Abstract

In graph theory, an Order-3 Maximal Independent Set (MIS) is a MIS whose cardinality cannot be enlarged by the utilization of up to 3 nodal additions and removals. This project aims to count Order-3 Maximal Independent Sets (MIS) and investigate the behavior of patterns found therein on graphs G of the type $G = \left(\otimes_{i=1}^{a} P_{n_i} \right) \times \left(\otimes_{j=1}^{b} C_{m_j} \right)$, $\forall\, i, j\ (n_i \geq 2, m_j \geq 3)$. Normal MIS are used in network theory to spread vertices over networks, for applications such as communication systems, fraud detection, infrastructural systems, and marketing, but typical MIS lack measurable properties to distinguish the density and uniformity of vertex packings. Order-3 MIS provide a measure of packing optimality to distinguish MIS which remain independent following small nodal perturbations. Computer simulations were used to acquire data for torus, lattice, and cylinder graphs, and then inverse matrices were used to reverse-engineer inductive patterns in the generated sequences when enlarging the path component of cylinders and lattice graphs. Pure exponential growth was observed in some special scenarios, while inductive patterns were observed in others. The number of Order-3 Maximal Independent Sets in these graphs allows analysis of different graph types and offers improvements to the existing applications of MIS, like network design, scheduling, and computation theory.