MA 3257 – SPRING 2023 C-TERM
HOMEWORK VII (OPTINOAL: DUE MAR 3RD, 2023)

Problem 1. (15 points) Let $f(x) = x^2 - 6$ and $p_0 = 1$. Clearly $f(x)$ has two roots.

(1) (4 points) Implement Newton's method p_k to find the root of f. Which one does it converge to, when the initial guess is as given?
(2) (4 points) Implement a simple fixed-point iteration q_k with the same initial guess $q_0 = 1$. Which root does this algorithm converge to?
(3) (4 points) Implement the secand method s_k with $s_0 = 1$ and $s_1 = 1.5$.
(4) (3 points) Compare the three methods by printing

k	Newton p_k	Fixed Point q_k	Secant s_k
0	1	1	1
1	.	.	1.5
2	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.

(5) (Bonus 5 points) Do a rate of convergence study (similar to HW 6).

Problem 2. (10 points) The nonlinear system

$$\begin{aligned} 5x_1^2 - x_2^2 &= 0 \\ x_2 - 0.25(\sin(x_1) + \cos(x_2)) &= 0 \end{aligned}$$

has a solution near $\left(\frac{1}{4}, \frac{1}{4}\right)^{\mathrm{T}}$.

(1) (2 points) Find a function $\boldsymbol{G}$ and a set D in $\mathbb{R}^2$ such that $\boldsymbol{G} : D \to \mathbb{R}^2$ and $\boldsymbol{G}$ has a unique fixed point in D.
(2) (4 points) Implement a functional iteration (using $\boldsymbol{G}$) to approximate the solution to within 10^{-5} in the l^∞ norm.
(3) (4 points) Implement the Gauss-Seidel version of part (2), i.e., within each iterative step, the algorithm updates the current coordinate using the immediate information obtained from previous coordinates. Does this modification accelerate convergence?