

# Convolution Properties.

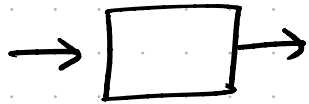
## Mathematical Properties.

- Convolution is

the order in which two signals are  
difference; the result is

makes

IF



THEN

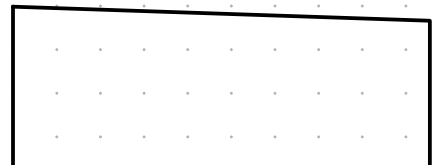


$$\underline{\text{IF}} \quad y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$$\text{let } l = \quad \Rightarrow \quad k =$$

THEN

↓  
for a sum the order  
doesn't matter



It is possible to convolve 3 or more signals?

→

$x[n]$  want to convolve with both  $h_1[n]$   $h_2[n]$

convolve two of the signals to produce an intermediate signal

intermediate signal

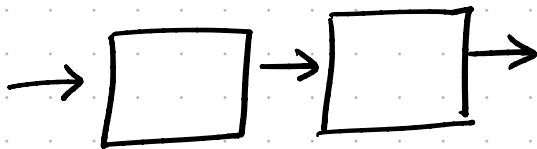
then we convolve the intermediate signal to the third signal

What should be the order of  $h_1[n]$   $h_2[n]$

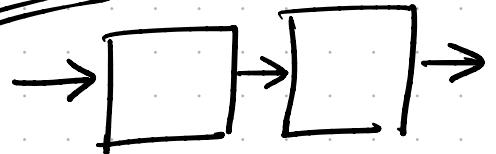
• convolution is

→ used in system theory to describe how systems behave

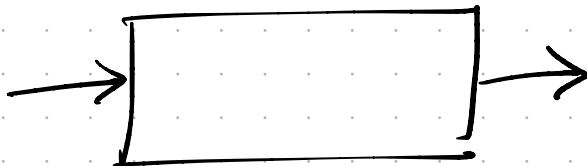
IF



THEN



ALSO



$$(x[n] * h_1[n]) * h_2[n] =$$

example → allows us to make bandpass filters as a sum of multiple high, low pass filters.

• Convolution is

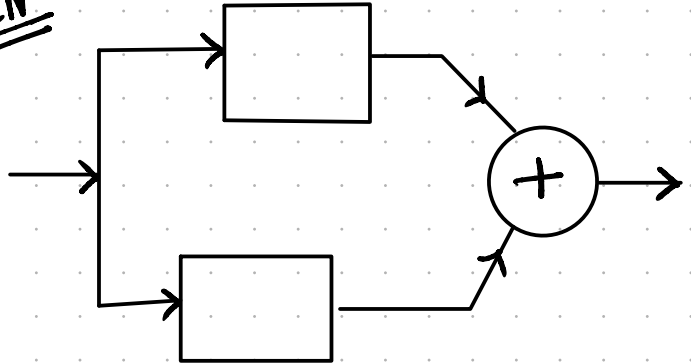
→ describes the operation in parallel system with addl output.

$$x[n] * (h_1[n] + h_2[n]) =$$

IF



THEN



Test if a linear Time Invariant is or not

↓  
Do not get an output  
prior to the input that  
caused it ( )

LTI system is iff

To See

$$y[n] = \sum_{-\infty}^{\infty} h[k] \cdot x[n-k]$$

Thus  $h[n < 0] = 0$ .



• LTI systems is stable. or bounded input results in bounded output

iff

implies  $\rightarrow$

as

also

if  $x[n]$  is finite duration  $y[n] \rightarrow$

as  $n \rightarrow$

# Finite Impulse Response (FIR)

If over finite range

AND

Then

system is a



when you apply an to the system  
you get an impulse response / an output

that is only for a

amount of time

so the response to the impulse is only for

a

- System has
- easy to achieve

↓  
just need the coefficients of the system to be

## Infinite Impulse Response

if  $h[n]$  has range

— system has memory

→ if you have an input to the system, the effect of that system is there forever and it never goes away.

— harder to achieve

# Why do we use recursion/feedbacks in digital systems?

Recursion gives us where

we have to think more about

↓

It gives us a compact & better description of the system

Example cumulative sum of  $x[n]$  starting at  $n=0$

$$y[n] =$$

system with an  
that is a

Issues →

## Efficient Recursive Formulation

$$y[n] =$$

## Linear Constant Coefficient Difference Equation

$$y[n] = -a_1 y[n-1] - a_2 y[n-2] - \dots \\ + b_0 x[n] + b_1 x[n-1]$$

$a$ 's are  $\rightarrow$  co-efficients

$b$ 's are  $\rightarrow$  co-efficients.

$y[n]$

If  $a$ 's and  $b$ 's are constants.

# Determining Impulse Response of a Linear Constant Co-Efficient

## Difference Equation

To determine  $h[n]$ :

1. Apply  $\delta[n]$  to resting system

2. Assume

Example Cumulative Sum

$$y[n] = \begin{cases} 0 & , n < 0 \\ y[n-1] + x[n] & , n \geq 0 \end{cases}$$

So,

$$h[n] =$$

$$\text{Thus, } h[0] =$$

$$h[1] =$$

$$h[2] =$$

on

## FIR vs IIR via Difference Equation

Find  $h[n]$  when  $a's = 0$

Then  $y[n] =$

Find impulse response  $h[n]$

- Apply impulse to it

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• If  $a's = 0$  & no feedback

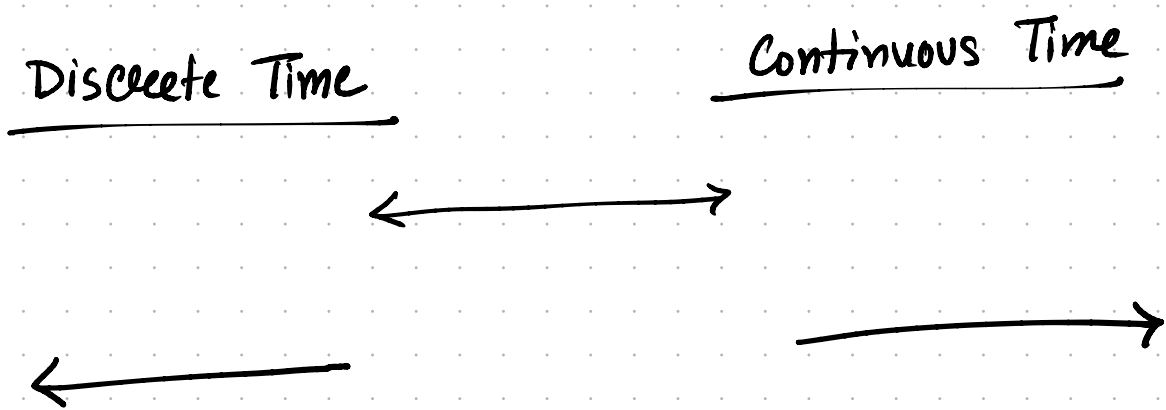
• else

## Solving Difference Equation

↳ we are not going to solve in the  
but look into the

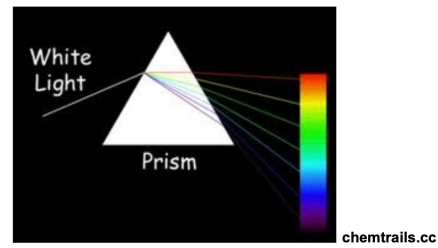
## # Why do the frequency transforms?

- Direct (time domain) solution to difference equation is



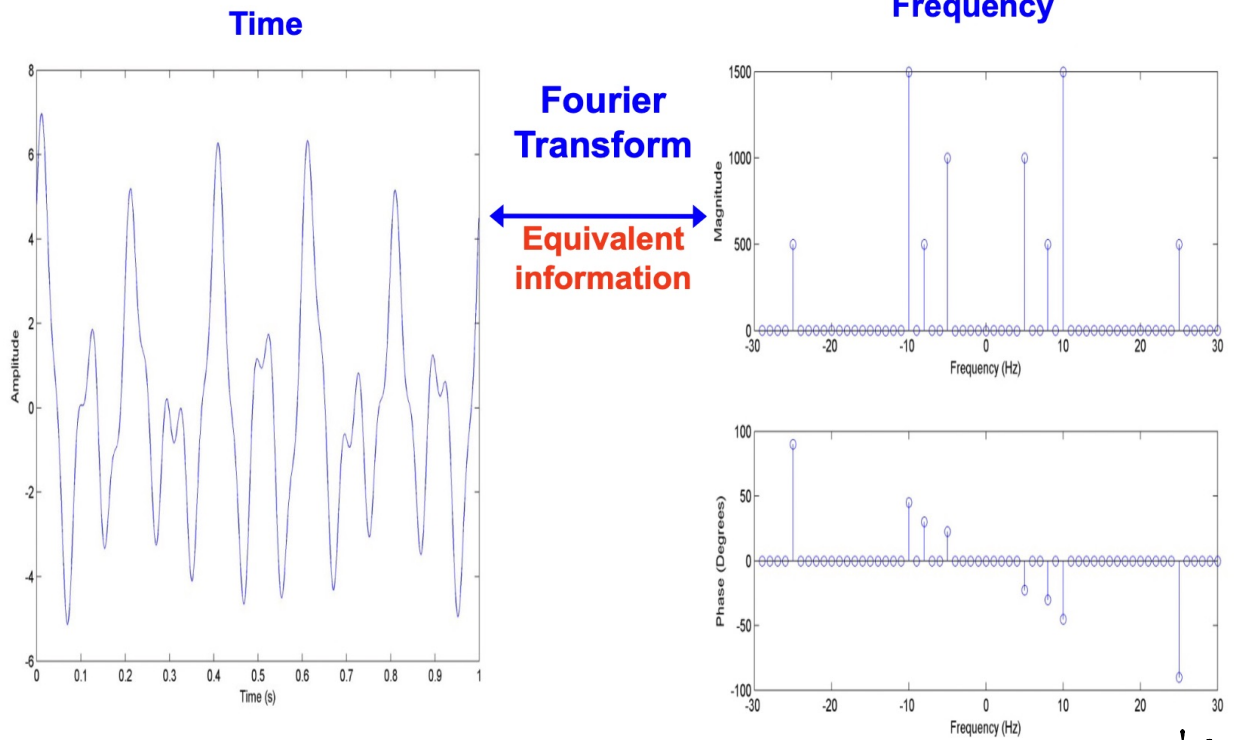


# #Analogy of light through Prism



- Prism separates light into its component colors
  - Each color is an electromagnetic wave of distinct frequency
- Combinations of component colors give other colors
  - (100% Red) + (100% Green) = Yellow
  - (100% Red) + (50% Green) = Orange

# #Time Domain vs Frequency Domain



Time Domain

Frequency domain

- good for  
occurring at specific  
information

- good for localizing information  
occurring at specific