MA 3257 – SPRING 2023 C-TERM
HOMEWORK IV (DUE FEB 10TH, 2023)

Problem 1. (5 pts) Consider the system

$$0.03x_1 + 58.9x_2 = 59.2,$$
$$5.31x_1 - 6.10x_2 = 47.0.$$

This has an exact solution $[10, 1]$. Use three-digit chopping arithmetic and the following methods to solve it:

(1) (2 pt) Naive Gaussian Elimination (no row swaps).

(2) (2 pt) Gaussian Elimination with Partial Pivoting.

(1 pt) Find the LU decomposition of the matrix A that represents this system. Hint: part (2) should help you immensely.

Problem 2. (5 pts) Read Example 3 in Section 6.5, and use it to determine a factorization in the form of $A = \left(P^{\mathrm{T}}L\right)U$ for the matrix

$$A = \begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 2 \\ 1 & -1 & 4 \end{bmatrix}.$$

Problem 3. (5 pts) Find l^∞ and l^2 norms of the following vectors. Determine if each sequence has a limit as $k \to \infty$ under both norms, and if so, find it.

(1) $\boldsymbol{x}^{(k)} = \left(\sin k, \cos k, 2^k\right)^{\mathrm{T}}$ for a fixed positive integer k.

(2) $\boldsymbol{x}^{(k)} = \left(\frac{4}{k+1}, \frac{2}{k^2}, k^2 e^{-k}\right)^{\mathrm{T}}$ for a fixed positive integer k.

Extra credit (5 pts): does convergence in norm imply convergence in l^2, i.e. if

$$\left\|\boldsymbol{x}^{(k)}\right\|_2 \to \|\boldsymbol{x}\|_2\,,$$

does this imply that $\left\|\boldsymbol{x}^{(k)} - \boldsymbol{x}\right\|_2 \to 0$? If so, prove it. If not, provide a counterexample.

Problem 4. (5 pts) Verify that the function $\|\cdot\|_1$, defined on $\mathbb{R}^n$ by

$$\|\boldsymbol{x}\|_1 = \sum_{i=1}^{n} |x_i|$$

is a norm on $\mathbb{R}^n$.

Extra credit (5 pts): prove that for all $\boldsymbol{x} \in \mathbb{R}^n$, $\|\boldsymbol{x}\|_1 \geq \|\boldsymbol{x}\|_2$.

Problem 5. (5 pts) Find the eigenvalues and their associated eigenvectors for the following matrices.

(1) (2 pts) $A = \begin{bmatrix} 2 & 2 \\ -1 & 2 \end{bmatrix}$.

(2) (3 pts) $B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$.