MA556, Applied Bayesian Statistics — Spring 2018

Homework Assignments

Homework #4 **Assigned: 10/20/19** **Due: 10/?/19**

1. Let $X \sim \text{Poisson}(\lambda), \lambda \sim \text{Gamma}(\alpha, \beta)$, where α and β are known. Find the marginal likelihood of X. Find an upper bound for the marginal likelihood.

2. Show that the Cauchy density,

$$f(x) = \frac{1}{\pi(1+x^2)}, -\infty < x < \infty,$$

is unimodal but not logconcave. What part of the support is it logconcave? [Logconcave densities are unimodal, but a unimodal density might not be logconcave.]

3. In a logistic regression model, the posterior density,

$$p(\nu \mid \underset{\sim}{a}) \propto \frac{e^{3\nu}}{\prod_{i=1}^{n}(1+a_i e^{\nu})} e^{-\nu^2/2}, -\infty < \nu < \infty,\ a_i \geq 0$$

arises naturally. Show that this density is logconcave. Use the adaptive rejection sampling (ARS) algorithm to draw samples from it, and to obtain the posterior mean and standard deviation of ν. You may take $n = 15$ and the values, $1, 3, 5, 4, 2, 2, 5, 6, 1, 8, 7, 6, 5, 5, 3$, for the a_i. Also, find 95% credible interval and HPD interval. [You need to use a software with the ARS e.g. R. Please do not program the algorithm; simply apply the algorithm!]