MATH 111-007 RECITATION 4

A well-known result for the exponential function is

$$\frac{d}{dx}\left(e^{x}\right)=e^{x},$$

namely, the function and its derivative (and thus any order of derivatives) are equal.

Problem 1. Compute $\frac{d}{dx}\left(e^{2x}\right)$ without using the Chain Rule (which we haven't learned).

Problem 2. Find the derivative of $y=\frac{(x-1)\left(x^{2}-2x\right)}{x^{4}}$. Use the **best** method in your opinion (which means you should try several).

Problem 3. Consider the free fall

$$s\left(t\right)=\frac{1}{2}gt^{2}$$

where $s\left(t\right)$ is the vertical displacement in meters, of a free falling object. g is the gravitational acceleration with unit m/s^{2}, though kept general here (so, not specifically the Earth's). Find t_0, the time it takes for the downward velocity of the object to reach some level v_0. How does t_0 depend on g?

Problem 4. Consider a projectile on an airless planet (so no air friction considered) with unknown gravitational acceleration constant g_s. The height of the projectile as a function of time t follows

$$s\left(t\right)=15t-\frac{1}{2}g_{s}t^{2}.$$

The projectile reached its maximum height 20 seconds after being launched. What was the value of g_s?

Problem 5. Determine <u>all</u> vertical asymptotes, and find the derivative of the following functions:

(1) $\tan\left(x\right)$.
(2) $\cot\left(x\right)$.
(3) $\csc\left(x\right)$.
(4) $\sec\left(x\right)$.

In each part, does it make sense that the vertical asymptotes are where they should be?

Problem 6. Find the derivative of $y=\frac{\cos^{2}(x)}{1+\sin(x)}$. Determine the **best** method.

Problem 7. Consider the piece-wise function

$$f\left(x\right)=\begin{cases}x+b, & x<0,\\ \cos\left(x\right), & x\geq 0.\end{cases}$$

Is there a value of b that will make $f\left(x\right)$ continuous at $x=0$? Differentiable at $x=0$ (Think about this. Think!)? Justify.