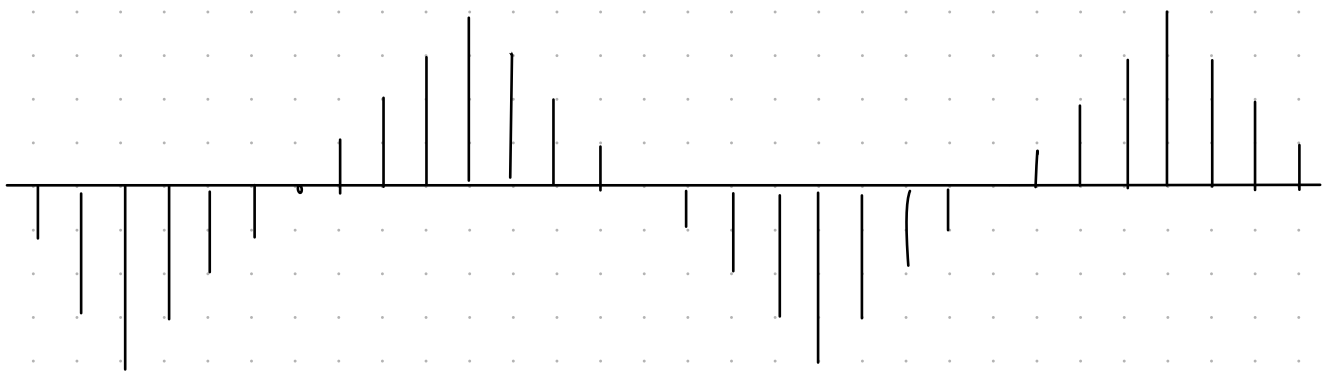
- A discrete time signal $x[n]$ is with period if



- The fundamental period is the smallest positive integer N for which the above equation holds and

- Set of all discrete-time complex exponential signals that are periodic with period N:

; $k = 0, \pm 1, \pm 2, \cdots$

- Discrete-time complex exponentials which in frequency by a multiple of are

• Representation of more general periodic sequences in terms of linear

of the sequence

$x[n] =$

varies over a range of N successive integers

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\left(\frac{2\pi}{N}\right)n} \rightarrow$$

$\rightarrow$

• Discrete-Time Fourier series pair:

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\omega_0 n} = \sum_{k=0}^{N-1} a_k e^{jk\left(\frac{2\pi}{N}\right)n}$$

$$a_k = \frac{1}{N} \sum_{k=0}^{N-1} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{k=0}^{N-1} x[n] e^{-jk\left(\frac{2\pi}{N}\right)n}$$

- a_k repeats as the signal is periodic.

$$a_{k+N} = a_k$$

$\boxed{\text{proof}}$ $$a_{k+N} = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j(k+N)\left(\frac{2\pi}{N}\right)n}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\frac{2\pi}{N}n} \cdot e^{-j\frac{2\pi}{\cancel{N}}n.\cancel{N}}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\frac{2\pi}{N}n} \cdot e^{-j2\pi n}$$

But, $$e^{-j2\pi n} = \underbrace{\cos(-2\pi n)}_{1} + \underbrace{j\sin(-2\pi n)}_{0} \quad ; \; n = 0, 1, \ldots, [N-1]$$

$$= 1$$

So, $$a_{k+N} = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\left(\frac{2\pi}{N}\right)n}$$

$$= a_k$$

a_k is periodic in N

- consecutive samples contain information.
- Usually select $k =$ → frequencies on

↓sampling freq.

Example

Consider the signal $x[n] = \sin \omega_0 n$. What are the fourier coefficients.

- $x[n]$ is periodic only if

- expanding the signal as sum of two complex exponentials.

$$\boxed{\sin x = \frac{e^{jx} - e^{-jx}}{2j}}$$

The fourier co-efficients are.

Example

$$x[n] = 1 + \sin\left(\frac{2\pi}{N}\right)n + 3\cos\left(\frac{2\pi}{N}\right)n + \cos\left(\frac{4\pi}{N}n + \pi/2\right)$$

find a_k values.

- expand as sum of two exponentials.

$$\boxed{\begin{aligned} \sin x &= \frac{e^{jx} - e^{-jx}}{2j} \\ \cos x &= \frac{1}{2}\left(e^{jx} + e^{-jx}\right) \end{aligned}}$$