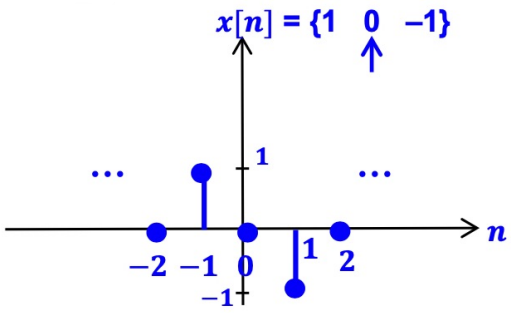
Graphical Convolution

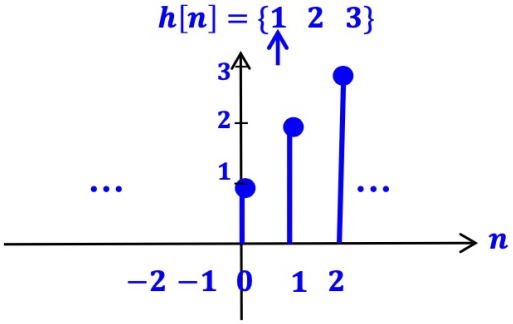
To implement convolution at $n = n_0$

1. Fold / Reflect
2. Delay / Advance
3. multiply
4. sum

For another n_0, shift (delay/advance) again

Example





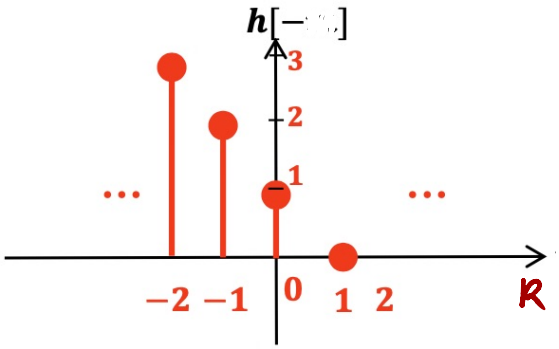
$$x[n] = \begin{cases} 1, & n = -1 \\ -1, & n = 1 \\ 0, & \text{otherwise} \end{cases}$$

$$h[n] = \begin{cases} 1, & n = 0 \\ 2, & n = 1 \\ 3, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

$= \{1, \underset{\uparrow}{2}, 3\}$

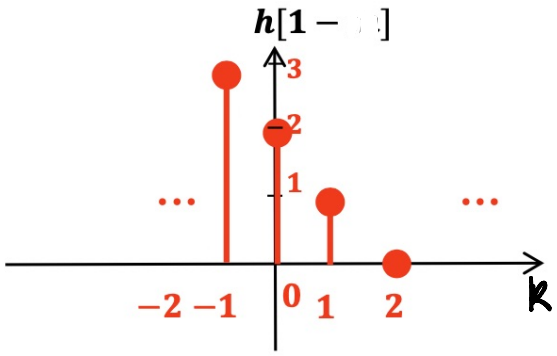
• Find $y[n] = x[n] * h[n]$ for $n = 1$

① Reflect

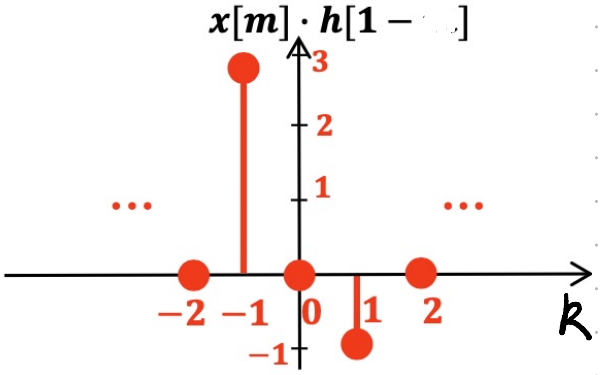


② shift by $n_0 = 1$

[positive n_0 → shift right for $h[-k]$



③ multiply $x[k]$ by $h[n_0 - k]$



④ sum

CHECK

so $y[1] = x[-1] \cdot h[1-(-1)] + x[0] \cdot h[1-0] + x[1] \cdot h[1-1]$

manual convolution computation

Convolve: $x[n] = \{1 \underset{\uparrow}{2} 3\}$ with $h[n] = \{\underset{\uparrow}{-1} 4\ 2\ 3\}$

Solution

First overlap ($n = -1$)

1 2 3
↑

Second overlap ($n = 0$)

$x[k]$ 1 2 3
↑

$h[0-k]$ ---

Third overlap ($n = 1$)

$x[k]$ 1 2 3
↑

$h[1-k]$ ---

Forth overlap (n = 2)

$x[k] = \{1, \underset{\uparrow}{2}, 3\}$ $h[-k] = \{3, 2, 4, \underset{\uparrow}{-1}\}$

$x[k]$ 1 $\underset{\uparrow}{2}$ 3

$h[2-k]$ _____

Fifth overlap (n=3)

$x[k]$ 1 2 3
$\uparrow$

$h[3-k]$ _____

Sixth Overlap (n=4)

$x[k]$ 1 2 3
$\uparrow$

$h[4-k]$ _____

Result: $x[n] * h[n] = \{$ $\}$

Sequence Duration of Convolution Sum

IF $x[n] \rightarrow$ finite duration of length :

$h[n] \rightarrow$ finite duration of length :

THEN $x[n] * h[n] \rightarrow$ finite duration of length :

$$x[n] = \{ x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_{L_x} \}$$

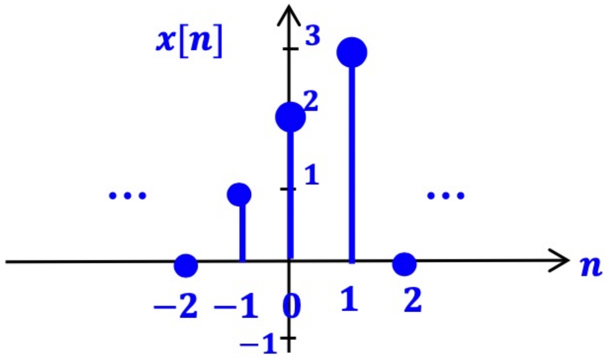
$$h[n] = \{ h_1 \quad h_2 \quad h_3 \quad \cdots \quad h_{L_h} \}$$

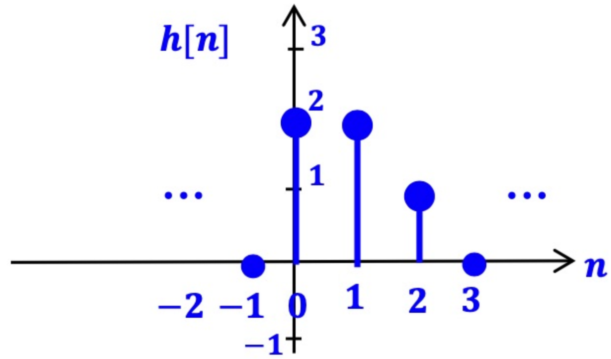
From previous example

$$x[n] = \{ 1 \underset{\uparrow}{2} \; 3 \}$$

$$h[n] = \{ -1 \underset{\uparrow}{4} \; 2 \; 3 \}$$

example 2





$x[n] = \{1 \ \underset{\uparrow}{2} \ 3\}$ $\qquad$ $h[n] = \{\underset{\uparrow}{2} \ 2 \ 1\}$

Solution $y[n] = x[n] * h[n]$

Ly

overlaps.

$y[-1]$ $\quad 1 \ \underset{\uparrow}{2} \ 3$

$\uparrow$

$y[2]$ $\quad 1 \ \underset{\uparrow}{2} \ 3$

$\uparrow$

$y[0]$ $\quad 1 \ \underset{\uparrow}{2} \ 3$

$\uparrow$

$y[3]$ $\quad 1 \ \underset{\uparrow}{2} \ 3$

$\uparrow$

$y[1]$ $\quad 1 \ \underset{\uparrow}{2} \ 3$

$\uparrow$

$y[n] = \{ \qquad \}$

example3 $x[n] = \{-1 \underset{\uparrow}{0} 1\}$ $h[n] = \{\underset{\uparrow}{2} 2\}$

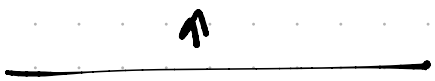
solution: $y[n] = x[n] * h[n]$

$h[-k]$

$L_y = L_x + L_h - 1 =$

$y[-1]$: $-1 \quad \underset{\uparrow}{0} \quad 1$

$y[0]$: $-1 \quad \underset{\uparrow}{0} \quad 1$



$y[1]$ $-1 \quad \underset{\uparrow}{0} \quad 1$

$y[2]$ $-1 \quad \underset{\uparrow}{0} \quad 1$

example 4

$$y[n] = \mu[n] * \mu[n]$$

Solution

$$\mu[k] = \{\underset{\uparrow}{1}\ 1\ 1\ 1 \cdots\}$$

For $n < 0$

For $n = 0 \rightarrow$

For $n = 1 \rightarrow$

IN GENERAL