MATH 1024 DL01
Spring 2025 D Term
Final Exam
05/07/25
Time Limit: 50 Minutes

Name (Print): ____________________

Discussion Section: 9 AM / 10 AM

This exam contains 10 pages (including this cover page) and 7 problems (including 2 bonus problems). Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any **calculator** on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- This is not the end of the world.
- In Polar, Cylindrical and Spherical Coordinates, the polar angle **always** uses the symbol θ.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- Manage the empty space wisely.
- Pay attention to the point weights of each subproblem and pick your battles wisely.
- Every syntax error is 1-point off, e.g. equal signs, imply signs (" $\Longrightarrow$ "), and other relevant symbols.

Problem	Points	Score
1	18	
2	30	
3	20	
4	12	
5	20	
6	0	
7	0	
Total:	100	

1. (18 points) **Set up** an integral for the area of the following regions R in **Polar Coordinates**.

   (a) (6 points) The semicircle of radius 2 in the first and second quadrant.

   $\iint_R dA =$

   (b) (6 points) The triangle with vertices $(0,0),(0,1)$ and $(1,1)$.

   $\iint_R dA =$

   (c) (6 points) The annulus between circles of radius 1 and 4, both centered at the origin.

   $\iint_R dA =$

2. (30 points) Let R be the region bounded by $x = 1$, $y = 0$ and the unit circle centered at $(1, 0)$.

   (a) (5 points) **Sketch** R and **find its area** by a simple geometric argument.

   Area =

   (b) (10 points) **Set up** two double integrals in **Cartesian coordinates** that compute the area of R. Justify by **sketching** the region and applying a suitable needle argument.

   $\iint_R dydx =$

   $\iint_R dxdy =$

(c) (10 points) **Set up** a double integral in **Polar Coordinates** that computes the area of R. Justify by **sketching** the region and applying a suitable needle argument.

$\iint_R dA =$

(d) (5 points) Compute either part (b) or (c) and confirm that it is the same as part (a).

3. (20 points) Given a point (x, y, z) in Cartesian coordinates,

   1. Sketch the cylindrical and spherical systems, labeling each variable;
   2. Write the equations relating (x, y, z) to the new variables and state their ranges.

   (a) ((sketch+equations+range) 4+3+3 points) Cylindrical Coordinate System (r, θ, z):

$x =$
$y =$
$z =$

Radial Distance r:
Polar Angle θ: $\_\_ \leq \theta \leq \_\_$
Vertical z:

(b) ((sketch+equations+range) 4+3+3 points) Spherical Coordinates System (ρ, θ, ϕ):

$x =$
$y =$
$z =$
Radial Distance ρ:
Polar Angle θ: $\_\_ \leq \theta \leq \_\_$
Azimuthal Angle ϕ: $\_\_ \leq \phi \leq \_\_$

4. (12 points) Express the regions in Cylindrical or Spherical Coordinates.

   (a) (6 points) (**Spherical**) A sphere of radius 1 centered at $(0, 0, 1)$.

   Expression in **Spherical**:

   (b) (6 points) (**Cylindrical**) The Cone $z = \sqrt{x^2 + y^2}$.

   Expression in **Cylindrical**:

   (c) (6 points) (**Spherical**) The plane $z = 1$.

   Expression in **Spherical**:

5. (20 points) Consider the spherical cap E of a sphere with radius 2 centered at the origin, sliced by the plane $z = 1$. Set up a triple integral to compute its volume using the following coordinate systems. Justify your answers with localized sketches. No support = 0 points.

   (a) (4 points) Rough Sketch.

   (b) (4 points) **Set up** in Cartesian Coordinates. Do NOT evaluate here.

   $\iiint_E dV =$

   (c) (4 points) **Set up** in Cylindrical Coordinates. **Justify** your bounds by sketching the projected shadow in the xy-plane. Do NOT evaluate here.

   $\iiint_E dV =$

(d) (4 points) **Set up** in Spherical Coordinates. **Justify** your bounds for the polar angle θ by **sketching** the side view. Do NOT evaluate here.

$\iiint_E dV =$

(e) (4 points) Evaluate **one** integral of your (wise) choosing from parts (b), (c) or (d).

$\iiint_E dV =$

6. (10 points (bonus)) Consider the double integral

$$\int_0^1 \int_0^{1-x} \cos(x-y)\sin(x+y)\,dydx$$

   (a) (5 points) Choose a convenient (linear) change of variable to simplify the integrand (you may use u and v as the new variables).

   (b) (2 points) Write down the Jacobian matrix, $\frac{\partial(x,y)}{\partial(u,v)}$, of this change of variable.

   (c) (1 point) Fill in the blank: $dydx = \underline{\hspace{2em}}dudv$.

   (d) (1 point) Sketch the original region of integration in the xy-plane, and the new region of integration in the uv-plane.

   (e) (1 point) Evaluate the integral using this change of variable.

7. (5 points (bonus)) Pick your most memorable topic learned in this course, and introduce it to me. Show some math and some discussion on why it is memorable/interesting to you, and possibly, to your major.