RECITATION IX

Problem 1. For each of the following series, convert to a proper sigma notation with $n = 1$ as the starting point. Write out the first four terms of the sequence of partial sums. Does it converge?

(1) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots$
(2) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$
(3) $\frac{5}{1\cdot 2} + \frac{5}{2\cdot 3} + \frac{5}{3\cdot 4} + \cdots$

Problem 2. Determine if the following geomtric series converges or not. It's helpful to write it in sigma notation first. If it converges, find its sum.

(1) $1 + \left(\frac{2}{5}\right) + \left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^3 + \cdots$
(2) $\frac{9}{4} - \frac{27}{8} + \frac{81}{16} - \frac{243}{32} + \cdots$

Problem 3. Consider the series

$$\sum_{n=1}^{\infty} \frac{1}{n\,(n+1)}.$$

(1) Write out the first few terms of the series (like the statement of problem 1 and 2).
(2) Then, perform the method of partial fractions to $\frac{1}{n(n+1)}$. Then, write out the first few terms again directly without doing any addition or subtraction. Does the sum become easier to visualise now? Derive a formula for the sequence of partial sum S_N and find its limit (and hence the sum of the series).

Problem 4. Consider the geometric series introduced in class

$$a + ar + ar^2 + ar^3 + \cdots = a\sum_{n=1}^{\infty} r^{n-1}$$

(1) Write out explicitly the N^{th} partial sum

$$S_N = a\sum_{n=1}^{N} r^{n-1} = ?$$

(2) Multiply r to both side of the above equation ($S_N = ?$) to obtain $rS_N = r?$.
(3) Subtract rS_N from S_N (both sides).
(4) Factor, rearrange, and solve for S_N in terms of r, a and N.
(5) For $|r| < 1$, find $\lim_{N\to\infty} S_N$.