Problem Statement:

In the problem, Player A will move first and fill up a row of their grid with letters A or B. Proceeding them is Player B, who will fill in their next tile with either an A or B. The final goal of the game differs between the players, with Player B needing to have a unique string of As and Bs that does not match up with any of Player As rows. If any of Player A's rows match up with Player B's rows, Player A wins. To make the game more interesting, both players can see each other's boards at any time. The question now is which player we would rather be and why.

Process & Solution:

After receiving the problem, we started off trying to model it. We, Ruchir, Isaac, and Abhi, started off playing the game. At first, we just tried to play to see who would win. The flow of the game more or less went like so:

Player A would begin filling up their first row with As and Bs. We started with all As.

Player A

1	A	B	A	A	B	A
2						

3						
4						
5						
6						

Player B

1	2	3	4	5	6
B					

After Player A filled their first row with all As, Player B would fill their first box with a letter. Since Player A chose to put an A in their first tile, Player B chose to do a B as that ensures that they are not matching with one of Player A's rows and thus eliminating worries of matching rows and thus losing.

Once Player B makes their move, Player A will proceed with their 2nd move. As they are aiming to match Player B' s row, it only makes sense that they use Player B's current grid, so they ensure they are matching everything thus far. As such, Player A's second move will include a B in the first tile and the rest As. Then Player B will go, trying not to match they will play the opposite letter that Player A put in their tile, and thus play B in their 2nd tile. This pattern will logically go on until the very last one.

Player A

1	A	B	A	A	B	A
2	B	B	B	A	A	B
3	B	A	B	A	A	A
4	B	A	A	A	A	B
5	B	A	A	B	A	A
6						

Player B

1	2	3	4	5	6
B	A	A	B	B	

We will call this method of playing the inverse tile very simply the "inverse method."

The last turn is the deciding one, so let's analyze this position thoroughly through Player A's perspective.

As always, Player A is aiming to match any one of their rows with Player B's. Up until now, none of Player A's rows match with Player B's current grid, meaning it is impossible for Player A to win off them and thus they can be neglected. But this is an impossible position for Player A.

Proceeding with the normal flow, Player A is going to want to match their last row to whatever Player B currently has so they can match their rows and win. This gives Player A the following position:

Player A

6	B	A	A	B	B	

Player B

1	2	3	4	5	6
B	A	A	B	B	

Now Player A needs to place the right tile that matches Player B's last tile to win. But there's an issue: we don't know Player B's last tile, because Player B hasn't played it yet. In fact, Player B won't play their tile, until after Player A has played their final move.

Notice how through each iteration, Player A always gets somewhat close to player B's match, but Player B notices that and writes the opposite of what player A wrote previously.

This situation doesn't change here in the end game. It's the exact same position. When Player A plays their tile, Player B will just play the opposite tile to ensure a win. If Player A plays an A, Player B will play a B to win:

Player A

6	B	A	A	B	B	A

Player B

1	2	3	4	5	6
B	A	A	B	B	B

And conversely, if Player A plays a B, Player B can simply play an A to win:

Player A

6	B	A	A	B	B	B

Player B

1	2	3	4	5	6
B	A	A	B	B	A

As such, as long as Player B keeps playing the opposite of Player A's *n*th tile (n being the current turn), the position will progress into one where Player A's last row is the only relevant one, allowing Player B to play the opposite final tile of Player A's 6th tile and win 100% of the time. We realized that the only way Player A can win is if Player B plays the same tile as Player A. Thus, if Player A plays strategically, Player A can cover both possibilities of tiles that Player B will be able to play in their move. This looks something like this:

Player A

1	A	A	A	A	A	A
2	B	A	A	A	A	A
3	B	B	A	A	A	A
4	B	B	B	A	A	A
5	B	B	B	B	A	A
6	B	B	B	B	A	B

Player B

1	2	3	4	5	6
B	B	B	B	A	

As you can see in this position, Player B chose to follow the inverse method up until turn 5, where they made the fundamental error of picking the same letter in the 5th tile as player A did. In doing this, Player B has completely trapped themselves. In the 5th move, Player A covered Player B's current grid, meaning going into the last turn, Player A's 2nd to last row is relevant. As Player A chose to place an A in the last slot of turn 5, in the last turn Player A will be able to use the same exact pattern, instead replacing the last A for a B. This means that when Player B goes to play, they only have 1 of 2 options, but it doesn't matter what they pick, because both final rows will have already been played by Player A, ensuring that they win.

Player A's win only came about due to Player B not using the inverse method, implying that so long as Player B does not use the inverse method, Player A will be able to win. While this statement is true, Player A's win condition is even more selective.

Observe the following position:

Player A

1	A	A	A	A	A	A
2	A	A	A	A	A	A
3	A	A	A	A	A	A
4	A	A	A	A	A	A
5	A	A	A	A	A	A
6	A	A	A	A	B	

Player B

1	2	3	4	5	6
A	A	A	A	B	

In this position, Player B does not use the inverse method until move 5. Blindly following the previous solution, we would assume Player A could win in this situation, but the truth is that this is an impossible position for Player A. Player A has 2 options for the last tile, allowing Player B to play the opposite move and win.

However, if Player A played the earlier game with a better strategy. By playing the opposite letter of their previous row in the *n*th tile, Player A can force Player B to either match with their previous row or their current row. Thus, allowing Player A to force a win as long as Player B fails to use the inverse method. As you can see below:

Player A

1	A	A	A	A	A	A
2	A	B	A	A	A	A
3	A	A	B	A	A	A
4	A	A	A	B	A	A
5	A	A	A	A	B	A
6	A	A	A	A	A	

Player B

1	2	3	4	5	6

A	A	A	A	A	

As you can see, because Player A utilized a different strategy after Player B didn't use the inverse strategy, Player A had the opportunity to guarantee a win. No matter where Player B decided to match Player A's board, Player A could have changed their next moves to win the game.

This means that the ONLY possible way Player A will EVER win, is if Player B fails to play the inverse move, allowing Player A to cover both possibilities Player B has in the last turn.

In conclusion, we found out that Player B is always able to win against Player A since they can use the strategy of writing the opposite letters in each box, allowing them to always win. For example, in the first row, when A goes first and finishes writing all the letters in the first row, Player B can write the opposite letter in their first tile than that of Player A's first tile. This process can occur for all six rows and tiles, which is why Player B always wins no matter the circumstances given Player A goes first.

With this clear advantage, we would choose to play as Player B every single time.

Extensions

Some extensions that could be derived from this problem is the question of what would happen if Player B went first and Player B is trying to match Player A. In short, the roles are swapped. However, after some consideration, we realized that this would result in Player B having the obvious disadvantage, as Player A could simply chose to not match Player B's first letter.

Another extension that could be included is if no players are able to see the board and there is a scribe who writes the players' choices. However, players can hear each other; therefore, if both players cannot hear each other, it would be a game of chance and therefore very hard to win.

Another extension could be included if there were more letters to choose from. For example, both players could choose from A, B, or C. However, player B can still not choose what player A chooses and wins every time.

Finally, another extension could be where player B has to match the diagonal of player A's box. However, it would be the same scenario, meaning that for each cell in B's box, player B can choose a letter different from the next diagonal of A's box. Therefore, the loophole still exists.