# MATH 111-007 QUIZ 4

OCT 4TH, 2021

**Problem 1.** Differentiate $f(x) = \tan^2(3x^2)$.

**Solution.** First, inner function is $g(x) = \tan(3x^2)$ and outer function is $h(x) = x^2$. Thus,

$$f'(x) = h'(g(x))\,g'(x) = 2\tan(3x^2)\frac{d}{dx}\tan(3x^2).$$

To deal with the second term, we find its inner function to be $3x^2$ and outer $\tan(x)$. Hence,

$$\frac{d}{dx}\tan(3x^2) = \sec^2(3x^2)\cdot 6x.$$

Altogether,

$$f'(x) = 12x\tan(3x^2)\sec^2(3x^2).$$

Now, if you choose differently, such as inner function is $g(x) = 3x^2$ and outer is $h(x) = \tan^2(x)$, you will still be fine.

$$f'(x) = h'(g(x))\,g'(x)$$

where you realise

$$h'(x) = 2\tan(x)\sec^2(x) \implies h'(g(x)) = 2\tan(3x^2)\sec^2(3x^2)$$

(find your inner and outer for $h(x) = \tan^2(x)$) and

$$g'(x) = 6x.$$

Patching them all together,

$$f'(x) = 12x\tan(3x^2)\sec^2(3x^2).$$

*Remark.* Note that even though $\frac{d}{dx}\tan(x) = \sec^2(x)$, it does NOT mean $\frac{d}{dx}\tan^2(x) = \sec^4(x)$ – otherwise you are claiming $\left(\frac{d}{dx}\tan(x)\right)^2 = \frac{d}{dx}\tan^2(x)$, which is absurd. To find $\frac{d}{dx}\tan^2(x)$, you do power chain rule.

**Problem 2.** Find $\frac{dy}{dt}$ when $x = 1$ if $y = x^2 + 7x - 5$ and $\frac{dx}{dt} = \frac{1}{3}$.

**Solution.** By the chain rule,

$$\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt} = \frac{1}{3}\frac{dy}{dx}.$$

At the same time,

$$\frac{dy}{dx} = 2x + 7 \implies \frac{dy}{dx}\Big|_{x=1} = 9.$$

Altogether,

$$\frac{dy}{dt}\Big|_{x=1} = \frac{1}{3}\frac{dy}{dx}\Big|_{x=1} = \frac{1}{3}\cdot 9 = 3.$$

*Remark.* Note, $\frac{dy}{dt} = \frac{d}{dt}y(x(t))$ is an explicit function of $t$, while $\frac{dx}{dt} = \frac{d}{dt}x(t)$ is also one. However, $\frac{dy}{dx} = \frac{d}{dx}y(x)$ is an explicity function of $x$, and has nothing to do with $t$.

$$\frac{dy}{dt} = \frac{d}{dx}(y(x(t))) = \frac{d}{dx}y(x)\times\frac{d}{dt}x(t)$$

is really what's happening.

**Problem.** (Bonus) Prove that $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$. (Hint: use the Power Chain Rule and then realise something.)

*Proof.*

$$\begin{aligned}
\frac{d}{dx}\sec(x) &= \frac{d}{dx}\frac{1}{\cos(x)} \\
&= \frac{d}{dx}(\cos(x))^{-1} \\
&= -\cos(x)^{-2}(-\sin(x)) \\
&= \frac{\sin(x)}{\cos^2(x)} \\
&= \frac{1}{\cos(x)}\frac{\sin(x)}{\cos(x)} \\
&= \sec(x)\tan(x)
\end{aligned}$$

□