MATH 1024 DL01
Spring 2025 D Term
Exam 1
04/10/23
Time Limit: 50 Minutes

Name (Print): ______________________

Discussion Section: 9 AM / 10 AM

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any **calculator** on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- Manage the empty space wisely.
- Pay attention to the point weights of each subproblem and pick your battles wisely.
- Every syntax error is 1-point off, e.g. missing limit signs (wherever appropriate), equal signs, imply signs (“ $\Longrightarrow$ ”), and other relevant symbols.
- Do NOT write in the table on the right.

Problem	Points	Score
1	25	
2	20	
3	15	
4	20	
5	20	
Total:	100	

1. (25 points) Consider the surface $z = f(x, y) = 4 - x^2 - y^2$.

   (a) (6 points) Specify the domain and range of this function in set notations.

   Domain: | Range:

   (b) (8 points) Compute $D_{\mathbf{u}} f$ at $(1, 1)$ where $\mathbf{u} = \langle 1, 1 \rangle$.

   $D_{\mathbf{u}} f(1, 1)$:

   (c) (8 points) Find the equation of the tangent plane on this surface at $(1, 1)$.

   Eqn. of Tangent Line:

   (d) (3 points) Find the unit direction perpendicular to the level curve through the point $(1, 1)$.

   Answer:

2. (20 points) Consider the function

$$f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2}, & (x,y) \neq (0,0); \\ 0, & (x,y) = (0,0). \end{cases}$$

(a) (15 points) Show that it is continuous at every point except the origin.

(b) (3 points) Compute the partial derivatives $f_x(0,0)$ and $f_y(0,0)$. [Hint: there is an easier way than the quotient rule.]

$f_x(0,0) =$
$f_y(0,0) =$

(c) (2 points) Can you linearize f at the origin (yes/no)? If so, find the linearization. If not, briefly explain why.

Yes/No?

Linearization/Reasoning:

3. (15 points) Consider the curve C: $\sqrt{x} + \sqrt{\frac{y}{x}} = 2$.

   (a) (5 points) Identify a function $f(x, y)$ such that C is a level curve of f.

   $f(x, y) =$

   (b) (10 points) Find the equation of the tangent line at $(1, 1)$.

   Eqn. of Tangent Line:

4. (20 points) Let $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0$, $y = 1$ and $y = x$ in the first quadrant. Sketch the region and label the boundary pieces to facilitate an organized analysis (5 points). Identify all local and global extrema, and saddle points (if any) of f in this bounded region (15 points).

   Write an ordered pair or pairs (x, y) in the answer box, or <u>N/A</u> if it doesn't apply. Do NOT write anything else in the box.

Local max:

Local min:

Global max:

Global min:

Saddle(s):

5. (20 points) You run a donut shop that makes two types of donuts: glazed and chocolate. Let

$$x = \text{number of dozen of } \textbf{glazed} \text{ donuts}$$
$$y = \text{number of dozen of } \textbf{chocolate} \text{ donuts}$$

Your profit is a function of the sales

$$P(x, y) = 6x + 15y - \frac{1}{2}x^2 - y^2$$

which you wish to maximize but are limited by your ingredient supply, which can only support exactly 30 dozens of donuts in total.

(a) (12 points) Set up the maximization problem by identifying the objective and the constraint.

Objective:
Constraint:

(b) (8 points) There are two ways to solve this problem. Pick one and proceed to find the optimizer (x^*, y^*).

$x^* =$
$y^* =$