MATH 111-007 RECITATION 1027

The following problems don't necessarily have to do with Chapter 4. **Any answer without an attempted explanation (when requested) receives automatically -1 for the recitation.**

Problem 1. (Odd and even functions and their derivatives)

(1) An odd function $f(x)$ satisfies $f(x) = -f(-x)$. Suppose further that f is differentiable. What can you say about $f'(x)$?
(2) An even function $f(x)$ satisfies $f(x) = f(-x)$. Suppose further that f is differentiable. What can you say about $f'(x)$?

Problem 2. Show that $y_1 = \sin(x)$, $y_2 = \cos(x)$ and $y_3 = a\cos(x) + b\sin(x)$ where a and b are constants, all satisfy the equation

$$y'' + y = 0.$$

How would you modify the functions above so that they satisfy $y'' + 4y = 0$? How about $y'' + ky = 0$ for $k \neq 0$?

Problem 3. Consider the function $f(x) = x^3$. Construct an interval such that

(1) both absolute extrema exist. Give their values.
(2) only absolute minimum exists, but not absolute maximum.
(3) no absolute extrema.

Problem 4. Again, consider the function $f(x) = x^3$. Is there a local maximum at $x = 0$? Why or why not?

Support your claims by using the definition of local maximum directly, e.g. if you claim it is a local maximum, then you need to tick all the check points of the definition; on the other hand, if you claim it is not, then state exactly which check points have failed in the definition in this situation.

Theorem. *(Extreme Value Theorem) If f is continuous on a closed and bounded interval $[a, b]$, then f achieves both its absolute maximum and minimum on $[a, b]$. More precisely, there are numbers $x_1, x_2 \in [a, b]$ such that $f(x_1) = m$ and $f(x_2) = M$, and*

$$m \leq f(x) \leq M, \quad \text{for all } x \in [a, b].$$

Problem 5. (Sketch and explain) For the following functions, identify the domain D. Determine whether $f(x)$ achieves absolute extrema on D. Explain how your answer is consistent with the Extreme Value Theorem (given in lecture and above).

(1) $f(x) = \begin{cases} x + 1, & -1 \leq x < 0; \\ \cos(x), & 0 < x \leq \frac{\pi}{2}. \end{cases}$.
(2) $f(x) = \begin{cases} \frac{1}{x}, & -1 \leq x < 0; \\ \sqrt{x}, & 0 \leq x \leq 4. \end{cases}$
(3) $f(x) = \begin{cases} 1, & x = 0; \\ 0, & -1 \leq x < 0 \text{ and } 0 < x \leq 1. \end{cases}$