Math 1021 AL05 **Name (Print):** _____
Fall 2024 A Term
Midterm Exam
09/13/24
Time Limit: 50 Minutes

This exam contains 8 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- **Manage the empty space wisely.**
- The Quotient Rule (frankly, you don't need it for this exam)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{f'(x)\,g(x)-f(x)\,g'(x)}{[g(x)]^2}.$$

- The Inverse Function Theorem: if $g(x)=f^{-1}(x)$, then

$$g'(x)=\frac{1}{f'(g(x))}.$$

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	0	
Total:	100	

Do not write in the table to the right.

1. (20 points) For the following functions, identify all vertical, horizontal and oblique asymptotes. **Use appropriate limits to justify your answers.**

   (a) (7 points) $f(x) = \frac{3x-1}{2x+5}$.

   (b) (7 points) $g(x) = \frac{1}{(x-2)^2}$.

   (c) (6 points) $h(x) = \frac{3x^2+4x}{x+2}$.

2. (20 points) For each function below,

   - Sketch it (label your axes clearly, and potential point(s) of interest);
   - Is f continuous? If not, identify the location(s) and type(s) of discontinuities. Show work.
   - Is f differentiable? Show work.

   (a) (4+3+3 points)

$$f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ x^2, & x > 1 \end{cases}.$$

   (b) (4+3+3 points)

$$f(x) = \begin{cases} 3, & x \geq 1 \\ 3x, & x < 1 \end{cases}$$

3. (20 points) This problem concerns the Intermediate Value Theorem. **Define your function clearly.** [Note the different weights of each problem]

   (a) (15 points) Show that $x^3 - x^2 - 3x + 1 = 0$ has at least one real solution.

   (b) (5 points) Show that $x = \cos x$ has at least one real solution.

4. (20 points) Find the derivative of the following functions, using any appropriate differentiation rules we've learned so far. Show work.

   (a) (5 points)

$$h(x) = \left(3x^2 + 1\right)^9.$$

   (b) (5 points)

$$h(x) = \sin(x)\cos\left(x^3\right).$$

(c) (5 points)

$$f(x) = \cot(x).$$

(d) (5 points)

$$s(t) = \frac{t^3}{t^3 + 1}.$$

5. (20 points) Use the Inverse Function Theorem (see page 1) to find the derivative of the following functions. ****If you don't use the Theorem, you can still earn at most 4 points for each problem by using any differentiation rules we've learned so far.**

   (a) (10 points)

$$f(x) = \frac{x+2}{x}.$$

   (b) (10 points)

$$f(x) = x^{1/3}.$$

6. (5 points (bonus)) Is it possible to find a and b (**both nonzero**) such that

$$f(x) = \begin{cases} -ax^2 + x + 1, & x < 1; \\ bx + 1, & x \geq 1. \end{cases}$$

is differentiable everywhere?