

Math 1021 AL05
Fall 2024 A Term
Midterm Exam II
10/04/24
Time Limit: 50 Minutes

Name (Print): _____

This exam contains 9 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- **Manage the empty space wisely.**
- **Deep breath and pace yourself.**
- If you know how to start, namely, know the punch-line, you earn 60% of the points.

Problem	Points	Score
1	21	
2	20	
3	20	
4	20	
5	19	
6	0	
7	0	
Total:	100	

Do not write in the table to the right.

1. (21 points) Find the derivative of the following functions using any differentiation rules learned in class.

(a) (7 points) $y = xe^{-x^2}$.

(b) (7 points) $y = \frac{5^x}{\ln 5}$.

(c) (7 points) $y = x^x$, i.e., x to the power of x .

2. (20 points) Consider the implicit equation $x^2 + y^2 = 2x + 2y$.

(a) (12 points) Find dy/dx .

(b) (4 points) Find the set of points (x, y) such that the tangent line is **horizontal**.

(c) (4 points) Find the set of points (x, y) such that the tangent line is **vertical**.

3. (20 points) Consider the function $f(x) = x^{\frac{1}{x}}$ (i.e., x to the power of $1/x$) on the domain $(0, 3]$.
- (a) (7 points) Calculate the derivative $f'(x)$.

(b) (7 points) Confirm that $x = e$ is the only critical point in this interval.

(c) (3 points) Is $x = e$ a local max on this interval? Absolute max? Provide your reasoning via either 1st derivative test or 2nd derivative test.

(d) (3 points) Does $f(x)$ have an absolute **minimum** on $(0, 3]$? Why or why not?

4. (20 points) Approximations and Related Rates.

(a) (10 points) Estimate $\sqrt{8.98}$ using either linearization or differentials.

(b) (10 points) The side of a cube increases at a rate of 1 m/sec. Find the rate at which the volume of the cube increases when the side of the cube is 4 m.

5. (19 points) Sketch the curve $f(x) = x^4 - 2x^2$ by answering the following questions.

(a) (1 point) Determine the end behaviour of f , i.e., evaluate $\lim_{x \rightarrow \pm\infty} f(x)$.

(b) (2 points) Find x and y intercepts.

x -intercept(s):

y -intercept(s):

(c) (2 points) Find all critical points of $f(x)$.

(d) (4 points) Find all intervals where $f(x)$ is increasing/decreasing.

(e) (2 points) Identify all local extrema of $f(x)$.

Problem 5 continued: $f(x) = x^4 - 2x^2$.

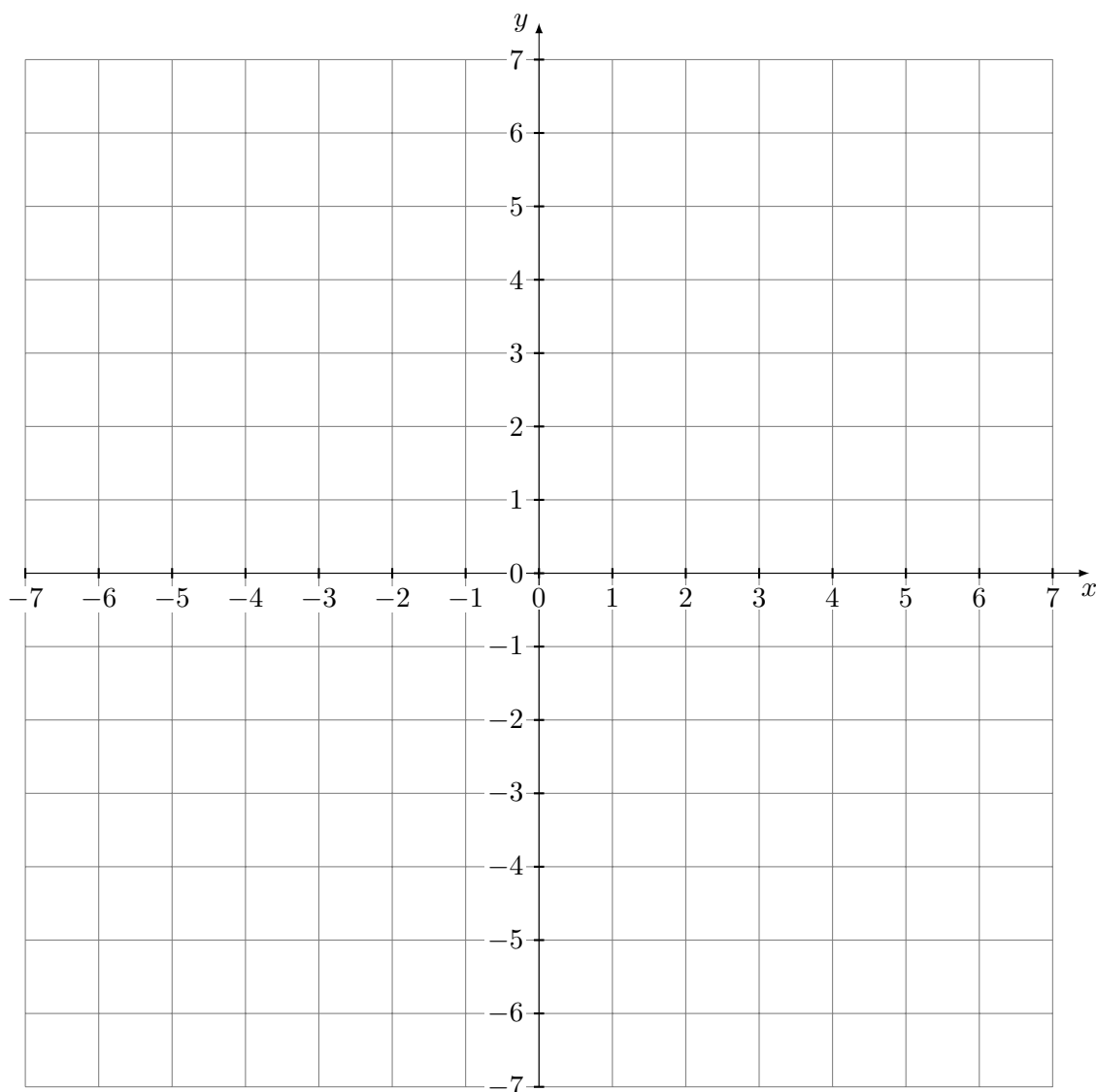
(f) (2 points) Find all intervals where $f(x)$ is convex (concave up).

(g) (2 points) Find all intervals where $f(x)$ is concave (concave down).

(h) (2 points) Find any inflection points of $f(x)$, if any.

- (i) (3 points) Sketch the curve based on the answers above.

Sketch of $f(x) = x^4 - 2x^2$



Use the space here to organize the information in previous parts to help you sketch.

6. (3 points (bonus)) Find the derivative of $f(x) = \frac{x\sqrt{2x+1}}{e^x \sin^3(x)}$. [Good luck using the Quotient Rule.]

7. (3 points (bonus)) Evaluate $\sin^{-1}(x) + \cos^{-1}(x)$.