Abhinav K. Sharma Mrs. Chase Advanced Physics 20 September 2023

Section I: Analysis (Data and Calculations with Explanations)

A. Data for Ramp Height of 4 cm (One Textbook)

B. Data for Ramp Height of 8 cm (Two Textbooks)

C. Calculations

Firstly, in order to calculate experimental acceleration for each ramp height, the starting distance of the cart from the sensor (m), that is, the displacement, was set equal to x. However, y was set equal to the square of the average velocity of the cart at each starting distance (m^2/s^2). That way, when calculating slope, a value in the units of acceleration would be generated. This is demonstrated below, with the focus being only on the units involved in calculating slope:

$$
m_{slope} = \frac{m^2}{s^2}
$$

$$
m_{slope} = \frac{m^2}{s^2} \left(\frac{1}{m}\right)
$$

$$
m_{slope} = \frac{m}{s^2}
$$

The units result in meters per second squared. Thus, the slope of the line is the experimental acceleration of the cart. Therefore: $x = \Delta x$;

Using these values for x and y, the graphs above were created, and from them, the following trendlines were generated (subscript of one is used for the ramp height of 4 cm, subscript of two for the $y = v^2$

$$
y_1 = 0.5455x_1 + 0.0154
$$
 ; $y_2 = 1.2753x_2 - 0.0061$

Now, using the value assigned to x and y, v^2 and Δx can be plugged into both equations.

$$
v_{t1}^2 = 0.5455\Delta x_1 + 0.0154 \; ; \; v_{t2}^2 = 1.2753\Delta x_2 - 0.0061
$$

This equation is in fact one of the kinematic equations used to express time, distance, velocity, and acceleration, namely the one that excludes time:

$$
v_t^2 = v_0^2 + 2a\Delta x
$$

All variables have an important meaning within the context of the experiment. For all trials, since the cart started moving from rest, the initial velocity is zero. Indeed, the y-intercept for both equations, v_0^2 , are both close to 0 m^2/s^2. Since v_t^2 represents the average velocities that were squared while generating the graphs, and Δx represents the displacement of the cart, that means the slope is $2a$. Hence, to calculate the experimental acceleration for both ramp heights, the slope for each of their trendlines must be divided by two:

$$
2a_1 = 0.5455
$$
; $2a_2 = 1.2753$
 $a_{exp1} = 0.273 \frac{m}{s^2}$; $a_{exp2} = 0.638 \frac{m}{s^2}$

After the experimental accelerations for each ramp height were determined, the theoretical acceleration in both cases was calculated. The theoretical acceleration is equal to the force due to gravitation multiplied by the sine of angle theta, where angle theta is the angle formed between the ramp and the table.

$$
a_{theor}=g\sin(\theta);
$$

For both ramp heights, the gravitational constant was multiplied by the ratio between the height of the textbooks and the length of the ramp.

$$
a_{theor1} = 9.8 \left(\frac{4}{104}\right) \; ; \; a_{theor2} = 9.8 \left(\frac{8}{104}\right)
$$

$$
a_{theor1} = 0.377 \frac{m}{s^2} \; ; \; a_{theor2} = 0.754 \frac{m}{s^2}
$$

From this, the percent error for each ramp can be calculated. In this case, it is the experimental acceleration subtracted by the theoretical acceleration, all divided by theoretical acceleration:

$$
\% \ error_{1} = \frac{a_{\exp 1} - a_{\text{theori}}}{a_{\text{theori}}} \quad ; \quad \% \ error_{2} = \frac{a_{\exp 2} - a_{\text{theorz}}}{a_{\text{theorz}}}
$$
\n
$$
\% \ error_{1} = \frac{0.273 - 0.377}{0.377} \quad ; \quad \% \ error_{2} = \frac{0.638 - 0.754}{0.754}
$$

% $error_1 = -27.8\%$; % $error_2 = -15.4\%$

Section II: Conclusion

The data from this experiment indicate that the ramp with the lower height (containing one textbook, measured to be four centimeters) had an experimental acceleration of about 0.273 meters per second squared. Meanwhile, for the ramp with the higher height (containing two textbooks, measured to be eighter centimeters), was over twice that, having an experimental acceleration of about 0.638 meters per second squared. This suggests that when there is an increase in the height of the ramp, there is an increase in the acceleration that the cart experiences. However, for both rates of acceleration, the experimental values are substantially lower than the theoretical values predicted, as the percent errors are –27.8% and –15.4% for the shorter and taller ramps, respectively.

There are many possibilities for sources of error in this experiment that could have played a role in the considerably low underestimates observed. Firstly, the expected value for acceleration assumes that there is no air resistance. When the cart travels down the ramp, with its velocity increasing due to gravity, the force it exerts on the air, and the resistance it faces from air molecules in turn, increases, slowing the velocity of the cart. The theoretical value does not account for this, but air resistance is reflected in the data, meaning the actual data is lower than what the expression for theoretical acceleration accounts for. As such, this may be a possible source for the very low underestimations.

Additionally, there were two major factors relating to carrying out the procedure itself that could have influenced the nature of these results. Firstly, for data points of the starting distance 0.6 meters from the sensor on the taller ramp, the sensor was misaligned, leading to improper readings for the velocity. After readjusting, it appeared that some measurements in velocity may have been lower than beforehand. Among the three trials, one velocity was lower than the other two. This would have lowered the average velocity, and in turn, the acceleration, contributing to the considerable underestimates observed. Secondly, a common technique on many trials, especially on the taller ramp, was to place a hand on the cart immediately after it passed through the sensor to prevent it from colliding with the end barrier of the ramp. However, if a hand was placed on the cart too early, which was often the case, then the cart would slow down prematurely, leading to lower velocity readings, and consequently a greater amount underestimation relative to the theoretical acceleration.

Moreover, the underestimations observed may be due in part to the many variations and approximations in measurements and calculations. Some variation is negligible. For example, the starting position of the cart from the sensor varied only ever so slightly among different trials. By contrast, measuring the height and length of the ramps would have influenced the value of the theoretical acceleration. These dimensions were only measured to the nearest whole centimeter, omitting the exact values for the sine ratio. Depending on the more exact measurements of the height and length of the ramp, the error introduced by measuring only to the nearest whole centimeter may have increased or decreased the percent error of the experimental acceleration.

However, approximation and variation in the measurement of the ramp dimensions would play a noticeable role in influencing the amount of percent error than other possible sources of variation. Omissions of decimal points in the form of having the slopes only going to four decimal places and experimental and theoretical accelerations going to three decimal places introduces another source of error whose impact on the percent error is similar to the impact of approximating the measurement of ramp dimensions. Overall, developing a more precise way of determining theoretical acceleration, along with improving the execution of the procedure and minimizing variation, must be done to improve the accuracy of results.