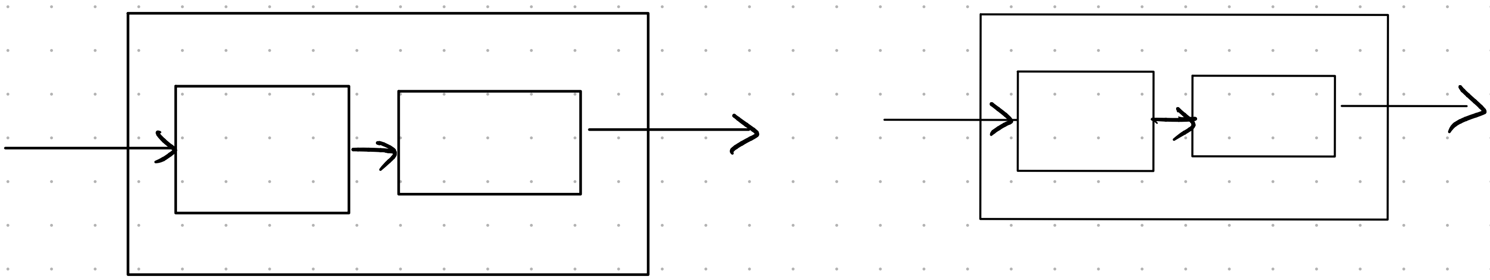
Convolution Sum

Decomposition

linearity is → order of cascading system

can be without the characteristics



Superposition ↓

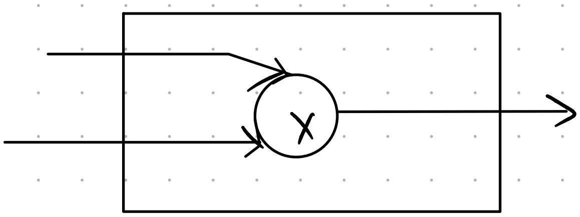
when two signals are added together & fed to a

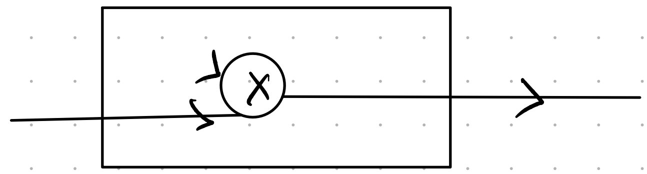
the output is the same as if one had put

through the system seperately & then added the output

Multiplication in Linear System

multiplication in linear system } based on the





Imagine a sinusoid multiplied with another sinusoid.

Systhesis → the process of combining signals through scaling & addition

Decomposition → a signal is broken into or more components.
of synthesis.

Decomposition is more involved than synthesis, as there are possible decomposition for any given signal

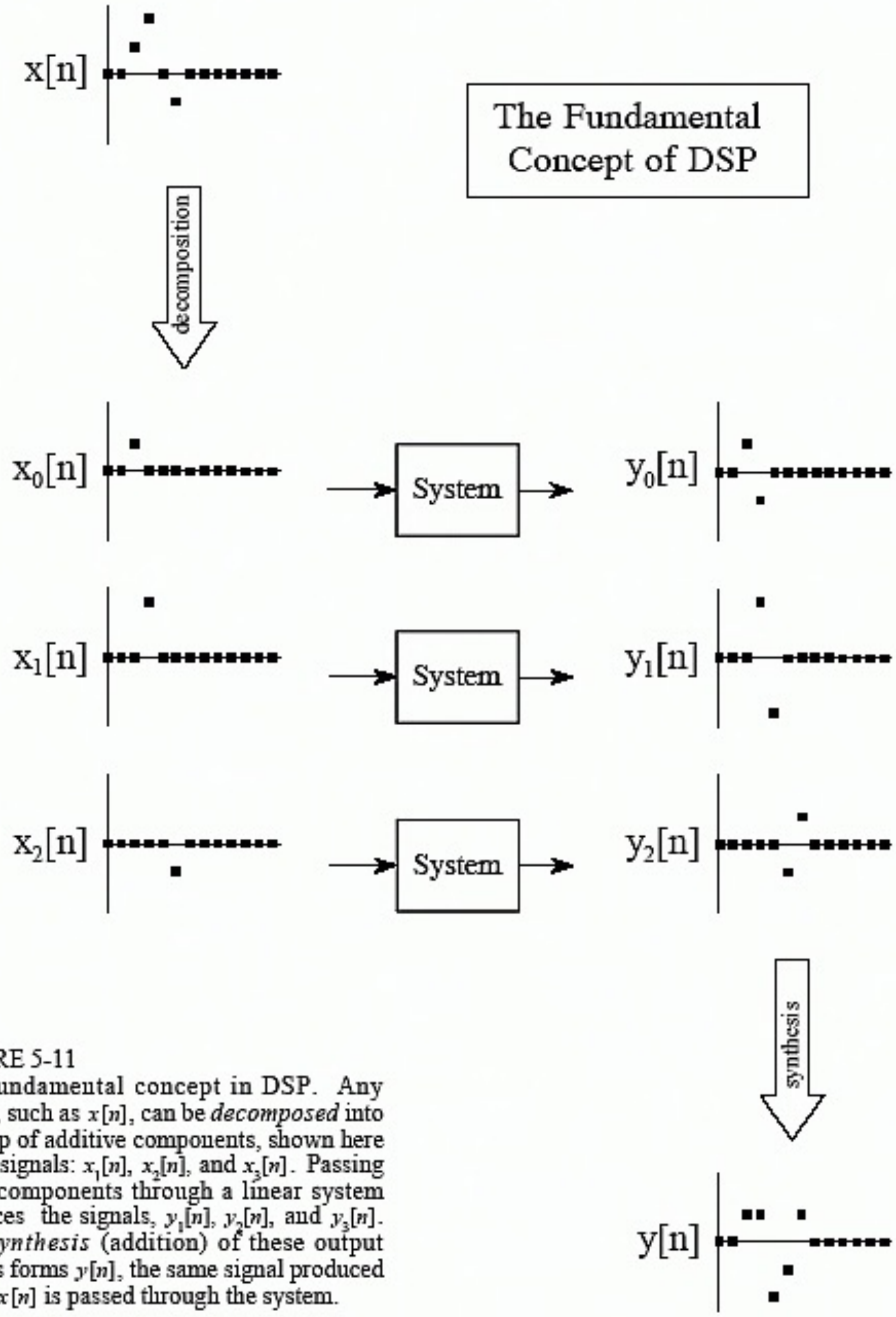


FIGURE 5-11
The fundamental concept in DSP. Any signal, such as $x[n]$, can be *decomposed* into a group of additive components, shown here by the signals: $x_1[n]$, $x_2[n]$, and $x_3[n]$. Passing these components through a linear system produces the signals, $y_1[n]$, $y_2[n]$, and $y_3[n]$. The *synthesis* (addition) of these output signals forms $y[n]$, the same signal produced when $x[n]$ is passed through the system.

The output signal obtained by this method is to the one produced by directly passing the input signal through the system.

Decomposition

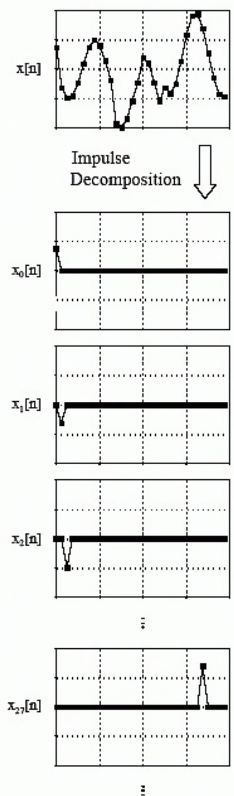


FIGURE 5-12
Example of impulse decomposition. An N point signal is broken into N components, each consisting of a single nonzero point.

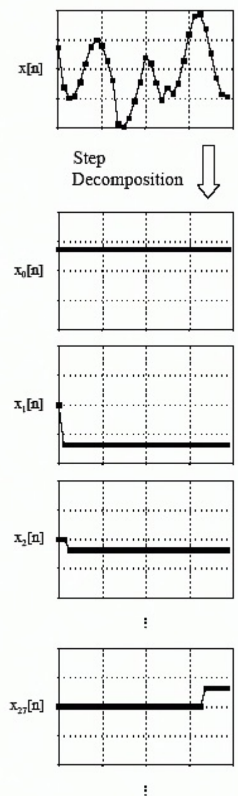


FIGURE 5-13
Example of step decomposition. An N point signal is broken into N signals, each consisting of a step function

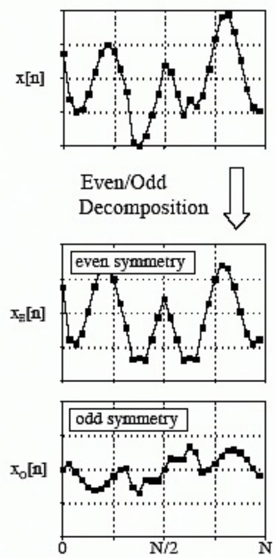


FIGURE 5-14
Example of even/odd decomposition. An N point signal is broken into two N point signals, one with even symmetry, and the other with odd symmetry.

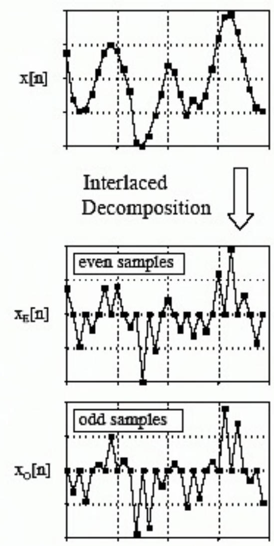


FIGURE 5-15
Example of interlaced decomposition. An N point signal is broken into two N point signals, one with the odd samples set to zero, the other with the even samples set to zero.

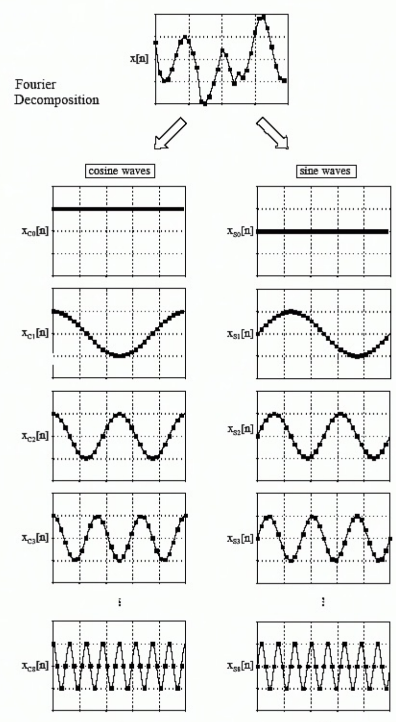


FIGURE 5-16
Illustration of Fourier decomposition. An N point signal is decomposed into N+2 signals, each having N points. Half of these signals are cosine waves, and half are sine waves. The frequencies of the sinusoids are fixed; only the amplitudes can change.

Convolution

Mathematical way of combining two signals to form a

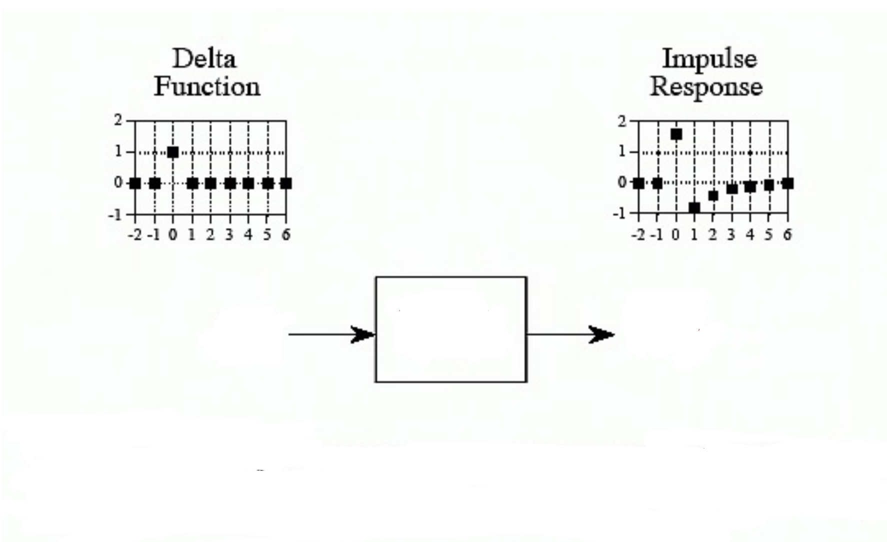
Use the strategy of $\rightarrow$ impulse

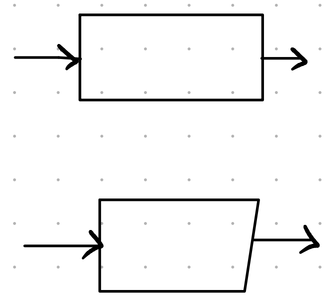
$\downarrow$

way to analyse signals one sample at a time

When impulse decomposition is used, the procedure can be described by the mathematical operation

$\delta[n]$





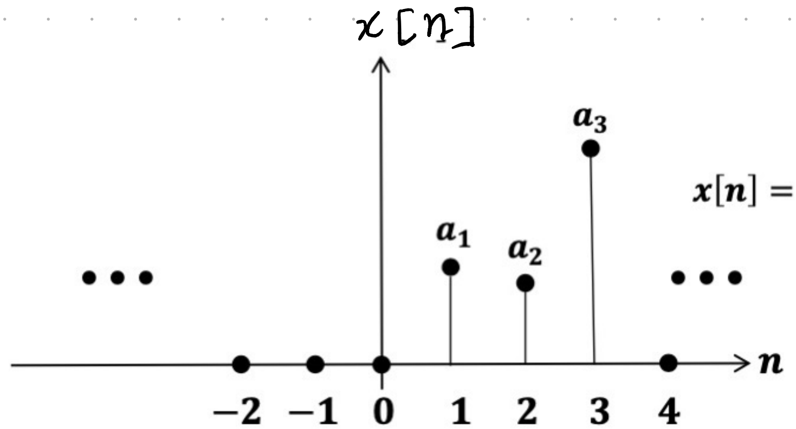
if $F_1(\cdot)$ $F_2(\cdot)$

then $h_1[n]$ $h_2[n]$

any impulse can be represented as a shifted and scaled unit impulse.

Signal $x[n] \rightarrow$ composed of all zeros except sample number 8, which has value -3

Can we show $x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] \quad -\infty \leq n \leq \infty$



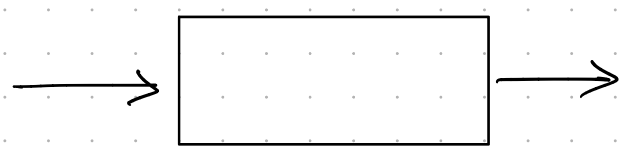
Impulse Response of LTI System

System output (response) if input $= \delta[n-k]$

- General definition of impulse resp se

$$h[n, k] = F(\delta[n-k])$$

If sample is shift-invariant



For filters impulse response

Image Processing $\rightarrow$ 1

How a linear system change the input signal to an output signal?

input signal

↓

into set of

(shifted and scaled unit impulse)

↓

$F(\cdot)$

↓

output resulting from each input impulse is

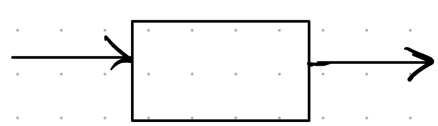
input impulse

↓

these scaled & shifted impulse responsed

↓

In system is used to describe the relation between three signals



Convolution is used as low-pass filter, high pass filter, inverting attenuator, discrete derivative

Derivation of LTI Convolution sum

- General discrete system :

- Rewrite $x[n]$ as a sum of δ's :

- Assume linearity

- Superposition summation

- Assuming Time-Invariance

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

any instance in time n
we are taking the impulse
response of the function
& because there is a
we are it
around the

shift that reversed impulse response
by

then multiply point point x event
& sum it over

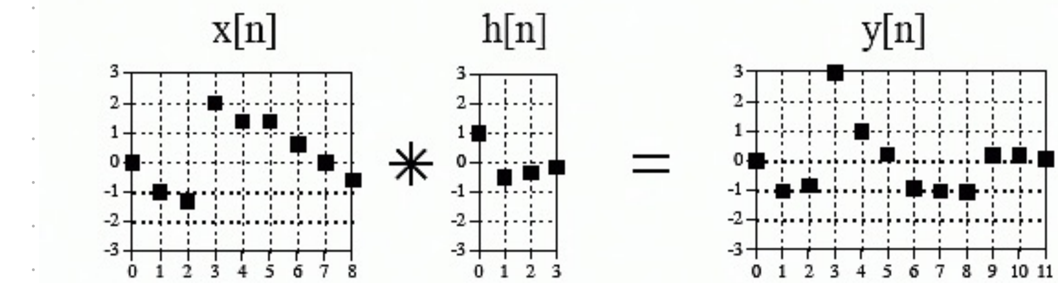


FIGURE 6-5
Example convolution problem. A nine point input signal, convolved with a four point impulse response, results in a twelve point output signal. Each point in the input signal contributes a scaled and shifted impulse response to the output signal. These nine scaled and shifted impulse responses are shown in Fig. 6-6.

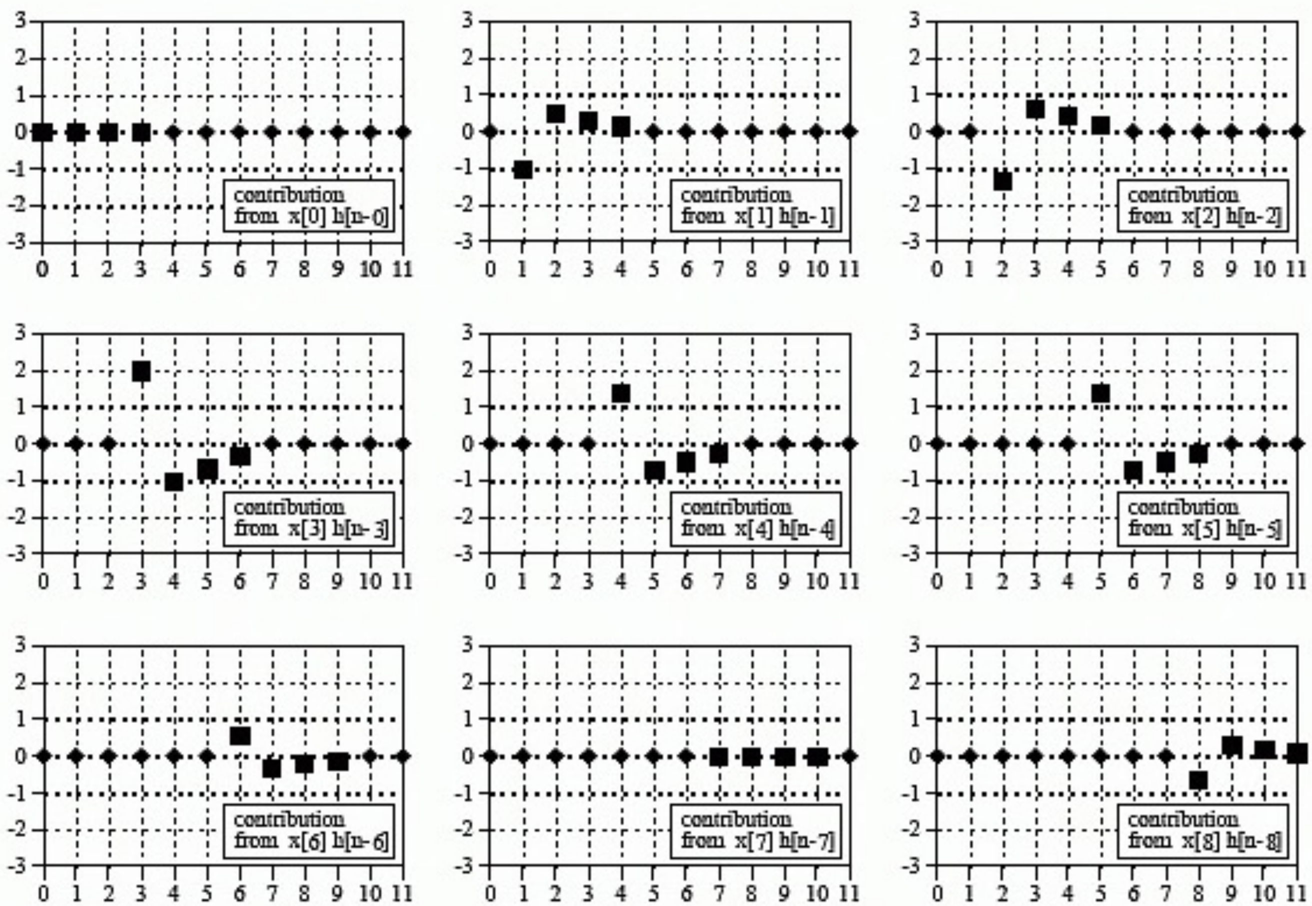


FIGURE 6-6
Output signal components for the convolution in Fig. 6-5. In these signals, each point that results from a scaled and shifted impulse response is represented by a square marker. The remaining data points, represented by diamonds, are zeros that have been added as place holders.

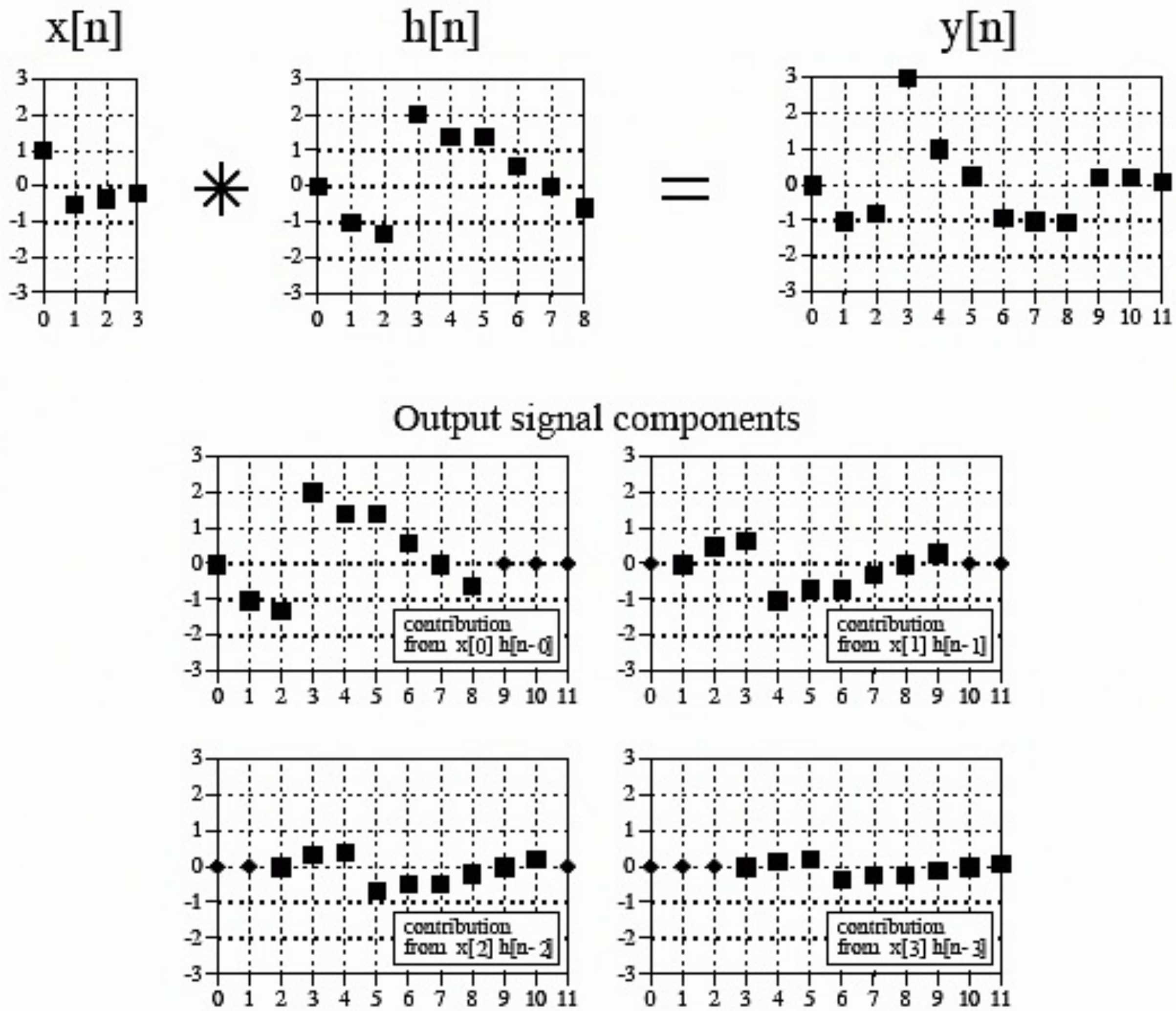
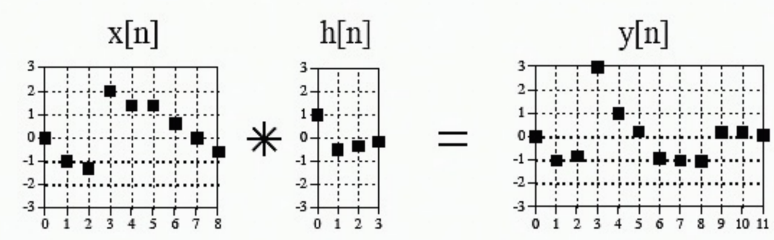


FIGURE 6-7
A second example of convolution. The waveforms for the input signal and impulse response are exchanged from the example of Fig. 6-5. Since convolution is commutative, the output signals for the two examples are identical.



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Convolution is

Next →