MA4632/541 **Spring 2019**

HOMEWORK ASSIGNMENTS

Homework #6 **Assigned: 4/4/19** **Due: 4/25/19**

1. (a) Let $X_1, \ldots, X_n \mid \theta \stackrel{iid}{\sim} \text{Uniform}(0, \theta)$. Obtain an equal-tail $100(1-\alpha)\%$ confidence interval for θ. Find the shortest $100(1-\alpha)\%$ confidence interval for θ. Present a numerical comparison of the two intervals for $n = 10$ and 90%, 95% and 99% coverage.

(b) Let $X_1, \ldots, X_n \mid \theta \stackrel{iid}{\sim} \text{Uniform}(0, \theta)$. Show that $X_{(n)}$ is a consistent estimator of θ. Discuss how you might use the sample mean to obtain a consistent estimator of θ. Compare these two estimators for both small and large samples.

2. (a) Problem 9.2 in the Casella and Berger textbook.

(b) Problem 9.12 in the Casella and Berger textbook.

3. (a) Problem 9.13 in the Casella and Berger textbook.

(b) Problem 9.16 in the Casella and Berger textbook.

4. Let $X_1, \ldots, X_n \mid p \stackrel{iid}{\sim} \text{Bernoulli}(p)$ and suppose that $p \sim \text{Beta}(a, b)$, where a and b are known. By using the relation between the beta and the F-distributions give a $100(1-\alpha)\%$ credible interval for p. Discuss how to obtain a $100(1-\alpha)\%$ HPD interval for p.

5. Let $X_{i1}, \ldots, X_{in_i} \mid \mu_i, \sigma_i^2 \stackrel{ind}{\sim} \text{Normal}(\mu_i, \sigma_i^2), i = 1, 2$. Construct a $100(1-\alpha)\%$ confidence interval for σ_2^2/σ_1^2.

6. Let $X_1, \ldots, X_n, X_{n+1} \mid \theta \stackrel{iid}{\sim} \text{Uniform}(0, \theta)$. Suppose you observe $X_1, \ldots, X_n$. Find a $100(1-\alpha)\%$ prediction interval for X_{n+1}. [Hint: Construct a pivot that is a function of X_{n+1} and ancillary for θ.]

7. (a) Let $X_1, \ldots, X_n \mid \theta \stackrel{iid}{\sim} f(x \mid \theta)$. Find the asymptotic relative efficiencies of the sample median versus the sample mean for estimating the location parameter in (i) logistic density and (ii) the double exponential.

(b) Let $X_1, \ldots, X_n \mid \theta \stackrel{iid}{\sim} \text{Uniform}(0, \theta)$. Show that $X_{(n)}$ is a consistent estimator of θ. Discuss how you might use the sample mean to obtain a consistent estimator of θ. Compare these two estimators for both small and large samples.

8. (a) Casella and Berger, Problem 10.3.

(b) Casella and Berger, Problems 10.34.

9. Let $X \mid p \sim \text{Binomial}(n, p)$. Find the $100(1-\alpha)\%$ of p asymptotic confidence interval of p that is based on the pivot $\frac{\hat{p}-p}{\sqrt{p(1-p)/n}}$, where $\hat{p}$ is the MLE of p.