Math 1023 BL02
Fall 2023 B Term
Exam 1
11/07/23
Time Limit: 50 Minutes

Name (Print): ____________________________

This exam contains 7 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- Manage the empty space wisely.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	0	
7	0	
Total:	100	

1. (20 points) True or False. If true, give a brief reason for your answer, such as a Theorem, a definition or a direct but concise argument. If false, provide a concise reason, correct the statement, or give a counterexample.

   (a) (2+2 points) The Integral Test may be applied to the series $\sum_{n=2}^{\infty} \sin(n)$.

   (b) (2+2 points) $\lim_{x\to 0} \frac{x}{1+x}$ yields an indeterminate form.

   (c) (2+2 points) If one wants to prove convergence of the improper integral $\int_1^\infty f(x)\,dx$ where $f(x) > 0$ and continuous, using the Direct Comparison Test, then one must look for another continuous function $g(x) > 0$ such that $g(x) \leq f(x)$ where $\int_1^\infty g(x)\,dx$ converges.

   (d) (2+2 points) When a limit yields an indeterminate form of $0 \cdot \infty$, one may apply L'Hopital's rule.

   (e) (2+2 points) If a_n converges to 0, then $\sum_{n=1}^{\infty} a_n$ also converges.

2. (20 points) Determine the following limits, if exist, using any methods taught in class.

   (a) (7 points)

$$\lim_{x\to 0^+} \sqrt{2x}\ln 2x$$

   (b) (7 points)

$$\lim_{x\to\infty} \frac{x^7 - 4x^3 - 6}{2x^7}$$

   (c) (6 points)

$$\lim_{x\to 0^+} \left(1 + \frac{1}{x^2}\right)^x$$

3. (20 points) Using Direct Integration or a Comparison Test (DCT or LCT) to establish convergence/divergence of the following improper integrals. If Direct Integration is used, find the exact limit. Make sure to check the **condition(s)** of the test before using it.

   (a) (7 points)

$$\int_0^\infty 2xe^{-x^2}dx$$

   Method:

   (b) (7 points)

$$\int_5^\infty \frac{x}{x^4+1}dx$$

   Method:

   (c) (6 points)

$$\int_2^\infty \frac{1}{\sqrt{x^4-1}}dx$$

   Method:

4. (20 points) Identify if the following infinite series is a geometric series or not. Study the convergence/divergence of the series. If the series ia convergent geometric series, find its sum. Possible methods are: n^{th}-term divergence test, geometric series, and the integral test.

   (a) (7 points)

   $$\sum_{n=1}^{\infty} \sin(n)$$

   Method:

   (b) (7 points)

   $$5 - \frac{25}{4} + \frac{125}{16} - \frac{625}{64} + \cdots$$

   Method:

   (c) (6 points)

   $$\sum_{n=1}^{\infty} \frac{\ln n}{n}$$

   Method:

5. (20 points) Determine if the following **sequences** converge or not. If the **sequence** converges, find the limit.

   (a) (7 points)

$$a_n = \frac{n^2 + 2n + 1}{n + 1}, \quad n = 0, 1, 2, \ldots$$

   (b) (7 points)

$$b_n = \frac{\ln \sqrt{n}}{\sqrt{n}}, \quad n = 1, 2, 3, \ldots$$

   (c) (6 points)

$$c_n = \left(1 + \frac{a}{n}\right)^n, \quad n = 1, 2, 3, \ldots,$$

   where a is any real number.

6. (5 points (bonus)) Find convergent geometric series $\sum a_n = A$ and $\sum b_n = B \neq 0$ and no b_n equals 0, which illustrate the fact that $\sum a_n/b_n$ may converge without being equal to A/B.

7. (5 points (bonus))

$$\sum_{n=1}^{\infty} \frac{100}{(1+t)^{n-1}}.$$

For what t does this series converge? Hint: write out a few terms to see what each term looks like and recognize the type of series.