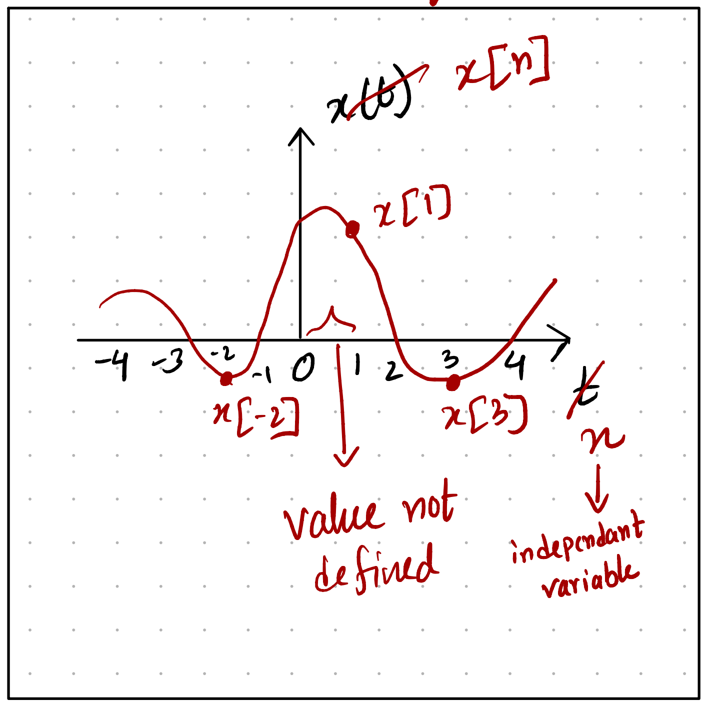
Classes of Signals.

Continuous Signals are represented as $x(t)$

General Sequence.



Digital Signal

Digital signals are represented as.

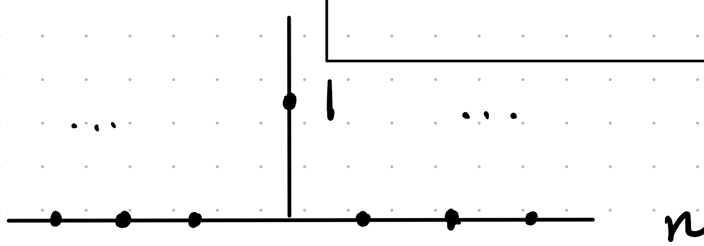
$$x(n) = \begin{cases} 2^{-n}, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$-\infty \leq n \leq \infty$

special DT signals

unit impulse

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



Kronecker delta, unit pulse (impulse)

a function of two variables (usually non-negative integers)

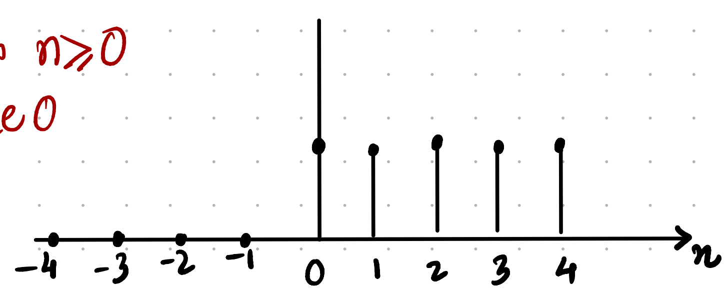
The function is 1 if the variables are equal. Otherwise zero.

value is 1 for $n = 0$
else value is 0

Unit step

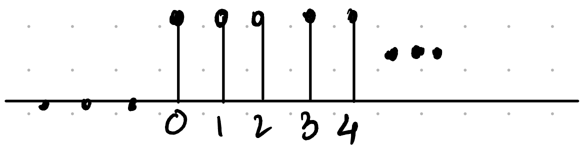
value 1 for $n \geq 0$
else value 0

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

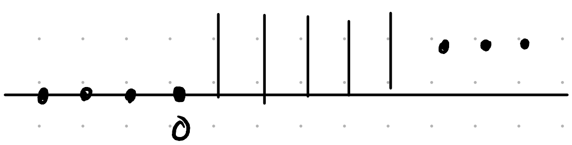


unit step and unit impulse are closely related.

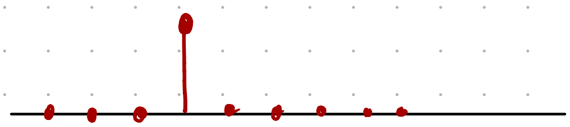
- The unit impulse sequence as the first backward difference of the unit step sequence



$\mu[n]$

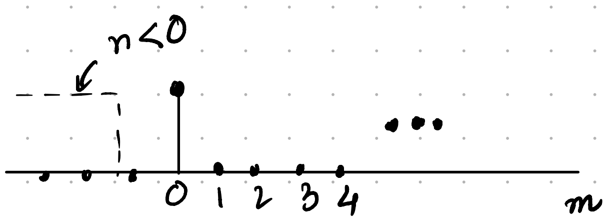


$\mu[n-1]$



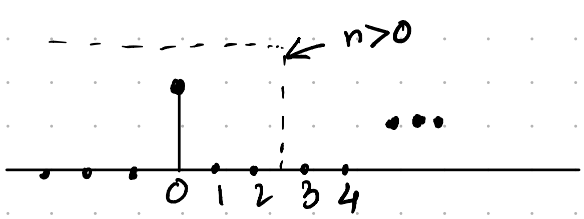
$$\mu[n] - \mu[n-1] = \delta[n]$$

- The unit step sequence as a running sum of the unit impulse.



$\delta[m]$

$$\mu[n] = \sum_{m=-\infty}^{n} \delta[m]$$

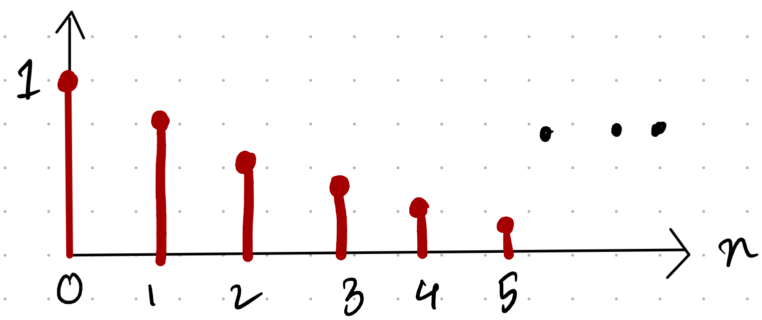


$\delta[m]$

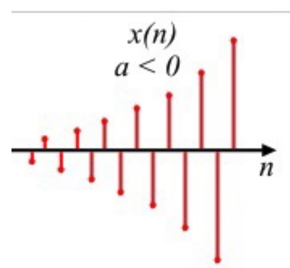
Real Exponential Sequence

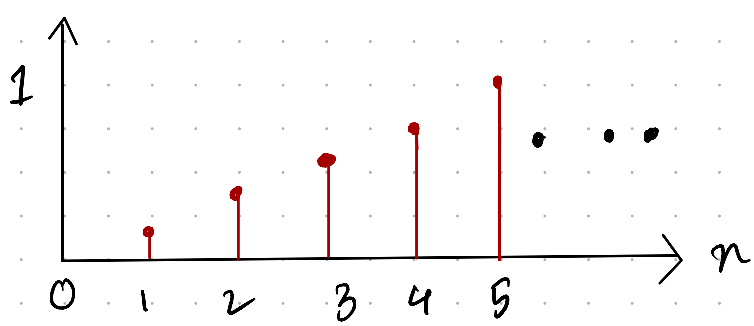
$$x[n] = a^n, \quad a \in \mathbb{R} \neq 0 \quad , -\infty \leq n \leq \infty$$

ⓐ $x[n]$, $n \geq 0$, $0 < a < 1$



ⓑ $x[n]$, $n \geq 0$ $a > 1$





For $a < 0$ polarity alternates.

"a" can be complex → treat as real, imaginary parts

or magnitude, phase

Complex exponential signal is defined by

$$x[n] = a^n e^{j(\omega_0 n + \varphi)} = a^n \cos(\omega_0 n + \varphi) + j a^n \sin(\omega_0 n + \varphi)$$

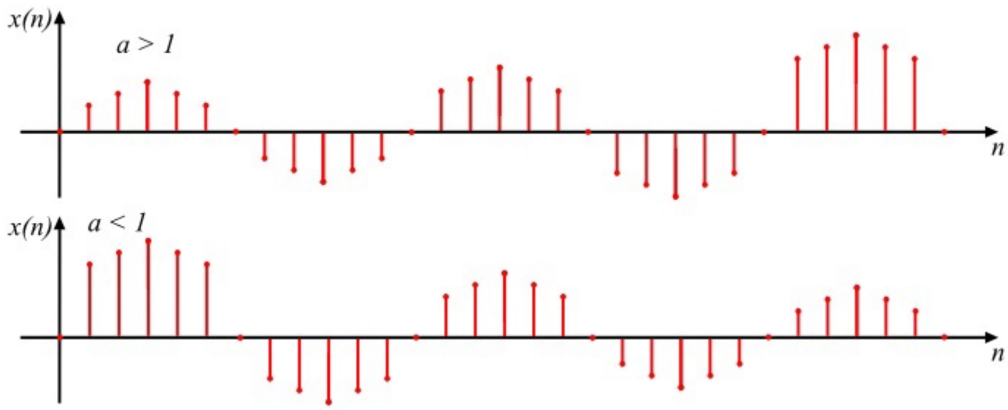
frequency
phase
time

Euler's formula

$|a| = 1 \rightarrow$ both real & imaginary parts are sinusoidal

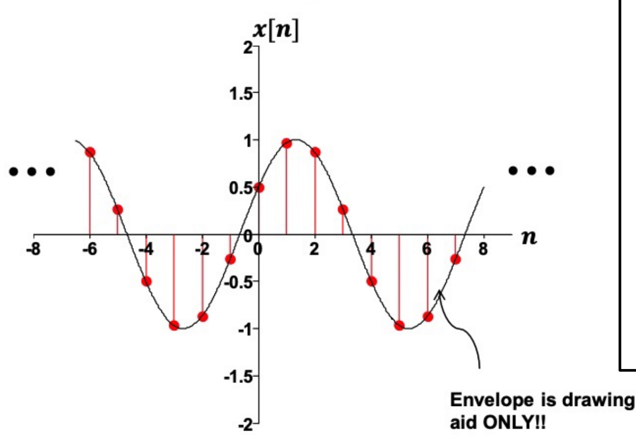
$|a| > 1 \rightarrow$ amplitude of the sinusoidal sequence increase exponentially

$|a| < 1 \rightarrow$ amplitude of the sinusoidal decreases exponentially



Sinusoidal Signal

$$x[n] = A \cos(\omega_0 n + \varphi)$$



Complex exponential & Sinusoidal signals are used to determine dynamic response of a system

For continuous-time sinusoid $\cos(\omega t + \varphi)$

$$\omega = 2\pi f \qquad f = \frac{\omega}{2\pi} \qquad T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Periodic Signal

a discrete-time signal $x[n]$ is periodic

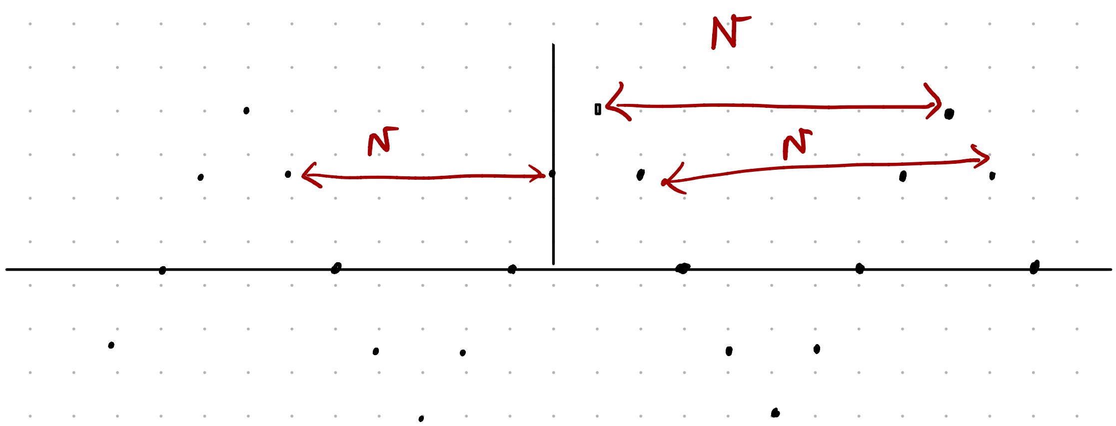
iff $x[n] = x[n+N]$ for $\forall n$ & smallest N

→ $N > 0$, real integer

→ $N \rightarrow$ period.

a sinusoidal signal is periodic

$$A\cos[\omega_0 n + \phi] = A\cos[\omega_0 (n+N) + \phi]$$



Since cos( ) invariant to $2\pi \cdot m$ phase shift (for m integer)

above cosine periodic iff

$\omega_0 N = 2\pi \cdot m$, for some m (or) $\left[\frac{\omega_0}{2\pi} = f_0\right] = \frac{m}{N}$

Thus.

if $f_0 = \frac{\text{int1}}{\text{int2}}$ → periodic otherwise not periodic

$\frac{m}{n}$ → rational

Let's check periodicity of the following signal.

- $x[n] = -7\cos(0.6\pi n + \frac{\pi}{3})$

$$f_0 = \frac{\omega}{2\pi} = \frac{0.6\pi}{2\pi} = \frac{0.6}{2} = \frac{6}{20} = \frac{3}{10}$$

Periodic

- $x[n] = 1.6\cos(0.7n)$

$$f_0 = \frac{\omega}{2\pi} = \frac{0.7}{2\pi} = \frac{7}{20\pi}$$

not periodic

- $x[n] = 3\cos(7n)$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{7}{2\pi} \rightarrow \text{not periodic}$$

- $x[n] = 4.2 \sin(\pi n + 42^\circ)$

$$f_0 = \frac{\pi}{2\pi} = 1/2 \rightarrow \text{periodic}$$

- $x[n] = \cos\left(\frac{3\pi}{5} n + \pi/2\right)$

$$f_0 = \frac{3\pi/5}{2\pi} = \frac{3\pi}{10\pi} = \frac{3}{10} \rightarrow \text{periodic}$$

Even & Odd Signals

$\rightarrow$ we will come back after we learn some signal operations.

Signal Operations

Addition $\rightarrow$ $x_{sum}[n] = x_1[n] + x_2[n]$

Subtraction $\rightarrow$ $x_{Dif}[n] = x_1[n] - x_2[n]$

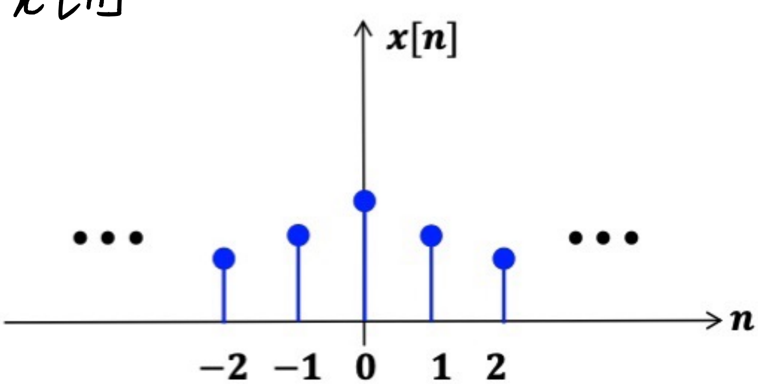
Multiplication $\rightarrow$ $x_{mul}[n] = x_1[n] \cdot x_2[n]$

Scaling $\rightarrow$ $x_{scale}[n] = a \cdot x_2[n]$, for a constant

---

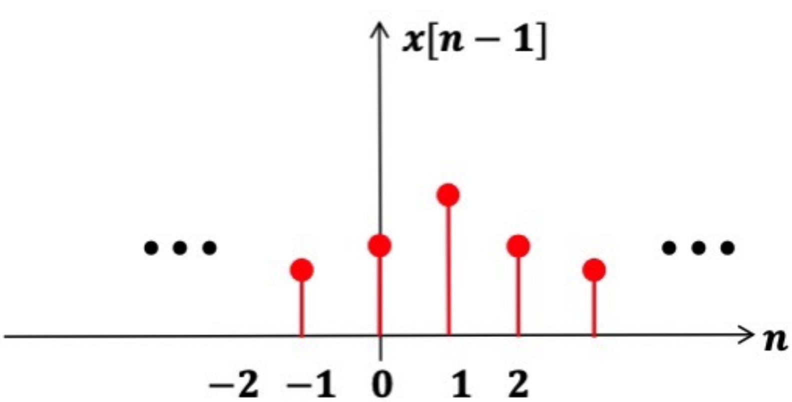
Time Shift $\rightarrow$ the shifting of signal in time

$x[n]$

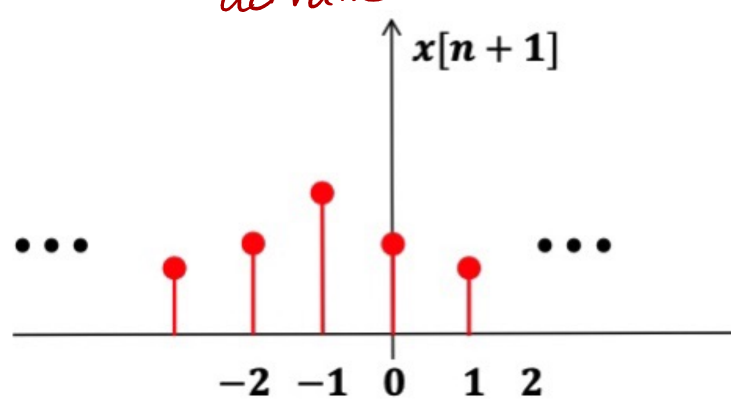


How to time shift in matlab.

$y[n] = x[n-1] \rightarrow$ delay

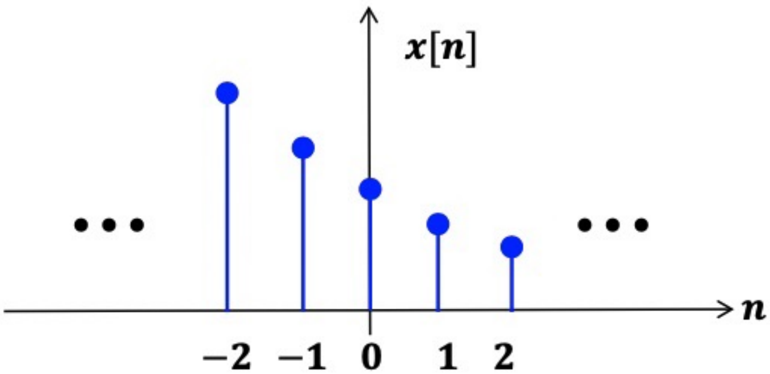


$y[n] = x[n+1]$
advance



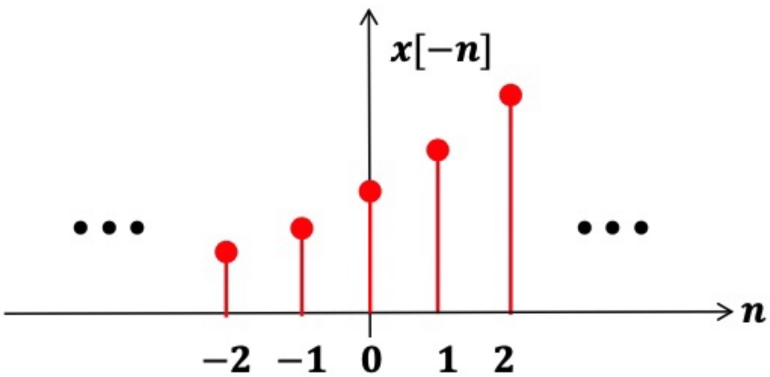
• Time Reversal

Reversing a signal in time



$x[n] = \{6 \;\; 3 \;\; 5 \;\; -1 \;\; 8\}$

$x[n] = \{8 \;\; -1 \;\; 3 \;\; 5 \;\; 6\}$



Reflection about $n=0$

• Time Scale

→ multiplying a scalar (α) to time variable (n) in argument of function.

streching / shrinking of time axis (n)

$$y[n] = x[a \cdot n]$$ "a" real

if a is positive integer → downsample

$a = -1$ → reflect sequence about y axis

a = negative integer → reflect & downsample

a non integer → Interpolation

Even & Odd Signals

A discrete-time signal $x[n]$ is an even signal if it is identical to its time-reversed counterpart, i.e., with its reflection about the origin

$$x[n] = x[-n]$$

A discrete-time signal $x[n]$ is odd signal if

$$x[n] = -x[-n]$$

A general sequence $x[n]$ can be seperated into its odd symmetric & even-symmetric parts such that

$$x_n = x_{even}[n] + x_{odd}[n]$$

where

$$x_{odd}[n] = \frac{x[n] - x^*[n]}{2} \qquad \text{odd symmetric}$$

$$x_{even}[n] = \frac{x[n] + x^*[-n]}{2} \qquad \text{even symmetrie}$$