Analyzing the Impact of In-rack Sprinklers in a Warehouse Fire: A Demonstration of the Role Optimization Has in Mitigating Damage

Andrew C. Trapp[a] and Ali S. Rangwala[b]

[a]School of Business , [b]Department of Fire Protection Engineering

Worcester Polytechnic Institute

100 Institute Road, Worcester, MA 01609, USA

Phone: +1 508 831 4935, Fax: +1 508 831 5720

atrapp@wpi.edu, rangwala@wpi.edu

Abstract

This study presents a tool that illustrates the potential and role that optimization can have in fire protection design and performance. Specifically, the tool uses validated engineering correlations to simulate a warehouse fire, and evaluates whether it is controlled by modeling the placement and number of sprinklers – both in-rack and ceiling. Using initial model inputs that include storage configuration, commodity type, water supply, and sprinkler type, we allow a fire to ignite and simulate its growth behavior and corresponding effects on a nearby target array. For any initial set of input parameters, our tool demonstrates the minimum number and location of available in-rack sprinklers to control the flame spread. With reasonable run times, our work yields new insights into the design of sprinkler systems and, more broadly, the usefulness of optimization in fire protection design.

Keywords: Warehouse Fire Modeling; Optimization; Fire Simulation; Sprinkler Configurations

[a]*Corresponding author*

1. Introduction

This study presents a tool that illustrates the potential and role that optimization can have in fire protection design and performance. Specifically, the tool uses validated engineering correlations to simulate a warehouse fire, and evaluates whether it is controlled by modeling the placement and number of sprinklers – both in-rack and ceiling. Using initial model inputs that include storage configuration, commodity type, water supply, and sprinkler type, we allow a fire to ignite and simulate its growth behavior and corresponding effects on a nearby target array. For any initial set of input parameters, our tool demonstrates the minimum number and location of available in-rack sprinklers to control the flame spread. With reasonable run times, our work yields new insights into the design of sprinkler systems and, more broadly, the usefulness of optimization in fire protection design – a possible complete loss of the warehouse. The advent and expansion of in-rack sprinkler systems offers much promise, but further compounds such interactions, making the situation very challenging to assess from a design perspective.

Ideally, all feasible design alternatives would be evaluated prior to settling on a final outcome. In practice, however, an ad-hoc method is often employed to solve engineering design problems: select a few feasible design alternatives, compare, and choose the best [1]. The lack of a systematic, integrated approach to analyze tradeoffs and make key decisions can have undesirable implications. Optimization as a design methodology can help. Broadly speaking, it is aimed at identifying optimal or near-optimal solutions to complex decision problems. Such problems often involve a system of equations or inequalities that implicitly characterize all feasible solutions, together with means to evaluate the quality of any given solution satisfying that system. In the context of engineering design, optimization and simulation optimization has proven useful in a wide variety of applications, including improving the designs of truss structures [2,3], Stirling engines [4,5], aircraft fuselage panels [6], water pumping systems [7], the built environment [8], and gas detector spacing [9], among others [10]. For more information on simulation optimization we refer to [11]. Thus we seek to answer the following question: How can optimization be leveraged to improve the design and layout of a sprinkler system in a warehouse setting?

The answer requires expressions of the fire growth as a function of time, the corresponding temperature and velocity field generated, the resulting sprinkler activation and finally the inter-action – i.e., the reduction of heat release rate (HRR) of the fire with the water spray. Such estimations can be obtained via engineering correlations [12], computational fluid dynamics (CFD) [13,14], or experimental data from several large scale experiments [12,15–19]. Among the many ways of representing this type of allowed to burn freely upwards. The approach can be extended to other ignition locations but it is not done here. The approach used is further illustrated in Fig. 2

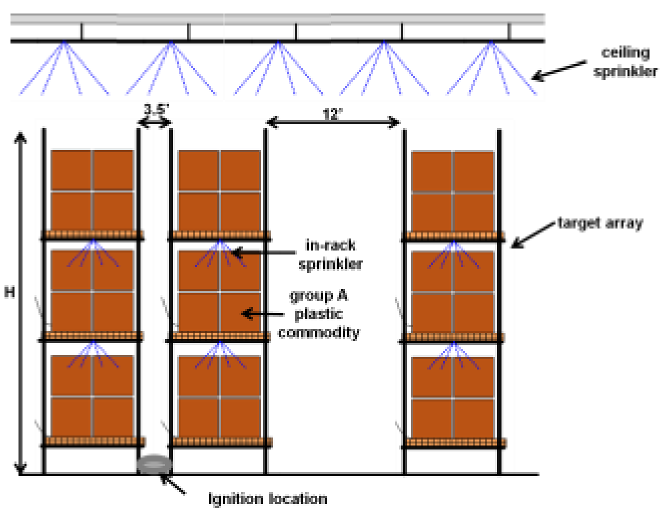

Fig. 1: An illustrative sketch of the problem considered in this study.

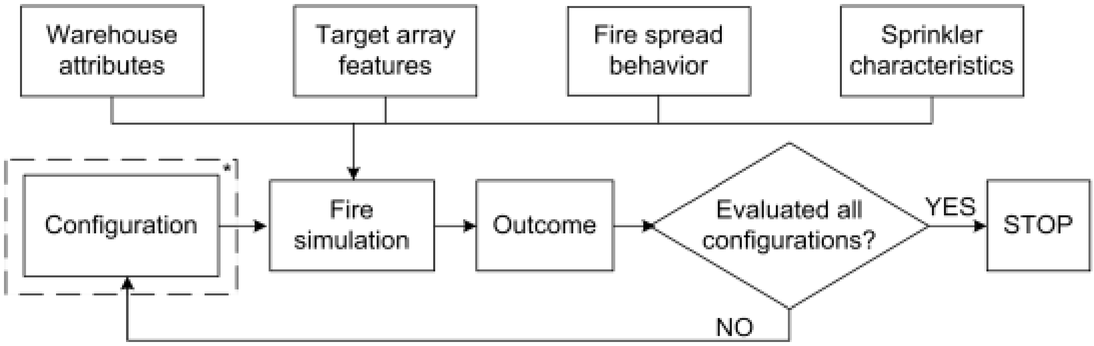

Fig. 2: Layout of warehouse model. In the current study we vary the number and placement of in-rack sprinklers to identify an optimal configuration.

Holding the warehouse attributes (e.g., commodity type, etc), target array features (e.g., perpendicular distance, heat transfer coefficient, etc.), fire spread behavior (e.g., heat release rate, etc.) and sprinkler characteristics (e.g., RTI, water application rate, etc.) constant, we vary the number and placement of in-rack and ceiling sprinklers with respect to the fixed location of the ignition source. To facilitate this, we define a sprinkler configuration as a possible arrangement of in-rack and ceiling sprinklers, where each tier may (or may not) host a single in-rack sprinkler, and where the ceiling may (or may not) feature a single sprinkler. As shown in Fig. 2, the configuration is changed, the warehouse fire simulated, and the corresponding outcome is checked based on the control criteria discussed earlier. The specific questions that are answered are:

1. *Is there a minimal-size configuration of in-rack sprinklers that can contain a fire to a single array such as illustrated in Fig. 1?*
2. *How does the minimal-size configuration change with overall height of storage (from 15 feet to 40 feet)?*
3. *How does a sprinkler's efficiency affect the outcomes?*

The answers to these questions go a long way towards improving warehouse fire suppression design. They also shed light on the tradeoffs between cost of in-rack sprinkler installation and maintenance versus damage due to a warehouse fire. Most importantly, the problem demonstrates the applicability of optimization to a critical problem in fire safety.

2. Mathematical Relationships Governing Fire Behavior

The fire model is a result of extensive experimental and mathematical modeling of the heat and mass transfer processes and is similar to models reported in Zalosh [12], Heskestad and Delichatsios [24], Beyler[25], and Schifiliti [26], where a sprinkler activation model based on the temperature and velocity profile around the sprinkler head is developed. Similar to Yu et al. [17] the impact of water on the fire is analyzed using a nonlinear reduction of the overall heat release rate. This revised heat release rate is then used to estimate the temperature at the target array through computing the flame's radiative heat flux.

2.1 Modeling Existence of Ceiling and In-rack Sprinklers

For an *N*-tiered array we allow for the possibility of up to *N* in-rack sprinklers, one per tier, as well as a ceiling sprinkler. For each of the tiers $i = 1, \ldots, N$ (with tier 1 being nearest the floor, and tier *N* being nearest the ceiling), we define a binary variable s_i as:

$$s_i = \begin{cases} 1 & \textit{if an in-rack sprinkler is located at tier } i, \textit{ and} \\ 0 & \textit{otherwise}. \end{cases} \quad (1)$$

In a similar manner we define a binary variable s_c to represent the presence (or absence) of a single ceiling sprinkler. We can mathematically represent such a configuration $\bar{s}$ using a vector of $N + 1$ positions, where the first *N* positions contain binary variables s_i, and the final position contain variable s_c, resulting in 2^{N+1} possible configurations. As an example, Fig. 3 depicts a 5-

tier array with in-rack sprinklers present at tier 3 and 5, and no ceiling sprinkler. We are interested in determining the minimum number of in-rack and ceiling sprinklers necessary, as well as their tier placements, to control an associated fire. The use of binary variables in this manner makes it straightforward to identify the number of in-rack and ceiling sprinklers for a given configuration. For any configuration $\bar{s}$, we define M as the number of present in-rack sprinklers, i.e., $M = \sum_{i=1}^{N} s_i$ (note that M does not include s_c).

$$\underset{1}{[0}\;\underset{2}{0}\;\underset{3}{1}\;\underset{4}{0}\;\underset{5}{1}\,|\,\underset{6}{0]}$$

Fig. 3: Binary vector representing configuration of 5-tier array with *no* ceiling sprinkler (location 6).

Furthermore, for each configuration we associate an outcome indicating whether the configuration is able to successfully control the fire (C), or fails to do so (F). It follows given the context of our study that the configuration [1 … 1| 1] (having all possible sprinklers) is assured of a control outcome (i.e., C), while the configuration [0 … 0| 0] (having no sprinklers) is assured of a failure to control outcome (i.e., F).

2.2 Heat Release Rate

The heat release rate is modeled using Eq. (B.3a) from NFPA 72 [20]:

$$\dot{Q}^{(t)} = \left(\frac{N}{2}\right)\left(\frac{1{,}055}{t_g^2}\right)(t - t_0)^2, \tag{2}$$

where $\dot{Q}^{(t)}$ is the heat release rate, N is the number of tiers, t_g is the growth time needed after ignition with a stable flame to attain a heat release rate of 1055 kW, and t_0 is the incubation time be-tween ignition and self-sustained burning. The radiative fraction is assumed to be 0.3. In this study, we set t_0 = 18 s for all cases. This equation is used to model the early phases of the fire growth prior to sprinkler activation.

2.3 Flame Height

Flame height is needed to determine the flame surface area and configuration (view) factor necessary to estimate the temperature of the target commodity shown in Fig. 1. The correlation developed by Heskestad [27] is used:

$$x_f = 0.23\dot{Q}^{(t)^{2/5}} - 1.02D_f, \tag{3}$$

where x_f is the height of the flame, and D_f is its diameter. In the current study, D_f is held fixed and equals the equivalent diameter of half of the cross-sectional area formed by four boxes as shown in Fig. 1. Note that the influence of lateral spread and corresponding increase in the effective diameter is not considered in Eq. (3). This assumption is reasonable for the early spread of the fire which is modeled in the current study.

2.4 Temperature and Velocity in the Plume and Ceiling Jet

A representation of the fire plume and ceiling jet is necessary for determining the time of activation of the in-rack and ceiling sprinklers. Several correlations are available in the literature enabling calculation of the temperature and velocity along the centerline of the plume and/or ceiling jet. At any time t, the plume centerline temperature $T_g^{(t)}$ at sprinkler i is determined using Kung et al.'s [16] correlation, given by:

$$T_g^{(t)} = T_\infty + 4.58\left[\frac{\dot{Q}^{(t)^{2/5}}}{z_i - \left(0.095\dot{Q}^{(t)^{2/5}} - z_{00}\right)}\right]^{5/3}, \tag{4}$$

while the plume centerline gas velocity $U^{(t)}$ can be expressed as:

$$U^{(t)} = 4.25(g\, T_{\infty}\rho_{\infty}c_p)^{1/3} \left[\frac{\dot{Q}^{(t)}}{Z_i - \left(0.095\dot{Q}^{(t)^{2/5}} - Z_{00}\right)}\right]^{1/3}. \quad (5)$$

The value Z_i represents the location along the vertical and Z_{00} is set equal to 1.6 m for up to two tier rack storage and equals -2.4 m above [12]. Gravity is denoted by g, the density of corrugated cardboard is $\rho_{\infty} = 115\ kg/m^3$, and c_p is the specific heat of corrugated cardboard and is set to $2000\, J/(kg\ K)$. Both Eqs. (4) and (5) are based on a series of 17 full-scale freeburn tests conducted to investigate the flows induced in growing rack storage fires [16]. Both standard plastic commodity and standard Class II commodity were used. Hence, Eq. (2) for heat release rate can be used in conjunction with Eqs. (4) and (5) to obtain a temperature and velocity field during the early stages of a growing rack storage fire. While the original correlations were developed for a maximum height of four tiers, we assume the equations to be applicable to the maximum height of eight tiers applied in the current study. This assumption may be reasonable considering that the velocity and temperature correlations were developed for fully turbulent fire induced flows.

2.5 Sprinkler Activation

A warehouse sprinkler is subjected to convective heating by the passing flow of hot gases. A heat balance equation is used to obtain the time of sprinkler activation given by [28]:

$$\frac{dT_i}{dt} = \frac{\sqrt{U^{(t)}}}{RTI}\left(T_g^{(t)} - T_i\right), \quad (6)$$

Where, at time t, T_i is the temperature of the in-rack sprinkler at tier i, and RTI is the response time index of the sprinkler, which we set to a constant. To approximate the temperature for the in-rack sprinkler at tier i at time $t + \Delta t$, we use an iterative Euler scheme together with the plume gas temperature $T_g^{(t)}$ and velocity $U^{(t)}$ from (4) and (5):

$$T_i^{(t+\Delta t)} = T_i^{(t)} + \frac{\sqrt{U^{(t)}}}{RTI}\left(T_g^{(t)} - T_i^{(t)}\right)t - \frac{C}{RTI}\left(T_i^{(t)} - T_\infty^{(t)}\right)t, \tag{7}$$

where we take $\Delta t = 1s$, and where the *RTI* is taken to be $148(ms)^{0.5}$. The link conduction parameter, C, is taken to be 0.7. The *RTI* is held constant throughout our study.

For both the in-rack sprinkler at tier i, as well as the ceiling sprinkler, the criterion for sprinkler activation is met at the time t when the sprinkler link temperature exceeds the solder melting point threshold taken to be $\theta_s = 347$ K [18] in our study. At this time, which we denote as t_i^* (t_c^* for the ceiling sprinkler), the sprinkler activates and begins dispensing water at a water application rate $\dot{m}_i$ and $\dot{m}_c$, respectively. We take $\dot{m}_i$ to be $0.0265m^2/s$ for the in-rack sprinklers based on the midpoint of lower and upper estimates from Yu et al. [17]; moreover, we take $\dot{m}_c = 2\dot{m}_i$. We furthermore allow for the possibility of a linear loss of sprinkler efficiency with increasing height using parameters η_i and η_c, respectively, further discussed in Section 4.

The effect of active sprinklers on the fire's heat release rate is modeled as:

$$\dot{Q}^{(t)} = \left(\frac{N}{2}\right)\left(\frac{1{,}055}{t_g^2}\right)(t - t_0)^2 \cdot e^{-\left(\sum_{i=1}^{N}(1-\eta_i)\dot{m}_i k_i s_i \max\{0, t-t_i^*\}\right)\dot{m}_c k_c s_c \max\{0, t-t_c^*\}} \tag{8}$$

Eq. (8) calculates the revised heat release rate $\dot{Q}^{(t)}$ after accounting for potential adjustments from any active in-rack (ceiling) sprinkler(s) for which the time t is greater than the initial activation time t_i^* (t_c^*). That is, only when a sprinkler is both present and activated, i.e., $s_i = 1$ ($s_c = 1$) and $t \geq t_i^*$ ($t \geq t_c^*$), does the corresponding water application rate act in suppressing $\dot{Q}^{(t)}$. Indeed, when all s_i and s_c variables are zero, Eq. (8) reduces to the unrestricted growth expressed in Eq. (2); otherwise, Eq. (8) experiences a net reduction proportionate to the difference in time since activation $t - t_i^*$ ($t - t_c^*$), if positive. The terms k_i and k_c are small positive constants derived from an existing correlation of Yu et al. [17]. While we assume the same RTI and solder melting points (347 K) for both in-rack and ceiling sprinklers, we take the ceiling sprinkler's flow rate to be twice that of the in-rack sprinklers.

It should be noted that Eq. (8) is hypothetical and no experimental evidence exists for its validation. It is based to some extent on work by Tewarson [29] and Magee and Reitz [30] where a minimum rate of water application necessary to extinguish a fire on different types of plastic material was correlated. A similar idea was also used in work by Grant et al. [31] and Schwille and Lupetow [32]. Yu et al. [17] used such correlations to obtain minimum delivered water fluxes required for suppression of Group A plastic commodity ($17 - 20 g/m^2 s$). We model the efficiency of the ceiling sprinkler as half of the efficiency of an in-rack sprinkler on the top-most tier (regardless of whether an in-rack sprinkler is actually present there). We believe that η_i and η_c represent reasonable reductions, as not all of the water from the sprinkler is actively used for fire suppression. Water is lost due to blockage by boxes, run off, evaporation, etc. These factors are combined into the single sprinkler efficiency parameters η_i and η_c.

2.6 Ignition of the Target Array

The temperature at the target array at time t can be derived from the radiative heat flux $\dot{Q}''_{rad}$, using a flame view factor F_v. It is assumed that the flame represents a vertical cylinder of fixed diameter and the target is located at a distance Y representing the horizontal gap. The view factor is expressed as per [33]:

$$F_v = \frac{1}{\pi}\left[\frac{1}{Y}\tan^{-1}\frac{x_f}{\sqrt{Y^2-1}} + \frac{x_f}{Y}\left\{\tan^{-1}\sqrt{\frac{Y-1}{Y+1}} - \frac{A}{\sqrt{A^2-1}}\tan^{-1}\sqrt{\frac{(A+1)(Y-1)}{(A-1)(Y+1)}}\right\}\right], \quad (9)$$

and where A is a function of the flame height x_f and Y:

$$A = \frac{x_f^2+Y^2+1}{2Y}. \quad (10)$$

The radiative heat flux $\dot{Q}''_{rad}$ can be determined using:

$$\dot{Q}''_{rad} = EF_v\tau, \tag{11}$$

which is based on the view factor F_v, the Emissive power E, and transmissivity τ (which is set to unity). The emissive power is determined using:

$$E = \dot{Q}^{(t)}\frac{\chi_{rad}}{A_f}, \tag{12}$$

where we take $\chi_{\mathrm{rad}} \approx 0.3$ and, given the assumption that the flame is of roughly cylindrical form, the temperature of the target array at time t, $T_r^{(t)}$, can be estimated as:

$$T_r^{(t)} = T_\infty + \frac{\dot{Q}''_{rad}}{h}(1 - e^{-\alpha t}), \tag{13}$$

where α represents the thermal diffusivity of corrugated cardboard and equals $4.3 \times 10^{-7} m^2/s$ [21] and h is the convective heat transfer coefficient taken to be 30 W/m^2 based on a Nusselt number correlation of turbulent heat transfer on a vertical flat plate [34].

2.7 Fire Suppression Criterion

For any time $t \leq t_{max}$, the success of a given in-rack sprinkler configuration can be evaluated by its ability to maintain the target array temperature $T_r^{(t)}$ below a specified threshold temperature equal to the ignition temperature of corrugated cardboard ($\theta_r = 723$ K) [35].

3. Solution Methodology

The mathematical relationships discussed in Section 2 govern the warehouse fire behavior under consideration, modeling the possibility of sprinklers at tier $i, i = 1, \ldots, N$ (ceiling) through the incorporation of binary variables s_i (s_c). The inclusion of these variables is important for two reasons. Principally, they allow us to evaluate the effect of such sprinkler's presence (and alternatively, absence) on controlling the fire. Moreover, they allow us to determine the number and placement of in-rack sprinklers in a minimal configuration capable of fire suppression. With this in mind, we next discuss our methodology to evaluate distinct sprinkler configurations.

3.1 Simulating Fire Behavior for a Single Configuration

As discussed in Section 2 and depicted in Fig. 2, we assign values to specific warehouse, fire, target array, and sprinkler attributes. For a given configuration $\bar{s}$ of in-rack and ceiling sprinklers, we then evaluate whether such a configuration is able to prevent it from spreading to the target array. Initially, all sprinklers present in configuration $\bar{s}$ are assumed to be inactive. Then for each value of $t = 1, 2, \ldots, t_{max}$, we determine the temperature and velocity of the plume at each existing sprinkler, in order to find the temperature at the sprinkler link. This link melts if it exceeds a certain temperature threshold, thereby causing the sprinkler to activate. Thus for any time t, we compute the current heat release rate $\dot{Q}^{(t)}$ using (8), which is reduced, where applicable, by the water application rates $\dot{m}_i, \dot{m}_c$ for any active sprinklers.

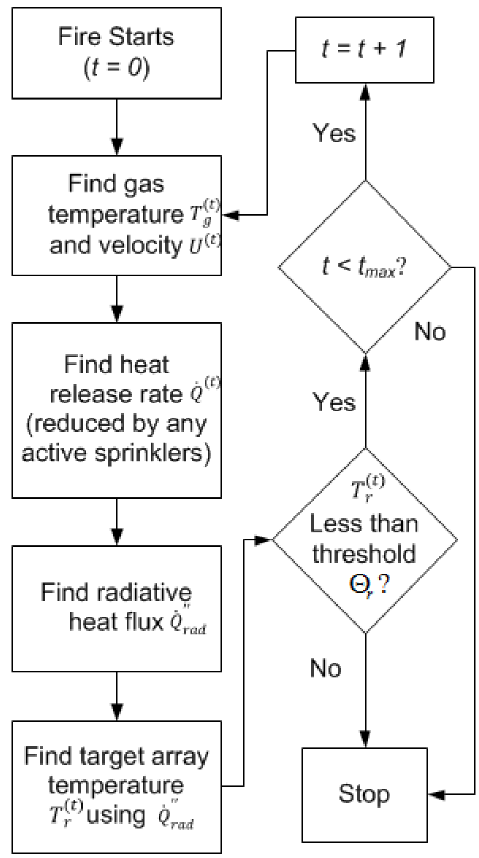

Fig. 1: Simulating warehouse fire behavior for a single configuration.

This modified $\dot{Q}^{(t)}$ affects the flame height and area and ultimately the temperature $T_r^{(t)}$ at the target array. As explained in Section 2.7, the configuration of in-rack sprinklers is deemed to be successful if, for the entire simulation run, $T_r^{(t)}$ remains below a threshold ignition temperature of corrugated cardboard which is set equal to $\theta_r = 723\,\mathrm{K}$. Otherwise, this configuration is considered a failure and the entire warehouse a loss. Fig. 4 depicts the logic-flow of the fire behavior simulation for a given configuration $\bar{s}$. These procedural steps are outlined in Algorithm 1.

3.2 Finite Termination

We call Algorithm 1 to evaluate every unique configuration $\bar{s}$ of in-rack and ceiling sprinklers. Aside from the two trivial configurations ($[0 \ldots 0|0]$ and $[1 \ldots 1|1]$, for an N-tiered array there are 2^{N+1} possible configurations to evaluate using Algorithm 1. Because each configuration is simulated over t_{max} discrete time steps, and due to the combinatorial yet finite total number of unique configurations, this procedure terminates in a finite number of steps. If the target temperature is reached during any configuration tested, the simulation can be stopped, or left to run, as desired, to gather additional output metrics.

Algorithm 1: Sprinkler Placement Algorithm

Input: Fixed configuration $\bar{s}$ of sprinklers; sprinkler activation temperature threshold θ_s.

Output: C (Control) or **F (Failure to Control)**.

1. Initialize warehouse, storage configuration, target array, fire, and sprinkler parameters.
2. Set time $t = 0$ (initial fire ignition) and $\dot{Q}^{(t)} = 0$.
3. While $t < t_{max}$:
    a. For all i such that $s_i = 1$ $(s_c = 1)$, use $T_g^{(t)}$ and $U^{(t)}$ to evaluate the link temperatures of these sprinklers and compare with θ_s for possible activation times t_i^* (t_c^*).
    b. Find revised heat release rate $\dot{Q}^{(t)}$ based on Eq. (8).
    c. Find radiative heat flux $\dot{Q}''_{rad}$ using Eqs. (11), (9), (10), and (12), and use Eq. (13) to estimate temperature at target array $T_r^{(t)}$.
    d. If $T_r^{(t)}$ remains below threshold θ_r, then $t = t + 1$, and go to Step 3.
    e. Else **STOP**, and RETURN **F**.

4. Computational Studies

The effectiveness of our proposed mathematical model and algorithmic approach is verified using a number of parametric cases. Our testing environment is a 2.50GHz Intel Core i7-2920XM processor with 16GB of RAM running Windows 7 Enterprise. The mathematical model and corresponding algorithm were coded in MATLAB [36]. The model is influenced primarily by the following four parameters:

- Number of tiers in array (N)
- Presence (or absence) of a ceiling sprinkler (s_c)
- The efficiency of the water application rate (η_i, η_c)
- Water application rates ($\dot{m}_i, \dot{m}_c$)

We varied the first three of these parameters to generate a set of test cases; specifically, we varied the number of tiers $N \in \{3, 4, 5, 6, 7, 8\}$, the presence of a ceiling sprinkler value $s_c \in \{0,1\}$, and the loss in efficiency of the water application rate in a linear fashion using the constant value $\eta \in \{0.01, 0.02, \dots, 0.1\}$. The efficiency for in-rack sprinkler at tier i was calculated as $\eta_i = 1 - i * \eta$ for $i > 1$, so that the tier nearest the floor has 100% efficiency, with subsequent tiers losing η in efficiency per tier. The loss in efficiency for the ceiling sprinkler, η_c, is modeled as an additional 50% loss in efficiency with respect to that of the highest tier. These efficiency values were chosen to simulate the fire behavior over a range of realistic decreasing water application levels.

5. Results and Discussion

We consider several computational tests to attempt to understand the underlying dynamics of the fire behavior. For many of the subsequent discussions, we chose $N = 8$ to more fully reveal the complexities of the underlying dynamics. Each individual case with a given set of parameters took no more than a few seconds for the simulation to complete.

5.1 Integration of Sub-models and Validation

The fire model used in the study is comprised of several engineering correlation and models based on large-scale experimental data for modeling heat release rate [12, 17, 37], flame height

[27], velocity and temperature profiles in the plume [15] and ceiling [15, 38], sprinkler activation [39], suppression [17, 30] and target array ignition [12]. Each sub-model has been validated independently using experiments that are rarely related with one another. The integration of the sub-models and their capability to model the highly-coupled interactions between the fire, plume and water spray is shown using a validation exercise. The experimental test data from a study by National Institute of Standards and Technology (NIST) [18] are shown in Fig. 5 in comparison with current model results. Several tests were performed to explore the influence of sprinkler spray (at different water application rates) on heat release rate. As observed in Fig. 5, our proposed model and the experimental data of [18] show similar profiles. Both the increase in heat release rate and the decrease after sprinkler activation are in good agreement. To test the sprinkler activation formulation, the first sprinkler activation times were compared with experimental test results reported by Troup [40], where 3–6 tier high commodity racks were tested, and reasonable agreement was observed.

5.2 Tracking Temperature and Heat Release Rate Over Time

Fig. 5(a) depicts the change in temperature at the target array over time for two sprinkler configurations ($N = 8$), as well as the threshold temperature for ignition θ_r. A ceiling sprinkler is present in both configurations ($s_c = 1$), and a linear loss in efficiency $\eta = 0.05$. The top curve depicts the configuration with an in-rack sprinkler in the bottom, second-from-bottom and fourth-from-bottom positions, while the curve at the bottom contains in-rack sprinklers at the bottom, first and second-from-bottom position. The change of this in-rack sprinkler position (third from bottom) is the key factor in the control of the fire for the current ignition configuration.

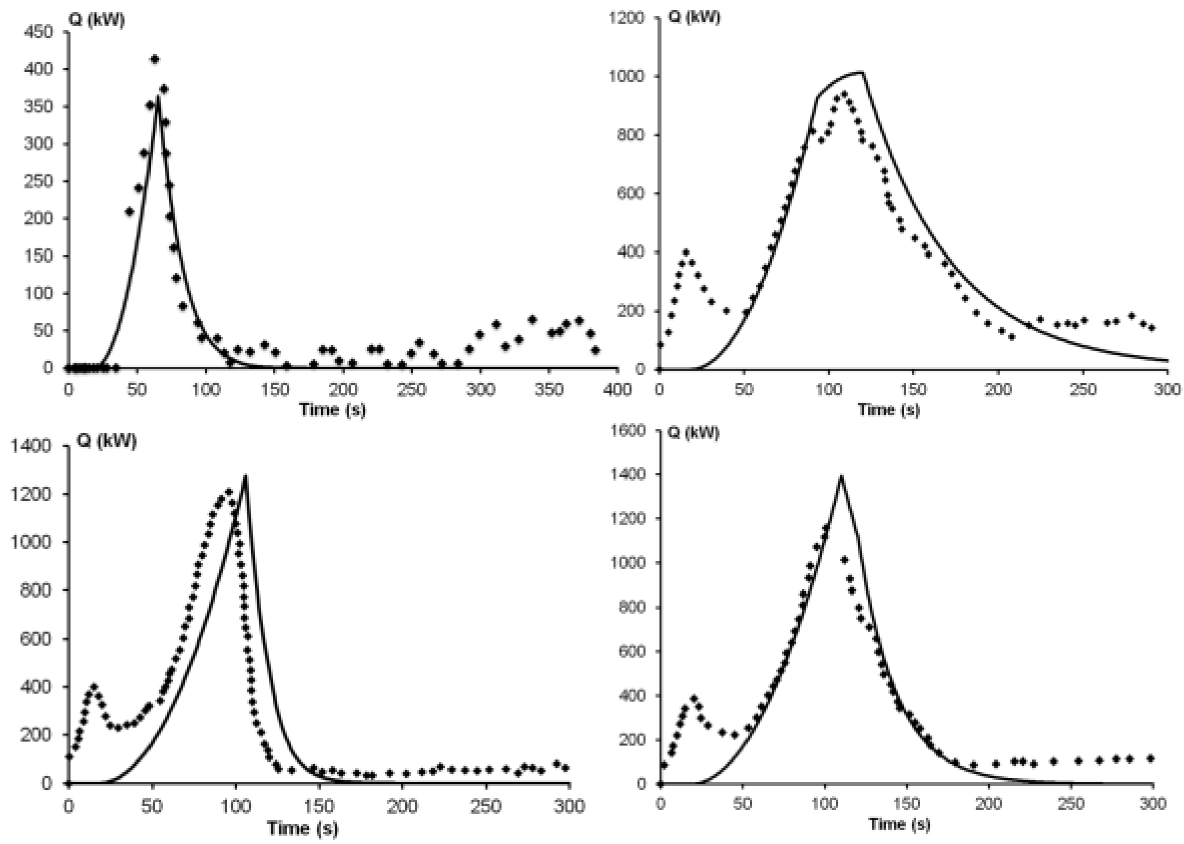

Fig. 5: Validation of model results (solid line) with experimental data from literature [18]

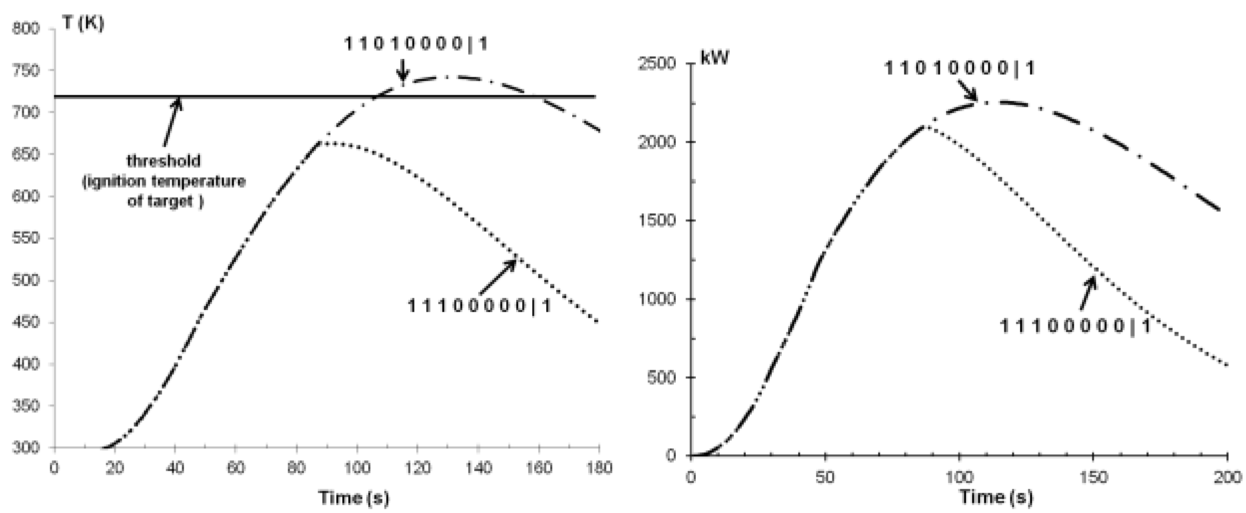

Fig. 5: (a) Changes in temperature over time for two configurations; (b) Changes in heat release rate over time for two configurations.

Fig. 5(b) depicts the change in the heat release rate $\dot{Q}^{(t)}$ over time for the same two sprinkler configurations as illustrated in Fig. 5. A similar increasing and subsequent decreasing trend is observed in the heat release rate over time.

5.3 Importance of Optimization with Respect to Identifying Best Configuration

This study considers first steps towards determining optimal layout configurations for in-rack sprinklers. As such, we consider a 2×2× N dimensional array with a floor fire ignition location. Even given these limitations, the case for optimization with a single 2×2× N array becomes apparent. For example, Fig. 7 below features the case with $N = 8$ tiers, no ceiling sprinkler ($s_c = 0$), and a linear loss in efficiency of 0.1, varying the number of present in-rack sprinklers M from 0 to 8 and plotting the corresponding best (largest) and worst (smallest) percent reduction from maximum $\dot{Q}^{(t)}$ among all such configurations having that value of M. For example, there are $\binom{8}{7} = 8$ configurations with $M = 7$ in-rack sprinklers.

Fig. 7 illustrates the wide range of percent reduction from the maximum level of $\dot{Q}^{(t)}$ for various levels of M. For instance, the placement alone of M=2 in-rack sprinklers can cause either no drop in the maximum level of $\dot{Q}^{(t)}$ (e.g., the fire continues to grow), or alternatively it can cause the maximum level of $\dot{Q}^{(t)}$ to drop by 85%. A similar range can be observed for other values of M. Thus, the configuration of the in-rack sprinklers is a critical component; depending solely on the layout of the sprinklers, the results can vary between little to no reduction in $\dot{Q}^{(t)}$, to a complete reduction. This difference will only become more pronounced with different fire ignition locations, for example floor versus ceiling ignition. This demonstrates the significance of optimization as a design tool to help inform sprinkler system designs. Further research can enhance the modeling of a fire in a more complicated environment such as a large-scale warehouse, as well as more sophisticated optimization methods that can efficiently process the volumes of information necessary to identify minimal configurations.

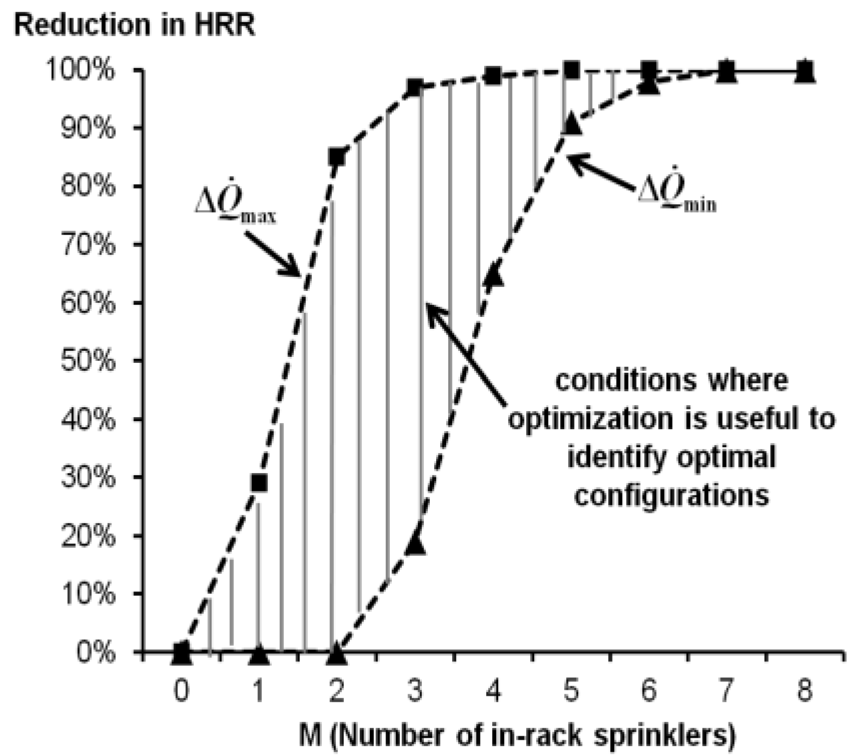

Fig. 7: The impact of sprinkler configuration on fire control.

6. Conclusions

We model an N-tiered, 2-by-2 array together with a target array to demonstrate the potential and role that optimization can have with respect to understanding how various configurations of in-rack and ceiling sprinklers affect the underlying dynamics of warehouse fires. While holding constant the commodity type, fire spread correlation and sprinkler attributes, we simulate a corresponding warehouse fire and aim to determine what the minimal-size configuration is for such a setup, in terms of number and location of sprinklers. Our approach is essentially simulation optimization with expensive function evaluation, where for the size of array configuration we consider, we can directly evaluate the effect of the variable assignments in the simulation without undue computational burden. By doing so, we demonstrate the importance of optimization in the context of in-rack and ceiling sprinkler configurations, illustrating that cases exist where, given the same number M of present in-rack sprinklers, fire control is completely dependent on the placement of the in-rack sprinklers.

Future extensions may include increasing to a $dx\ 2xN,\ d > 2$ array, as well as fires in multiple locations, for example fires starting in tiers higher than the initial ground ignition location. This would require modeling lower-tier deterioration as the fire grows in both size and temperature. Also merited are additional investigations to analyze the effect of sprinkler configurations on the inherent expense tradeoffs of up-front installation and ongoing maintenance, versus complete warehouse loss.

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8. Nomenclature

8.1 Symbols

A	Area (m^2)
c	Subscript used for indicating ceiling sprinkler
c_p	Specific heat of corrugated cardboard ($J/(kg\ K)$)
C	Link conduction parameter
D	Diameter (m)
D_f	Flame Diameter (m)
E	Emissive power (kW/m^2)
F_v	View factor
g	Gravity (m/s^2)
h	Heat transfer coefficient ($W/(m^2K)$)
i	Subscript used for indicating specific rack of an in-rack sprinkler
k	Constant related to water application rate [20]
$\dot{m}_i$	Water application rate for in-rack sprinkler i (or ceiling sprinkler c) (m^2/s)
M	Number of in-rack sprinklers present in a given configuration
N	Number of tiers in array
$\dot{Q}^{(t)}$	Heat release rate at time t (kW)
$\dot{Q}''_{rad}$	Radiative heat flux (kW/m^2)
RTI	Response Time Index ($(ms)^{0.5}$)
s	Decision variable (binary) indicating presence (absence) of sprinkler
T	Temperature (K)
T_∞	Ambient temperature (K)
T_F	Sprinkler frame temperature (K)
$T_g^{(t)}$	Temperature of plume centerline (K)
$T_r^{(t)}$	Temperature of target array (K)
t	Time (s)
t_g	Growth time needed after ignition with a stable flame to attain a heat release rate of 1,055 kW (s)
t_0	Incubation time between fire ignition and self-sustained burning (s)
t_i^*	Activation time for in-rack sprinkler i (or ceiling sprinkler c) (s)
$U^{(t)}$	Velocity of plume centerline (m/s)
x	Height (m)
x_f	Flame Height (m)
y	Width (m)
Y	Horizontal distance from local array to target array (m)
Z_i	Vertical distance above fire origin for in-rack sprinkler i (m)
Z_{00}	Tier-height constant from Kung and Stavrianidis (1982) [15]

8.2 Greek Symbols

α	Thermal diffusivity of corrugated cardboard ($4.3 \times 10^{-7} m^2/s$)
ε	Small positive constant
η	Parameter expressing percentage loss in sprinkler efficiency
η_i	Loss in efficiency for in-rack sprinkler i (or ceiling sprinkler c)
ρ_∞	Density of corrugated cardboard (kg/m^3)
τ	Transmissivity (atmospheric)
Θ_r	Threshold ignition temperature for corrugated cardboard
Θ_s	Threshold activation temperature for sprinkler
χ_{rad}	Constant used in emissive power representing ratio of radiative heat release rate to total heat release rate

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