Question: Do the relationships between force, mass, and acceleration of two variable-weighted carts on connected tracks of equal-slope inclined planes, oriented so the combined height of both carts remains constant, obey Newton's second law?

Hypothesis: The relationship between [the acceleration of the carts] and the [difference in the weights of the two carts (m_1-m_2)] will be linear. The slope of this line should be [gravity multiplied by the sin of the base angle] divided by [the sum of the masses].

Strategy:

- The inclination of the planes, the string used, the carts and weights used, and the pulley used, all stayed the same throughout the entire experiment.
- The mass of each cart was changed by moving weights from one cart to another, thus preserving the total weight of the system.
- Three trials were conducted where heavy hexagonal-prism-shaped masses were transferred from one cart to the other cart. The number of masses on each cart for each trial were [5 and 0], [4 and 1], and finally [3 and 2].
- The resulting acceleration was calculated through a built-in position sensor in each cart.
- The difference in weights was then graphed against the measured acceleration to verify that the slope was equal to [gravity multiplied by the sin of the angle] divided by [sum of the masses].

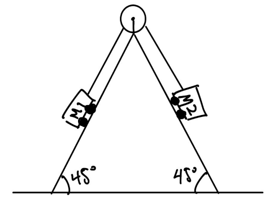

Fig:1 Modified Atwood's Machine

Data:

Total mass of the system: 1.23 kg

Mass of Car 1 (kg)	Mass of Car 2 (kg)	Difference in Masses(kg)	Acceleration (m/s^2)
0.678	0.552	0.126	0.873
0.804	0.426	0.378	2.588
0.93	0.3	0.63	4.443

The shown acceleration is the average of the three trials

Analysis:

The free body diagrams in Figure 2 show the forces on each cart in the modified Atwood's machine.

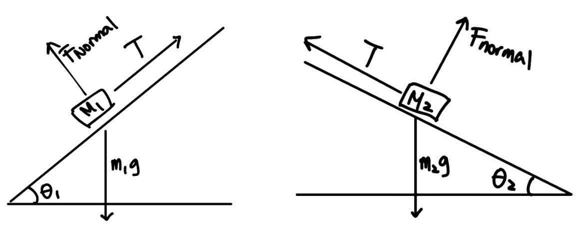

Figure 2: Free Body Diagrams for each cart

Friction between each cart and its track is negligible because the cart's wheels spin freely. The following equations are based on the free body diagrams. Positive motion is defined as down the ramp for the left cart (and thus up the ramp for the right cart.

$$(m_1\sin(\theta)g) - T = a\,(m_1+m_2)$$
$$(m_2\sin(\theta)g) - T = -a\,(m_1+m_2)$$

These two equations can be combined into the following:

$$\frac{m_1 \sin(\theta_1)g - m_2 \sin(\theta_2)g}{m_1 + m_2} = a$$

Rearranging,

$$m_1 - m_2 = \frac{a(m_1 + m_2)}{g\sin(\theta)}$$

This equation indicates that there is a linear relationship between the difference in the masses and the acceleration. The slope of this line should be [acceleration divided by gsin(θ)].

A graph of the difference in mass vs. [acceleration divided by gsin(θ)] data for this experiment shows that it is indeed linear, and that the slope is equal to 0.355 kg.

The expected value can be calculated by dividing 9.8*sin(65) by 1.23, the sum of the weights, to get an expected slope of 7.22. The actual slope was 7.08, which is only a 2% error!

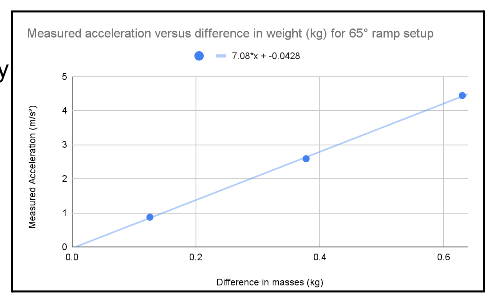

Figure 3: Measured acceleration vs. difference in masses for a 65-degree ramp.

The fact that it is too large indicates that the acceleration values were less than expected, and thus energy loss occurred. Some factors that might have caused this loss of energy could've been friction, air resistance, or inertial transfer into both car wheels and pulleys. Although we tried our best to reduce friction and use pulleys and wheels with low moments of inertia, physical constraints restricted our accuracy.