Math 1023 BL02
Fall 2023 B Term
Exam 2
11/21/23
Time Limit: 50 Minutes

Name (Print): ____________________

Discussion Section: ____________________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- Manage the empty space wisely.
- Good intuition itself does not constitute an answer and must be verified by mathematical details.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	0	
Total:	100	

1. (20 points) True or False. If true, give a brief reason for your answer, such as a Theorem, a definition or a direct but concise argument. If false, correct the statement or give a counterexample.

   (a) (2+2 points) If a series converges, it also converges absolutely.

   (b) (2+2 points) The power series of the function $\frac{1}{1-x}$ converges for every real number x.

   (c) (2+2 points) If one wants to prove divergence of the positive series $\sum a_n$ using the Direct Comparison Test, then one must look for another positive sequence b_n such that $a_n \leq b_n$ where $\sum b_n$ diverges.

   (d) (2+2 points) The alternating series test (AST) applied to the series $\sum_{n=2}^{\infty}(-1)^n \frac{\sin n}{n^2}$ shows that it converges.

   (e) (2+2 points) One may apply the Ratio Test to a power series to find the initial interval of convergence, without checking the endpoints of the found interval.

2. (20 points) Determine the convergence/divergence of the following series, using any **applicable** method learned in class.

   (a) (5 points) $\sum_{n=1}^{\infty} \frac{1}{n2^n}$. (Dying quickly.)

   (b) (5 points) $\sum_{n=2}^{\infty} \frac{n}{n^2-1}$. (Use the fastest method if possible.)

   (c) (5 points) $\sum_{n=0}^{\infty} \frac{(3n)!}{(2n)!n!}$. (One method prevails!)

   (d) (5 points) $\sum_{n=1}^{\infty} \frac{1}{\ln n}$. (Helpless, need a friend.)

3. (20 points) Definitions.
   (a) (2 points) State the definition of an absolutely convergent series.
   A series $\sum a_n$ is absolutely convergent if ____.
   (b) (2 points) State the definition of a conditionally convergent series.
   A series $\sum a_n$ is conditionally convergent if ____.

   Using these definitions, determine if the following series converges absolutely, converges conditionally, or diverges.

   (a) (8 points) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$.

   (b) (8 points) $\sum_{n=1}^{\infty} (-1)^{n+1} (0.8)^n$.

4. (20 points) This problem on power series has two parts.

(a) (15 points) Using the power series,

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n,$$

find the power series representation of the function $g(x) = \frac{1}{1+x^2}$.

(b) (5 points) Determine its interval of convergence.

5. (20 points) This problem uses the Maclaurin series

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}.$$

(a) (8 points) Find the interval of convergence of this series.

(b) (8 points) Find the Maclaurin series for $\cos\left(x^2\right)$. Hint: find the Maclaurin series of $\cos(x)$ and then use function composition.

(c) (4 points) Find the Maclaurin series for $-2x\sin(x)$. Hint: differentiate part(b) term-by-term.

6. (5 points (bonus)) Find convergent series $\sum a_n$ and $\sum b_n$ such that that $\sum a_n b_n$ diverge.