Question: Given an object on a flat, rough plane that intersects a smooth, inclined plane with an object connected to the first one, does a greater angle with the vertical perpendicular to the flat surface at its intersection with the incline (as seen in Figure 1), result in a lower acceleration of the entire system down the inclined plane?

Hypothesis: As the angle with the vertical increases, the acceleration will decrease, the rationale being that, as the cosine of this angle, which drives acceleration down the inclined plane (as shown in Figure 2), approaches zero.

Strategy:

- A wooden plank was placed along a flat plane, being elevated and stabilized by a trashcan and textbooks.
- A block, m_1, was placed on this plank. μ_k was calculated by dragging m_1 across the plank at as constant of a velocity as possible
- A ramp was placed next to the plank at an incline. A chair and a varying number of textbooks were put under the ramp to stabilize it and modify the angle with the vertical. Angles were measured with Apple's Measure app.
- A Vernier cart, m_2, whose sensor was used for measurements, was put on the ramp and attached to a pulley and m_1 with fishing wire.
- Each trial, m_1 was held in place. Then, it was released, allowing for the system to accelerate. Five trials were carried out for each angle.
- Acceleration was measured by using the slope of portions of the velocity vs. time graphs on Vernier Graphical Analysis.
- A theoretical linear regression model based off equations derived from the free body diagram of this experiment (Figure 1) was developed, and compared to one based on experimental data.
- Notably, x was defined as cosine(Θ). Thus, the output of the cosine function was on the horizontal axis, not the angle measure itself.
- An upper-tail t-test for linear regression at α = 0.05 was performed to verify the significance of the relationship empirically tested.

Data:

First, the masses of the system were measured:
m_1 = 0.1247 kg (given on label of the block)
m_2 = 0.3035 kg (measured with scale)

Then, μ_k was attained through the following equation (Figure 3 depicts free body diagram):

$F_{pull} - F_f = ma$; $(a = 0)$
$F_{pull} - \mu_k m_1 g = 0$
$\mu_k = F_{pull}/m_1 g$
$\mu_k = 0.942/((0.1247)(9.8))$
$\mu_k = 0.771$ (μ_k is hereafter referred to as μ)

Finally, the five trials were carried out at each angle, and average acceleration was calculated:

Θ (degrees)	Average Acceleration (m*s^-2)
30	3.687
53	1.067
60	0.732
66	0.500
69	0.320

Analysis:

Using the free body diagram in Figure 1, a theoretical linear equation between the angle with the vertical and the acceleration was developed. Positive motion for the system is defined as movement down the inclined plane. Due to the cart's wheels and the smooth surface of the ramp, it is assumed that m_2 does not experience friction:

$F_{net} = m_{total}a$
$m_2 g\cos\Theta - F_F = (m_1+m_2)a$
$F_F = \mu F_{N1}$; $F_{N1} = m_1 g$; $F_F = \mu m_1 g$
$m_2 g\cos\Theta - \mu m_1 g = (m_1+m_2)a$
$a = (m_2 g\cos\Theta - \mu m_1 g)/(m_1+m_2)$
Let $a = y$, and $\cos\Theta = x$
$y = (m_2 gx - \mu m_1 g)/(m_1+m_2)$
$y = (m_2 g/(m_1+m_2))x - (\mu m_1 g)/(m_1+m_2)$

The slope is the cart's force due to gravity and the y-intercept is the frictional force. Both quantities are divided by the total mass of the system. Plugging in the known values produces the following equation:

$y = ((0.3035)*(9.8)/(0.1274)+(0.3035))x - ((0.771)*(0.1247)*(9.8))/((0.1247)+(0.3035))$

$y = 6.903x - 2.200$

Let this now be compared to a linear regression of the experimental data:

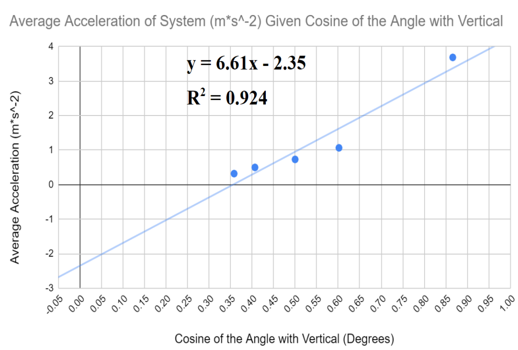


Figure 4: Linear Regression Model of Experimental Data

The equation derived from the experimental data is y = 6.61x - 2.35. The slope and y-intercept of the model were about 4.24% lower and 6.80% higher than their respective quantities in the theoretical model. Indeed, there were many sources of error associated with the procedure. As trials were successively carried out for each angle, the ramp had to be frequently adjusted to keep it aligned with the plank, with the ramp oftentimes becoming less steep. This would mean the cosine of the angle with the vertical was closer to 90∘, thus lowering the acceleration. The pulley and wood plank were readjusted often for the same reasons. Since they were not always fully level, the system may have incurred additional sources of friction. Additionally, the theoretical model does not take into account the weight of the string nor any possible friction from the pulley or air as the system travels down the incline. These were likely forces at play when the experiment was conducted, providing another factor that may have made the acceleration lower than predicted. Conversely, these constant readjustments and neglected sources of friction likely resulted in the underestimation of the y-intercept, as the present model only accounts for friction from m_1. Moreover, when m_1 was dragged across the wood plank to measure μ, it was not done at a completely constant velocity. Consequently, the μ value used in the model is likely inaccurate, contributing to the observed underestimation.

Nonetheless, the linear model of the experimental results provides compelling evidence in support of the initial hypothesis. Firstly, an upper-tail t-test for linear regression indicates that there is a statistically significant positive correlation between the cosine of the

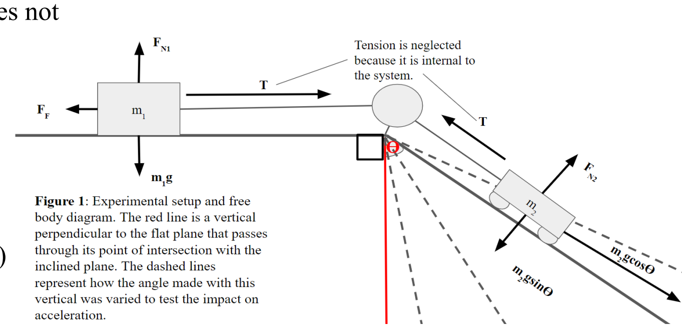


Figure 1: Experimental setup and free body diagram. The red line is a vertical perpendicular to the flat plane that passes through its point of intersection with the inclined plane. The dashed lines represent how the angle made with this vertical was varied to test the impact on acceleration.

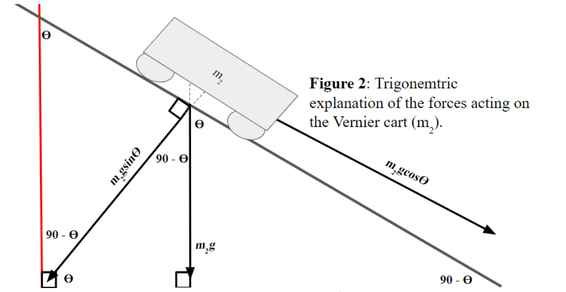
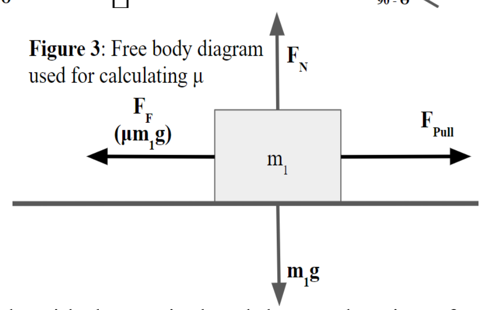


Figure 2: Trigonemtric explanation of the forces acting on the Vernier cart (m_2).

Figure 3: Free body diagram used for calculating μ



angle with the vertical and the acceleration of the system down the incline. Under a null hypothesis that there is no relationship between the two, and at significance level of $\alpha = 0.05$, given that $t_3 = 15.047$, $p < 0.001$. Therefore, the null hypothesis is rejected. The experimental data provide sufficient evidence that there is a positive relationship between the aforementioned variables. A greater output from the cosine of an angle indicates that its measure is closer to 0. Thus, as the angle with the vertical approaches 0, acceleration shall increase. Intuitively, this makes sense. As the ramp becomes steeper, approaching 0 degrees with the vertical and eventually becoming perpendicular to the plank, the cart travels faster. By contrast, as the ramp becomes less steep, approaching 90 degrees with the vertical and eventually becoming level with the plank, the cart travels slower, and eventually does not move due to too much friction. Additionally, the relationship between these two variables is very strong. An r^2 value of 0.924 indicates that 92.4% of the variation observed in the acceleration is due to its relationship with the cosine of the angle with the vertical. Taken together, this experimental procedure strongly supports the original hypothesis. However, as previously addressed, major improvements must be made with regards to accounting for additional sources of friction and having a more stable set-up for the experiment.

$H_0: \beta_1 = 0$ $y = 6.61x - 2.35$ $df = n-2$
$H_A: \beta_1 > 0$ $\beta_1 = 6.61$ $df = 5-2$
$\alpha = 0.05$ $SE(\beta_1) = 0.439$ $df = 3$

$t = (\beta_1 - 0)/SE(\beta_1)$ $p = tcdf(\beta_1, UB, df)$
$t = 6.61/0.439$ $p = tcdf(15.047, 999, 3)$
$t = 15.047$ $p < 0.001$

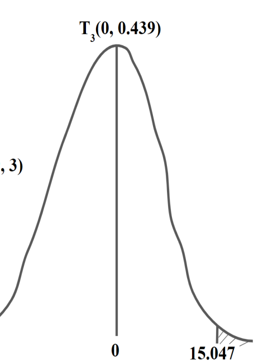


Figure 5: Calculations performed for t-test for linear regression.