Standard POW Write-up

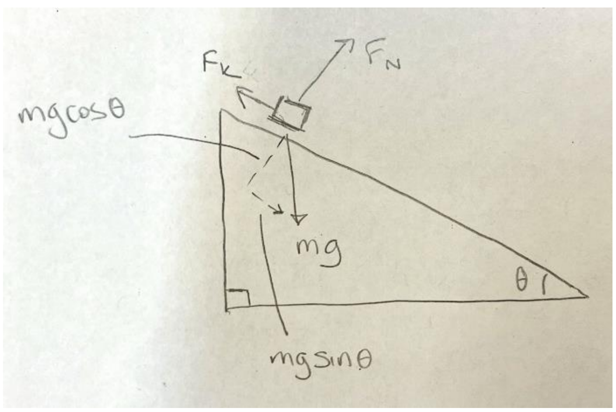
Problem Statement

A puck starts at rest and slides down an inclined ramp of 38° with respect to the horizontal. The puck is 72g. The ramp is 2.9 m long, and the coefficient of friction between the puck and the ramp is 0.17. The end of the ramp is 1.6 meters above the ground. Find:

a.) The x distance the puck travels (X_{BC}) after leaving the ramp
b.) The optimized angle for launch to maximize X_{BC}

Process

To start, we realized the initial diagram could be broken into two parts: a block on an inclined plane (from the dynamics unit) and a two-dimensional projectile motion (from the kinematics unit.) We would need the final velocity of the dynamics portion to put into kinematics equations for the kinematics portion, so we tackled the block on an inclined plane first. Additionally, we converted the mass of the puck from grams to kg to keep all our units as SI units.



1. This is our FBD with all forces acting on the object labelled. As it is a block on an inclined plane, mg needs to be broken up into its components: $mg\ sin\theta$ and $mg\ cos\theta$.
2. Use dynamics equations $\Sigma F_y = 0$ and $F_{net} = ma$

   A) $\Sigma F_y = 0$ can be rewritten to be $F_N = mgcos\theta$ for this problem. Putting in values for these, we get $F_N = (0.072)(9.8)\cos(38)$. Simplified, $F_N = 0.556$N

   B) $F_{net} = ma$ can be rewritten to $mgsin\theta - \mu F_N = ma$ for this problem. Putting in values for these returns $(0.072)(9.8)\sin(38) - (0.17)0.556 = 0.072a$. $a = 4.72 m/s^2$

3. Use no-T kinematics equation to find the final velocity of the block on an inclined plane. This will be the initial velocity for the 2D projectile motion portion of the problem.

A) As we know the block starts at rest, v_0=0. We also know the total length of the ramp is 2.9, therefore $\Delta x = 2.9$ m. The equation becomes $v^2 = 0^2 + 2(4.72)(2.9)$. Simplifying this equation, we get $v = 5.233$ m/s.

4. Now, the goal is to find Δx for the 2D projectile motion problem. We know $v_0 = 5.233$ m/s, $\Delta y = -1.6$ m, and $a = -9.8\ m/s^2$. We can put these values into the no-V equation to find time, which will allow us to solve for Δx .

A) It is important to note that the block enters the second portion of the problem falling at an angle, meaning its velocity is actually $v = -5.233\ sin\theta$, or $v = -3.222$ m/s.

B) Using the equation $\Delta y = v_0 t + \frac{1}{2}at^2$ and plugging in the known values returns $-1.6 = -3.22t - 4.9t^2$. $t = 0.33$ seconds

5. As we have time, we can use the no-A kinematic equation to find Δx as we have all the required values. As with 4A), it is important to note that the block is at an angle here as well, therefore, initial velocity is $5.233 \cos 38$, $v_0 = 4.12366$ m/s

A) $\Delta x = v_0 t$ can be rewritten to $\Delta x = 5.233 \cos 38(0.33)$ → $\Delta x = 1.36$ m

Solution

a.) We concluded that the puck lands 1.36 meters way from the base of the counter X_{BC}. We know that this solution is right because we cross-checked with other groups, who all got the same solution. Furthermore, we went through the problem step by step to check if we had any potential errors in our equations that may lead us to an incorrect answer.

b.) Using our excel spreadsheet, we found that the optimal angle for launch to maximize the distance traveled, which is 1.5176 m, was 27 degrees. We also cross-checked this solution with other groups. The change in X in the spreadsheet for 38 degrees was the same as our solution for part a, meaning our equation was correct. This ensured the validity of our spreadsheet. We also tested other angles by manually finding the change in X, and we found that they were all correct.

- Substituting our excel equation into Desmos Graphing Calculator, we obtain a graph with a vertex that is the optimal angle for traveling the greatest X_{BC} distance. The optimal angle that desmos gives is 26.65° and the greatest distance is 1.51784 m as this is the only vertex that is in the range of 0° and the equation is as follows:

$$\frac{\left(\sin(x)\cdot\sqrt{2\cdot\frac{\left((0.072\cdot 9.8\cdot\sin(x))-(0.17\cdot 0.072\cdot 9.8\cdot\cos(x))\right)}{0.072}\cdot 2.9}-\sqrt{\left(\sin(x)\cdot\sqrt{2\cdot\frac{\left((0.072\cdot 9.8\cdot\sin(x))-(0.17\cdot 0.072\cdot 9.8\cdot\cos(x))\right)}{0.072}\cdot 2.9}\right)^2+4\cdot 4.9\cdot 1.6}\right)}{2\cdot -4.9}\cdot\cos(x)\cdot\sqrt{\left(2\cdot\frac{\left((0.072\cdot 9.8\cdot\sin(x))-(0.17\cdot 0.072\cdot 9.8\cdot\cos(x))\right)}{0.072}\cdot 2.9\right)}$$

Extensions

1. If the height of the counter was not 1.6 meters but 3.2 meters, how would this affect the optimal angle of the ramp?
    a. Answer: The optimal angle of the ramp would increase. This is shown through the test case in our spreadsheet. The angle increased from 27 degrees when the height was 1.6 meters to 29 degrees when the height was 3.2 meters.
2. If gravity were to be different than -9.8, how would this affect the optimal angle of the ramp?