RECITATION 1007

Problem 1.

$$\int \frac{3x^2+2}{(x^2+3)(x-4)}dx$$

Solution. Since x^2+3 is an irreducible quadratic, it's partial fraction will contain a polynomial one degree fewer than itself (namely, a linear function).

$$\begin{aligned}
\frac{3x^2+2}{(x^2+3)(x-4)} &= \frac{Ax+B}{x^2+3} + \frac{C}{x-4} \\
&= \frac{Ax^2+Bx-4Ax-4B+Cx^2+3C}{(x^2+3)(x-4)} \\
&= \frac{(A+C)x^2+(B-4A)x+3C-4B}{(x^2+3)(x-4)}
\end{aligned}$$

which implies that the matching must yield

$$\begin{aligned}
A+C &= 3 \\
B-4A &= 0 \\
3C-4B &= 2
\end{aligned}$$

Solving it whatever way you want, we will obtain

$$\begin{aligned}
A &= \frac{7}{19} \\
B &= \frac{28}{19} \\
C &= \frac{50}{19}
\end{aligned}$$

Therefore, with some arithmetic (some is done to promote easier substitution)

$$\begin{aligned}
\int \frac{3x^2+2}{(x^2+3)(x-4)}dx &= \frac{1}{19}\int \left(\frac{7x+28}{x^2+3} + \frac{50}{x-4}\right) dx \\
&= \frac{7}{38}\int \frac{2x}{x^2+3}dx + \frac{28}{19}\int \frac{1}{x^2+3}dx + \frac{50}{19}\int \frac{1}{x-4}dx \\
&= \frac{7}{38}\ln\left|x^2+3\right| + \frac{28}{19\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + \frac{50}{19}\ln|x-4| + C
\end{aligned}$$

Problem 2. Estimate the integral by Trapezoidal rule with 4 subintervals: $\int_1^3 \frac{3}{x-\frac{1}{2}}dx$. Determined the error by comparing to the exact solution. (Sketch the graph and draw your subintervals)

Solution. The subinterval endpoints are then $\left\{1, \frac{3}{2}, 2, \frac{5}{2}, 3\right\}$ with spacing $\Delta x = \frac{1}{2}$. Our function $f(x) = \frac{3}{x-\frac{1}{2}}$. Thus, by factoring the 3 out from the function (since it is common),

$$\begin{aligned} T_4 &= \frac{\Delta x}{2}\left[f(1) + 2f\left(\frac{3}{2}\right) + 2f(2) + 2f\left(\frac{5}{2}\right) + f(2)\right] \\ &= \frac{3}{4}\left[\frac{1}{1-\frac{1}{2}} + \frac{2}{\frac{3}{2}-\frac{1}{2}} + \frac{2}{2-\frac{1}{2}} + \frac{2}{\frac{5}{2}-\frac{1}{2}} + \frac{1}{3-\frac{1}{2}}\right] \\ &= \frac{3}{4}\left[2 + 2 + \frac{4}{3} + 1 + \frac{2}{5}\right] \\ &= \frac{101}{20} \\ &= 5.05 \text{ (decimals only for comparison purposes)} \end{aligned}$$

Meanwhile, the exact result is

$$\int_1^3 \frac{3}{x-\frac{1}{2}} dx = 3\left[\ln\left|x - \frac{1}{2}\right|\right]_{x=1}^{x=3} = 3\left[\ln\frac{5}{2} - \ln\frac{1}{2}\right] = 3\ln\frac{5/2}{1/2} = 3\ln 5 \approx 4.8283$$

So we overestimated. However, if you do a proper sketch, you will notice that for monotone decreasing concave up functions like the one we have here, trapezoidal rule **ALWAYS** overestimates because the slope of each trapezoid lies on top of the curve, hence overestimating by a “moon wedge” at each subinterval.