

### MATH 111-007 QUIZ 3

SEPTEMBER 27TH, 2021

**Problem 1.** Find the derivative of the following functions. If you run out of time, write down the **best** derivative rule (among the ones we have learned so far) for each problem.

(1)  $f(x) = 6x^5(3x^2 + 1)$ .

**Solution.** Multiply out and do power rule.

$$f(x) = 18x^7 + 6x^5.$$

Then,

$$f'(x) = 18 \cdot 7x^{7-1} + 6 \cdot 5x^{5-1} = 126x^6 + 30x^4.$$

I strongly advise against product rule.

(2)  $g(y) = \sqrt[3]{y^2}(y-1)^{-1}$ .

**Solution.** Simplify the first term into a power and use quotient rule.

$$g(y) = \frac{y^{\frac{2}{3}}}{y-1}.$$

Then,

$$g'(y) = \frac{(y-1) \frac{d}{dy} y^{\frac{2}{3}} - y^{\frac{2}{3}} \frac{d}{dy} (y-1)}{(y-1)^2} = \frac{(y-1) \frac{2}{3} y^{\frac{2}{3}-1} - y^{\frac{2}{3}}}{(y-1)^2} = \frac{2}{3} \frac{y^{-\frac{1}{3}}}{(y-1)} - \frac{y^{\frac{2}{3}}}{(y-1)^2}$$

or whatever simplified form you arrive at.

**Problem.** (Bonus) Based on your knowledge of the product rule for two functions, guess (1 pt for correct guess) or prove (2 pts) the product rule for three functions, e.g.  $\frac{d}{dx}(f(x)g(x)h(x)) = ?$  (Hint: write down  $\frac{d}{dx}(f(x)g(x))$  is, first, then think how you can put yourself in position to use it when considering  $\frac{d}{dx}(f(x)g(x)h(x))$ . Do you have to consider all 3 functions simultaneously or not?)

*Proof.* Let  $I(x) = g(x)h(x)$ . Then,

$$\begin{aligned} \frac{d}{dx}(f(x)g(x)h(x)) &= \frac{d}{dx}(f(x)I(x)) \\ &= f'(x)I(x) + f(x)I'(x) \end{aligned}$$

At the same time, we know

$$I'(x) = g'(x)h(x) + g(x)h'(x).$$

Altogether,

$$\frac{d}{dx}(f(x)g(x)h(x)) = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x).$$

The pattern is that in each term, you keep two fixed and differentiate the other, then add up all the cases. □