

## Summary

Over the last few decades, the honeybee population has experienced a drastic and rapid decline in population. This alarming phenomenon has scientists asking “What went wrong, and *how can we fix it?*” Clearly, this is no easy question to answer.

Bees play a vital role in the environment. Researchers attribute them with the success of nearly 9 out of every 10 flowering plants, one third of our crops, and around 5% of all plants in the wild [8]. In addition, they are incredibly complex and advanced organisms, with elaborate social structures, intricate communication based on chemical signals, and other behaviors unique among the animal kingdom [11]. Pesticides, habitat destruction, predators, viruses, and degrading environmental conditions are believed to be the main causes of the bees’ peril, but to save this amazing species, understanding their population better is crucial.

Many factors play into the complexity of this unique organism, including life span, egg laying rates, hive social structure, availability of flowering plants as sources of food, predators, and seasonal variations in these elements. We use the most commonly widespread bee species, *Apis mellifera*, to examine the changes in a honeybee colony over time and create a model that can represent these changes based on data from a study that examined the seasonal changes in the population of a honeybee hive taken from hives in varying regions [20]. Our model is robust and takes into account many factors that can affect a bee’s population in the birth and death rates. We used biological equations and used data from a specific open-access study to test our model and proved its ability to realistically model a hive’s population. Our model takes into account the egg laying rate as birth rate, lifespan and death rate from varying causes and initial population to determine a hive’s final population after time. The birth and death rates we used also take into account that when an *Apis mellifera* colony reaches carrying capacity, as the queen and half of the worker bees leave to form a new colony, cutting the population size in half.

In addition, we determine the factors that create the greatest variation and have the most impact on the honeybee population by manipulating different factors of our model and examining the percent variation the output experiences. Furthermore, we look at how many hives are needed to support a specific, 20 acre parcel of land with crops, and create a model that can be used with different sized parcels of land to decide the number of hives that should be present on the property. This model was created using known information about bee efficiency and pollination capabilities.

Finally, we create an infographic that can be easily interpreted by the public to learn about honeybees. Our infographic sums together important information we determined throughout this project, including facts about *Apis mellifera*’s behavior and life cycle, seasonal variations, and the species’ importance to the ecosystem.

Our models can be used in a variety of situations to accurately determine the population of a hive, the factors that have the greatest influence on a colony’s population, while our infographic offers a summary of information that can help educate the public on *Apis mellifera*. By further examining *Apis mellifera*, our hope is that scientists will eventually *bee* able to answer how we can save this extraordinary species.

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# Introduction

## Requirements

1. Develop a model to determine the population of a honeybee colony over time.
2. Conduct sensitivity analysis on your model to determine which factors (e.g., lifespans, egg laying rates, fertilized/unfertilized egg ratios, or other factors) have the greatest impact on honeybee colony size.
3. Model and predict how many honeybee hives you will need to support pollination of a 20-acre (81,000 square meters) parcel of land containing crops that benefit from pollination.
4. Create a non-technical, one-page blog or infographic for a website that provides the information you developed.

## Assumptions

Assumptions are listed under the problem that they fall under, and are based on given data from the problem and additional research. Any requirements not listed do not have relevant assumptions.

### Problem 1:

- Assume that the queen bee lays the same amount of eggs each season, which we set to be 1,000 eggs per day based on sources.
- Assume that not all 1,000 eggs hatch depending on food supply and season, so the amount of eggs hatched can change per season.
- There are at least 10,000 bees in a hive in early spring

### Problem 2:

- Assume that the model already takes into account survival rates of bees in colder weather lifespans in the summer vs winter, and food availability, so the only other factor that must be taken into consideration is human impact.
- Assume that human impact includes pesticides, habitat destruction, insufficient crops, and disease.

### Problem 3:

- Assume that the parcel of land is a perfect square.
- Assume that the parcel of land has an ideal climate for the growth of the bee colony.
- Assume that the parcel of land has abundant crops, resources, and little negative human impact, so the bee population will be at its peak.
- Assume that the bee population will not travel outside of the parcel of land.
- Assume that the bees are able to pollinate more than one flower, and can pollinate a flower more than once a day
- Assume that the field is made up of flowers, taking an average amount of room per flower.
- Assume that there are at least 10,000 bees in a hive in early spring

### *Apis mellifera*

Our model is based on information specifically from the *Apis mellifera* species. This species is the most common of all honeybees, and is found in all regions globally [11][26]. They can survive in almost every climate, with the exception of polar areas [11]. Basing our model on *Apis mellifera* gave us more concrete and uniform research, allowing us to have a consistent model that uses data based on one specific species.

*Apis mellifera* generally live in hives of 20,000 and 80,000 bees. The hive consists of one queen bee, 95% worker bees, and around 5% drones [23]. Workers are responsible for foraging for food, tending to the bee eggs and queen, making honey and building the honeycomb, and protecting the hive [23]. At any time, roughly half of the workers are foraging [23].

The lifespan of *Apis mellifera* differs for the three types of bees. Queens can live anywhere between 2 and 5 years, while workers only live 16 days in peak pollinating season, and drones live around 24 days [11]. Due to their short lifespan, up to 1,000 bees can die in a single day, while the queen can lay up to 2,000 eggs per day [23]. If the hive reaches carrying capacity, or if the queen is about to die, a new queen bee egg is born, and a swarm occurs [6]. Sometimes referred to as the “split”, the old queen bee takes half of the workers with her and leaves the hive to form a new colony [10][6]. Honeybees play a vital role in the environment, and show incredibly complex social structures and behaviors. By examining the species’ population, we learn more about their declining population and are one step closer to saving them.

### Given Data

- Honeybees can travel up to 20 km, but typically stay within 6 km of their hive.
- A typical honeybee hive contains between 20,000 and 80,000 honeybees.
- A single honeybee can visit approximately 2,000 flowers or more in a single day.
- Because of the high workload during summertime, most honeybees work themselves to death, resulting in a shorter lifespan.
- During autumn and wintertime, honeybees may live a bit longer (four to six months).
- A honeybee’s level of activity, pollen consumption, and protein abundance impacts its lifespan.

# Approach and Model Description

## Modeling Population Size Over Time

$$P_i + 1000t(B - D) = P_f$$

$P_i$  = initial population size

$t$  = the change in time in days

$B$  = birth rate (decimal per 1000 bees)

$D$  = death rate (decimal per 1000 bees)

$P_f$  = the final population size

This equation is inspired by the population growth equation from AP Biology [2][3] shown below. The change in population size over time is equivalent to the difference of birth rate and death rate of the population. To find the population size at a specific time period, the initial population must be added to the change of the population over time. Because the change of population over time is equal to the difference of birth rate and death rate ( $B - D$ ) the modified equation can be written as  $P_i + t(B - D) = P_f$  where  $P_f$  is the final population. Instead of having  $t$  alone we multiplied it by 1000, since the data we used was accounting for the birth and death rates per 1000 bees, thus the final equation for modeling population size is

$$P_i + 1000t(B - D) = P_f$$

$$dN/dt = B - D$$

$dN$  = change in population size

$dt$  = change in time

$B$  = birth rate

$D$  = death rate

We modified the population growth equation from AP biology in order to better fit the data that we have collected regarding the population of honeybee.

The data we used for the model was acquired from a research article called *Dynamic modelling of honey bee (Apis mellifera) colony growth and failure* [20]. This research paper had many variables and created a model that accounted for larval death rate and food intake. However, we modified it to make the equations more approachable and more adaptable for taking into account death rates during certain seasons. Our modified equation also shows birth rates during the growing season which were the graphs below for  $L_s$  (**we called this B**).

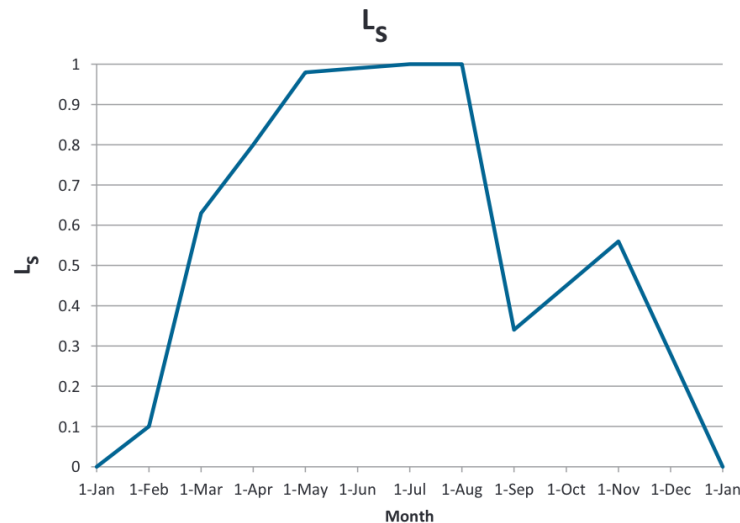
This graph displays the number of larvae that **survive per 1000 eggs laid**. 1000 eggs per day was the average amount of eggs laid by the queen each day, and this number was used by the research article to calculate birth rates of eggs hatched.  $M_s$  (**we called this D**) is a death rate

seasonality term, which gives the proportion of the maximum death rate that is applicable throughout the year [20].

The birth and death rates from this article took into account a multitude of factors. The model “focuses on how internal demographic processes within a colony interact with food availability and brood rearing to alter growth trajectories” [20]. In addition, the model took into account “the effects of the variation of many of the key parameters known to influence colony growth and mortality over a seasonal cycle, such as food availability, queen laying rate and [bee] mortality” [20]. This was beneficial for our model, since the death and birth rates we were using were already calculated using a multitude of different equations. The birth and death rates also took into account something called “the split” where when a colony reaches its carrying capacity (~80,000 to 100,000 bees [7]).

The researcher’s model took into account food storage as a parameter, but we did not add this to our equation as the birth rates and death rates calculated by the researchers and used by us had already included this parameter in the calculations. The researchers used computer modeling, which was not available to us, thus we trusted that their rates were accurate.

**Below are figures 2 and 3, showing the monthly birth rates and death rates, respectively:**



**Fig. 2.** Input to the model of the seasonal variation of the laying rate,  $L_s$ , over 12 months, beginning from 1 January. The pattern of laying repeats in a yearly cycle over a total of three years.

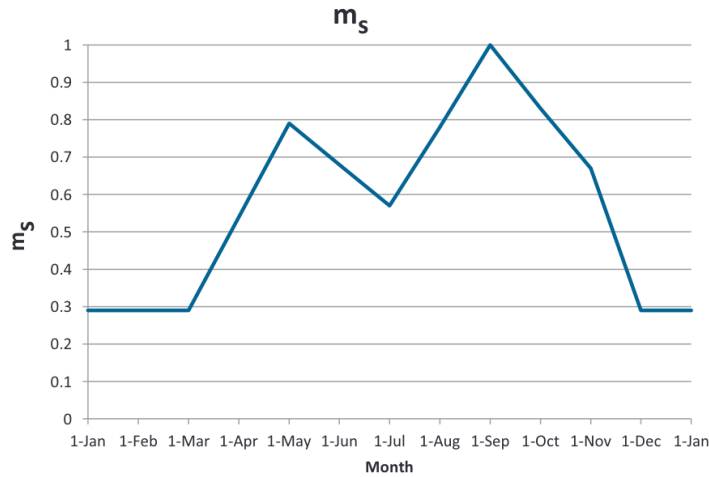


Fig. 3. Input to the model of the seasonal variation,  $m_s$ , of the forager death rate over 12 months, beginning from 1 January. The pattern repeats in a yearly cycle over a total of three years.

Now, the article we used for reference uses  $M_s$  as a seasonal death rate, so this is equal to  $D$  (death rate in our equation). The same is said for  $L_s$ , so we called this  $B$  in our equation.

Then:  $P_i + 1000t(B - D) = P_f$  can be written as  $P_i + 1000t(L_s - M_s) = P_f$

Exact values of the rates are difficult to discern from the graphs, so below is a table that summarizes the information in the charts.

The data from the article take into account seasonal changes that occur in the Northern Hemisphere that correlates to bee population. For example, it is known that in the fall and winter bees have a higher likelihood of surviving since they are not working to death and this is accounted for in our model since the death rate of bees in the winter months of December, January, and February are at the lowest of approximately 0.3.

**Northern Hemisphere Rates:**

Month	$M_s$ which is $D$ (death rate per 1000 bees)	$L_s$ which is $B$ (birth rate per 1000)	Number of days
January	0.29	0	31
February	0.29	0.1	28 or 29 (depends on leap year)

March	0.29	0.62	31
April	0.56	0.8	30
May	0.78	0.99	31
June	0.67	1	30
July	0.59	1	31
August	0.79	1	31
September	1	0.35	30
October	0.81	0.45	31
November	0.65	0.55	30
December	0.29	0.3	31

Our data models that during the months of September, October, and November the death rate is higher than the birth rate and this results in an overall decrease of bees. This is accurate, as it accounts for external reasons bees can die, for example weather and many colonies' loss of their "summer bees" which are not designed for colder temperatures [20]. Queens also decrease their laying rates, and male bees (drones) are killed in the fall because the queen does not need them anymore, as the egg laying rate is decreased overnight [15]. Additionally, many bees die of starvation in November since pollen is not as prevalent especially in more temperate climates like New England [7]. This is accounted for in our model effectively, since bee colonies are modeled to reach their peak around 80,000 and then decrease to 10,000 to 20,000 in winter, and most of the decline is made in the fall months [4].

We used external information to find that approximately 30,000- 40,000 bees are present in a hive by late spring [20.], which we said was the end of May. Then, during the summer it is peak season for the bees and their population can grow to almost **80,000 by the end of July**, but the population can grow anywhere from 50,000 to 100,000 [4] bees depending on the size of the hive. According to other sources, most hives at the end of winter are approximately 10,000 to 15,000 [4] bees, so we set this as the initial population of our model, and all calculations that we make are based off of that. If people want to use this model, then they can input their own initial populations. **To use this model correctly, each month is calculated separately, so that we obtain the individual month's change in population.**

In addition, the split (see Introduction) was taken into account by the data used from the article. If the birth and death rates do not take the split into account, this must be added in by the following. If the final population has a surface area greater than the area available in the hive, the



queen bee leaves the hive with half of the worker bees. This cuts the population in half. This can be calculated by multiplying the final population of bees by the bees' size, which is around 15 mm long (although this varies for the queen, workers, and drones) and width to find the total surface area they take up. If this surface area is greater than the surface area available in the hive, a swarm will occur, and the population will split in half. We wanted to include it here since as stated earlier, our model and the rates that we used from the article *Dynamic modelling of honey bee (Apis mellifera) colony growth and failure* already accounted for this swarming that occurs 1-2 times a year since the birth and death rates occur in a cycle of 3 years [20].

Now, here's an example of how our model works. If we started with 10,000 bees in a hive at the end of February, and wanted to calculate how many bees we would have in late spring (end of March) we would have to use the model to calculate March's growth, April's growth, and May's.

$$P_i + 1000t(B - D) = P_f$$

We can set  $P_i$  as **10,000** and then we can calculate each month's growth. When calculating the month's growth we do not add 10,000 to each one as that would be overestimating, we only add 10,000 to the first month we calculate and that counts for all of the initial bees in the hive.

**For March, the birth rate (B) is 0.62 and the death rate (D) is 0.29.** Thus, our equation became  $10000 + 1000 * 31(0.62 - 0.29) = P_f$  and this resulted in an addition of **10,230 bees** to the initial population of 10,000 so 20,230 bees after the month of March.

**For April, B is 0.8 and D is 0.56 and there are 30 days.** Our equation becomes  $1000 * 30(0.8 - 0.56) = P_f$  and this evaluates to **7,200** new bees during this month.

**For May, B is 0.99 and then the D is 0.78 and there are 31 days in May.** Our equation becomes  $1000 * 31(0.99 - 0.78) = P_f$  and this evaluates to **6,510** new bees during this month.

**Thus, the total population in late spring is 33,940 bees, relatively close to the average of 30,000 to 40,000 bees in a hive by the end of spring [20.], so we believe our model is accurate.**

**This model can be adaptable for any time of the year and any month to find the total amount of bees after a time frame.**

### Sensitivity Analysis of Factors that Affect Honeybee Population:

The second question asked us to conduct sensitivity analysis of our model. To do this, we will take into account some factors that may affect the number of bees in a hive: Our model already takes into account the survival rates of bees in colder weather and lifespans (being shorter in the summer and longer in the winter). Additionally, the birth and death rate proportions that we used had been derived from a study that took into account food availability and amount [20]. So we will specifically **focus on how diseases may affect a bee population**. Diseases can be caused by pesticides, and human interaction [9]. To do this, we will calculate the new birth and death rates that represent how the population may change due to those factors, and how different those are compared to our model's prediction. We will specifically say that all colonies of bees have a population of 10,000 during late winter/ beginning of spring, and calculate the total population at the end of spring. Our model and equation before any changes predicts a bee hive size of **33,940 bees, which falls to an average of 30,000 to 40,000 bees in a hive by the end of spring [20]**.

**Firstly, let's say a disease wipes out a whole of a bee colony by the end of spring.** So, if we halve the birth and death rates of bees in a colony, we should have no bees predicted to be left.

**Now, there are two ways that we can do this. We can either say that the disease affects the amount of eggs that hatch, or that the disease only affects adult bees. Thus, the birth rate halves or the death rate doubles.**

#### a) **Death rate doubles:**

We will go through the same method of calculating the late spring population by calculating the increase in population for the months of March, April, and May.

We know that for March **the original death rate was 0.29 and the birth rate was 0.62. However, we need to double the death rate**, so due to the disease that affects the adult and worker bees would be **0.58**. Our equation would then be  $5000 + 1000 * 31(0.62 - 0.58) = P_f$ , and this would result in a new population of **6,240** (5,000 + 1,240 new) bees, as the original 10,000 bees would be affected by the new disease since it affects non-larvae bees or eggs.

For April, the same process would be repeated, but we would not add 5,000 since those bees were accounted for in March. For April, **the original death rate was 0.56 and the birth rate was 0.8. But the new doubled death rate is 1.12 and there are 30 days.** Our equation then becomes  $1000 * 30(0.8 - 1.12) = P_f$  and this evaluates to **a decrease of 9,600** bees during this month.

**This would make our bee population negative (-3,360), but if we were to say that the original 10,000 bees were not affected by the new disease, then our bee population would be 11,240 minus 9,600 resulting in 1,660 remaining bees.**

**Now, let's calculate May. The original birth rate for May is 0.99 and then the death rate was 0.78 and there are 31 days in May. Our new death rate would be 1.56.** Our equation becomes  $1000 * 31(0.99 - 1.56) = P_f$  and this evaluates to **-17,670** bees being lost during this month.

Now, the total change of the adult bees population would be -17,670 minus 9,600 plus the 1,240 bee increase during March. This is a decrease of 26,030 bees from the initial 10,000. This is a strength of our model since we didn't account for human impact, or disease and we ended up having no bees left in the colony, which was the goal. We will expand on this in our conclusion and overall strengths and weaknesses model discussion.

#### **b) Birth rate halves:**

The second way diseases could affect a bee population, is by decreasing the birth rate or the number of laid eggs that hatch. To test this we are going to have the birth rates for each month we need to calculate.

We know that for March **the original death rate was 0.29 and the birth rate was 0.62. We then half the birth rate**, so the new rate of the number of eggs that survive would be **0.31**. Our equation would then be  $10000 + 1000 * 31(0.31 - 0.29) = P_f$ , and this would result in a new population of **10,620** (10,000 + 620 new) bees, as the original 10,000 bees would not be affected by the new disease since it affects only eggs and unborn bees.

For April, the same process would be repeated, but we would not add 10,000 since those bees were accounted for in March. For April, **the original death rate was 0.56 and the birth rate was 0.8. But the new halved birth rate is 0.4 and there are 30 days.** Our equation then becomes  $1000 * 30(0.4 - 0.56) = P_f$  and this evaluates to **a decrease of 4,800** bees during this month. **So the total bees present in the colony would be 5,820.**

**Now, let's calculate May. The original birth rate for May is 0.99 and then the death rate was 0.78 and there are 31 days in May. Our new birth rate would be 0.495.** Our equation becomes  $1000 * 31(0.495 - 0.78) = P_f$  and this evaluates to **-8,835** bees being lost during this month.

At the end of April, we had 5,820 bees left in the colony and then a decrease of 8,835 bees. This results in less than 0 bees but in reality there would be no bees left in the colony

which was what we were aiming for when we said that a disease wiped out the whole colony by the end of spring.

### Strengths and Weakness of our Model:

Overall some strengths of our model is the birth and death rates we used from a research article [20.] already were calculated using a multitude of factors, like fertilization rates, laying rates, and food supply. This helped us get the most accurate data as possible to test the model and the most accurate birth and death rates according to season. Our model also takes into account seasons, which is important since bee population sizes can fluctuate based on the temperature and season. Lastly, our bee population model accurately predicts how diseases can affect a bee population, and in two different ways depending on how a disease affects either the adult bees or the eggs and unhatched larvae.

A weakness of our model is that if a disease did not wipe out the whole population of bees, our model may predict that differently. We did not have enough time to test this theory out, but we believe that a disease kills off  $\frac{1}{4}$  of the adult bee population in a hive that will not be modeled correctly since we simply cannot have an inverse relationship with death rate and disease. This is because if we have  $\frac{1}{4}$  of the bees left after a certain amount of time, we can not simply just multiply the death rate by 4 as that would leave a negative amount of bees while a problem may state that  $\frac{1}{4}$  of the population is left. Thus, this aspect of our model needs further exploration, even if it may account for food intake and other natural factors.

### Honeybee Hives Needed to Support a 20-acre Parcel of Land :

For the modeling and prediction of how many honeybees are needed to support pollination of a 20-acre parcel of land containing crops that benefit from pollination, we take into consideration the distance that the bees can travel. We also made multiple assumptions, such as the parcel of land will contain abundant resources and ideal conditions such as climate for the bee population to reach its peak and perform at an optimal level. The bees are assumed to be able to pollinate a flower more than once a day. The parcel of land is assumed to be a perfect rectangle as well.

In the given information, we are told that 20 acres of land is approximately 81,000 square meters. Assuming this field is square, we can take the square root of 81,000 to figure out the length of one side of the field. This is 284.605 meters, which is equal to 0.284605 km.

To correctly determine how many bee hives should be in this field, we should first consider how far bees can travel. With the information that was given within the problem, it is known that bees can travel at most **20 km, but normally they only fly 6 km per day.**

Having a side length of 0.287 km, this is much less than 6 km so we know that the bees are able to effectively travel in this distance. Additionally, from the given information we know that each bee can visit up to 2,000 flowers. We also assumed the 20 acres are all flowering plants. On average, there are about 50,000 individual flowers in  $\frac{1}{4}$  of an acre. So, on average in 1 acre

there are 200,000 flowers [9]. Thus, in 20 acres there are about 4,000,000 flowers. Now, assuming each bee visits 2,000 individual flowers, and each bee is working to their maximum, 2,000 bees are needed to pollinate the 4 million flowers every single day. We also know that there are at least 10,000 bees in a hive during the beginning of spring when flowers start to produce pollen [4]. So, one hive is definitely more than enough to pollinate 20 acres of flowers.

**Model to find the number of beehives needed per acre of land:**

$(4,000,000 * a) / 2,000 =$  number of bees ( $q$ ) needed to pollinate  $a$  acres of fields

Number of hives needed =  $q / 10,000 *$

\*assuming there are at least 10,000 bees in a hive at all times

**Infographic**

## All About Apis

### Life Cycle of Apis Mellifera

Egg → Larva → Pupa → Adult

### Threats to Honeybees

- Pesticides
- Viruses
- Predators
- Habitat Destruction
- Environmental Conditions

### Effect of Honeybees In the Ecosystem

- They are effective pollinators for plants, and the reason why there are so much diversity in plantation.
  - Plants prevent soil erosion, food for wildlife, produce oxygen, keep waterways clean, counteract climate change, etc.
- The elimination of honeybees can lead to less prey, thus resulting in competition amongst predators
- A decline in honeybee population has lead to poor harvests and a lack of honey production, which is something that is widely used.

### How these threats affect the Honeybee Population

Seasonal variation in the laying rate,  $L_s$ .

Seasonal variation in the forager death rate,  $M_s$ .

Image Cited: Honeybee (蜂). In Clipart Library. [https://ih.bina.com/ih/IdR\\_bf2c8d4d6ba8082462103a375ac8077nk-vHDoX%28IyYJ5wwKru=htn%3a%2f%2fclipart-library.com%2fimage\\_gallery%2f138057.png&hkh=iH%2fQRWJEM\\_sFWYk7o7s0aXcaVaWkhhv0xHE3w%2fD%2bes%3d&rsj=&ncr=imnR&ncr=imnR](https://ih.bina.com/ih/IdR_bf2c8d4d6ba8082462103a375ac8077nk-vHDoX%28IyYJ5wwKru=htn%3a%2f%2fclipart-library.com%2fimage_gallery%2f138057.png&hkh=iH%2fQRWJEM_sFWYk7o7s0aXcaVaWkhhv0xHE3w%2fD%2bes%3d&rsj=&ncr=imnR&ncr=imnR)  
 Russell, S., Barrow, A. B., & Harris, D. (2013). Dynamic modelling of honey bee (*Apis mellifera*) colony growth and failure. *Ecological Modelling*, 265, 158–169. <https://doi.org/10.1016/j.ecolmodel.2013.06.005>  
<https://www.planetbee.org/why-we-need-bees>

## Conclusion

Overall, our models take into consideration the majority of factors that influence the honeybee's role in our environment including temperature, climate, predators, and a variety of pollination catalysts. We based our project off of the *Apis mellifera*, the most common species of honeybee, which can most accurately fit our testing of different climates and environments. Additionally, the birth and death rates that we used took into account many factors mentioned above, and the equations we used accounted for the split in population that occurs once or twice a year. Our model is very unique in this way, and can provide a more accurate process in saving the bees that have helped us progress so far throughout civilization, through pollinating various plant species and honey production.

However, there are still weaknesses to our model, such as in addressing problem #3, where the ideal conditions assumed for the parcel of land does not apply to every situation globally. In addition, the data that we got from a research paper does not take into account the southern hemisphere, but some temperatures in the northern hemisphere are similar to those in the southern hemisphere.

For further extension of our model, a java program can be created where the user inputs things such as the time period, original bee hive colony, and region, and the output given by the computer will be the population of bees. In addition, the surface area or overall volume of the hive can be taken into account, and more specific sensitivity analysis for different factors, such as survival rates of bees in the winter, carrying capacity (when the queen bee takes half of the population and leaves), and natural disasters that affect the bee population. Other bee species can be taken into consideration other than *Apis Mellifera*, and different life spans of the honeybee population can be considered into our model.

## References

- [1] *About Honey Bees | Types, races, and anatomy of honey bees.* (n.d.). About Honey Bees | Types, Races, and Anatomy of Honey Bees.  
<https://www.uaex.uada.edu/farm-ranch/special-programs/beekeeping/about-honey-bees.aspx>
- [2] *AP Biology Rate and Growth Notes.* (n.d.). Retrieved November 10, 2022, from  
<https://www.nausetschools.org/cms/lib/MA02212418/Centricity/Domain/204/Rate%20and%20Growth%20Notes%20AP%20Biology%202017.pdf>
- [3] *AP® BIOLOGY EQUATIONS AND FORMULAS.* (n.d.).  
<https://apcentral.collegeboard.org/media/pdf/ap-biology-equations-and-formulas-sheet.pdf>
- [4] *Bee Facts – Canadian Honey Council.* (n.d.).  
<https://honeycouncil.ca/industry-overview/bee-facts/>
- [5] *Bee Informed Partnership - National Management Survey Map.* (2019). Beeinformed.org.  
<https://research.beeinformed.org/loss-map/>
- [6] *Bee Swarms | Facts, Information and Habitat.* (2021, October 26). Animal Corner.  
<https://animalcorner.org/bee-swarms/?adlt=strict&toWww=1&redig=08C2E91AB26E48BC9CD933309E675F59>
- [7] Brad Schneider, R., Moosbeckhofer, R., & Crailsheim, K. (2010). Surveys as a tool to record winter losses of honey bee colonies: a two year case study in Austria and South Tyrol. *Journal of Apicultural Research*, 49(1), 23–30. <https://doi.org/10.3896/ibra.1.49.1.04>
- [8] Conocimiento, V. al. (2021, June 23). *The Devastating Consequences of a World Without Bees.* OpenMind.  
<https://www.bbvaopenmind.com/en/science/bioscience/consequences-a-world-without-bees/>
- [9] Greenwood, D. (2022, July 2). *How Far Do Bees Travel from Their Hive? | BeehiveHero.*  
<https://Beehivehero.com/>  
<https://beehivehero.com/how-far-do-bees-travel-from-their-hives/>
- [10] *Guide to Bee Swarming - Everything You Should Know.* (n.d.). Bees4life. Retrieved November 10, 2022, from  
<https://bees4life.org/bee-extinction/solutions/sustainable-beekeeping/swarming?adlt=strict&toWww=1&redig=6E1FB07C756848E8B3CF8679577C429C>
- [11] Hammond, G., & Blankenship, M. (n.d.). *Apis mellifera (honey bee).* Animal Diversity Web. Retrieved November 10, 2022, from  
[https://animaldiversity.org/accounts/Apis\\_mellifera/?adlt=strict&toWww=1&redig=7960C55994264BCABE128B52EFF2BDA3](https://animaldiversity.org/accounts/Apis_mellifera/?adlt=strict&toWww=1&redig=7960C55994264BCABE128B52EFF2BDA3)
- [12] *Honey Bee Genetics - PerfectBee.* (n.d.). Wwww.perfectbee.com. Retrieved November 10, 2022, from  
<https://www.perfectbee.com/learn-about-bees/the-science-of-bees/honey-bee-genetics?adlt=strict&toWww=1&redig=2364B12F982840DB9CEB1B5CE93CC786>

- [13] *Honey Bee Queens: Evaluating the Most Important Colony Member – Bee Health*. (2019, August 20). Bee-Health.extension.org.  
<https://bee-health.extension.org/honey-bee-queens-evaluating-the-most-important-colony-member/?adlt=strict&toWww=1&redig=7624FDDCDB87435398CF82464ACB1B2D>
- [14] Honey Bee | insect. (2019). In *Encyclopædia Britannica*.  
<https://www.britannica.com/animal/honeybee>
- [15] *How Honey Bees Keep Warm in Winter*. (n.d.). ThoughtCo. Retrieved November 10, 2022, from  
<https://www.thoughtco.com/how-honey-bees-keep-warm-winter-1968101#:~:text=Most%20bees%20and%20wasps%20hibernate%20during%20the%20colder>
- [16] *How Many Bee Hives Per Acre? - Beekeeper Tips*. (2021, December 14). Beekeeper Tips.  
<https://beekeepertips.com/how-many-bee-hives-per-acre/#:~:text=If%20we%20look%20at%20the%20foraging%20range%20of>
- [17] Lee. (2022, March 4). *Drones Vs. Worker Bees: What's The Difference? | Busy Beekeeping*. Busybeekeeping.com.  
<https://busybeekeeping.com/drones-vs-worker-bees/?adlt=strict&toWww=1&redig=1A18CA4749314C44B20D4DBF83EFAADA>
- [18] Lee, K. V., Steinhauer, N., Rennich, K., Wilson, M. E., Tarpy, D. R., Caron, D. M., Rose, R., Delaplane, K. S., Baylis, K., Lengerich, E. J., Pettis, J., Skinner, J. A., Wilkes, J. T., Sagili, R., & vanEngelsdorp, D. (2015). A national survey of managed honey bee 2013–2014 annual colony losses in the USA. *Apidologie*, 46(3), 292–305.  
<https://doi.org/10.1007/s13592-015-0356-z>
- [19] *Recognizing and Reporting Bee Kills – Pesticide Environmental Stewardship*. (n.d.). Pesticide Environmental Stewardship. Retrieved November 10, 2022, from  
<https://pesticidestewardship.org/pollinator-protection/recognizing-and-reporting-bee-kill/>
- [20] Russell, S., Barron, A. B., & Harris, D. (2013). Dynamic modelling of honey bee (*Apis mellifera*) colony growth and failure. *Ecological Modelling*, 265, 158–169.  
<https://doi.org/10.1016/j.ecolmodel.2013.06.005>
- [21] Station, G. (2015, September 29). *Understanding Colony Buildup and Decline: Part 4 - The Linear Growth Phase*. Scientific Beekeeping.  
<https://scientificbeekeeping.com/understanding-colony-buildup-and-decline-part-4/>
- [22] Switanek, M., Crailsheim, K., Truhetz, H., & Brodschneider, R. (2017). Modelling seasonal effects of temperature and precipitation on honey bee winter mortality in a temperate climate. *Science of the Total Environment*, 579, 1581–1587.  
<https://doi.org/10.1016/j.scitotenv.2016.11.178>
- [23] Ten, J. (2022, March 23). *How Many Bees Are in a Hive? (More Than You Might Think)*. LearnBees.  
<https://learnbees.com/how-many-bees-are-in-a-hive/?adlt=strict&toWww=1&redig=8D9B21F0226A4C0CAC1CB12AC7DC223B#heading1>



- [24] *Threats to Bees*. (n.d.). Museum of the Earth. Retrieved November 10, 2022, from [https://www.google.com/url?q=https://www.museumoftheearth.org/bees/threats&sa=D&source=docs&ust=1668103236642157&usg=AOvVaw1QE1-L1PSQ8t4\\_ZmcWcNja](https://www.google.com/url?q=https://www.museumoftheearth.org/bees/threats&sa=D&source=docs&ust=1668103236642157&usg=AOvVaw1QE1-L1PSQ8t4_ZmcWcNja)
- [25] *Top States: Number of Colonies About the Program NASS Highlights Honey production up 2 percent in 2018*. (2019). [https://www.nass.usda.gov/Publications/Highlights/2019/2019\\_Honey\\_Bees\\_StatisticalSummary.pdf](https://www.nass.usda.gov/Publications/Highlights/2019/2019_Honey_Bees_StatisticalSummary.pdf)
- [26] Tristan Copley Smith. (2018, November 2). *Types of Bees and Their Traits*. Osbeehives.com; OSBeehives. <https://www.osbeehives.com/blogs/beekeeping-blog/types-of-honey-bee-and-their-traits>
- [27] Victor, T. (2022, March 1). *Drone vs Worker Bee: What are the Differences?* AZ Animals. <https://a-z-animals.com/blog/drone-vs-worker-bee-what-are-the-differences/?adlt=strict&toWww=1&redig=8B675E153DE746CA864547C1C7652C7E>
- [28] *What Are the Different Climate Types?* | NOAA SciJinks – All About Weather. (2016). Scijinks.gov. <https://scijinks.gov/climate-zones/>
- [29] Rufus Isaacs and Zachary Huang, M. S. U. E. (2018, September 20). *Using bees for pollination of small fruit crops*. MSU Extension. Retrieved November 11, 2022, from [https://www.canr.msu.edu/news/using\\_bees\\_for\\_pollination\\_of\\_small\\_fruit\\_crops](https://www.canr.msu.edu/news/using_bees_for_pollination_of_small_fruit_crops)