MATH 111-007 RECITATION I

SEPTEMBER 15TH, 2021

Remark. Key concepts:

- Continuity of $f(x)$ at $x = a$ means
$$\lim_{x \to a} f(x) = f(a)$$
(it means you can plug in). It further implies that if $\lim_{x \to b} g(x) = a$, then
$$\lim_{x \to b} f(g(x)) = f\left(\lim_{x \to b} g(x)\right) = f(a).$$
This applies directly below.
- $\lim_{x \to 0} \frac{\sin(ax)}{ax} = 1$ for $a \neq 0$. A common technique to use this fact is by putting yourself in position with an expression that looks like it:

$$\begin{aligned}
\lim_{x \to 0} \frac{\sin(3x)}{\sin(x)} &\overset{\text{manipulation}}{=} \boxed{\lim_{x \to 0} \frac{\sin(3x)}{3x} \frac{3x}{\sin(x)}} \\
&\overset{\text{factor out constants}}{=} 3 \lim_{x \to 0} \frac{\sin(3x)}{3x} \frac{x}{\sin(x)} \\
&\overset{\text{product rule (since both limits exist)}}{=} 3 \left(\lim_{x \to 0} \frac{\sin(3x)}{3x}\right) \left(\lim_{x \to 0} \frac{x}{\sin(x)}\right) \\
&\overset{\text{by choosing } f(x)=\frac{1}{x}, g(x)=\frac{\sin(x)}{x}, b=0\text{; you also know } a=1}{=} 3 \cdot 1 \cdot \lim_{x \to 0} \left(\frac{\sin(x)}{x}\right)^{-1} \\
&= 3 \cdot \left(\lim_{x \to 0} \frac{\sin(x)}{x}\right)^{-1} \\
&= 3 \cdot 1 \\
&= 3
\end{aligned}$$

The boxed step is the most important step.

Problem 1. (Limits)

(1)
$$\lim_{x \to 0} \frac{\tan(3x)}{\sin(x)}.$$
(Hint: express $\tan(3x)$ differently.)

(2)
$$\lim_{x \to 2} \frac{x^3 - 4x}{x^2 - 2x}.$$
(Hint: plugging in won't work.)

Problem 2. (Continuity) Determine the value of k such that the following function is continuous
$$f(x) = \begin{cases} x^2 + 2kx + 1, & x \leq -1 \\ -x - 4, & x > -1 \end{cases}.$$
Sketch $f(x)$.

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Problem 3. (Asymptotes) Find all horizontal, vertical and oblique asymptotes for the following function,

$$f(x) = \frac{x^2 - x}{x - 2}.$$

Problem 4. (Intermediate value theorem) Show that the following equation

$$x^3 - 2x + 1 = 0$$

has at least one real solution. (Hint: for the test values, choose the simplest possible.)