Section B
Jacks
POW #1 - Happy Birthday!
10/13/2023

## Problem Statement

Society revolves around the concept of time. Knowing the day of the week is important, as our lives are scheduled on a weekly basis. Given the days of the week for several famous birthdays, the calendar for October 2023, and the number of days in each month, how can one calculate the day of the week for any given date between 1900 to 2100 (both exclusive)?

## Process

I. Pattern Identification and Cycle Development

The first step of our process was pattern identification. We recognized the following three patterns:

- Going from one year to the next, the day of the week shifted one day forward.
  - Ex. October 12, 2022 was a Wednesday and October 12, 2023 was a Thursday.
- Going from a year before a leap year to a leap year, for days after February 29th, the day of the week shifted forward by two days.
  - Ex. March 2, 1903, a year before a leap year, was a Monday, and March 2, 1904, a leap year, was a Wednesday.
- Going from a leap year to a year after a leap year, for days before February 29, the day of the week shifted forward by two days.
  - Ex. January 31, 2016, a leap year, was a Sunday, and January 31, 2017, the year after the leap year, was a Tuesday.

Either way, the pattern is as follows: every year, shift the day one day forward, but every fourth year, shift it two days forward. After identifying this to be the case, we found that every 28 years, the cycle of how the days of the week shift repeats, as is shown below. At this point, we had the following mapping: Monday = 0, Tuesday = 1, …,  Sunday = 6. This applies when sticking to one static date.

0, 1, 2, 3 | 5, 6, 0, 1 | 3, 4, 5, 6 | 1, 2, 3, 4 | 6, 0, 1, 2 | 4, 5, 6, 0 | 2, 3, 4, 5 | 0, 1, …

II. Manually Calculating a Starting Point

Seeing as there is a four year cycle for every leap year, we initially decided to create a four-year cycle, with every day therein receiving a distinct numerical designation. Given that there are (365.25 * 4), or (365 + 365 + 365 + 366), that is, 1461 days, in a cycle of four years, days were numbered 0 through 1460, and March 1 of the leap year was considered the first day. To illustrate this model, consider the following table:

| Date | Day in Cycle |
|---|---|
| March 1, 2016 | 0 |
| … | … |
| February 28, 2020 | 1459 |
| February 29, 2020 | 1460 |
| March 1, 2020 | 0 |
| March 2, 2020 | 1 |
| … | … |
| February 29, 2024 | 1460 |

Although the 1461 day-cycle itself was not used, it established March 1 as a starting point, which was to be a good primer for the calculations for our actual model. Combined with the fact that we had established a 28-year cycle and that the scope of the model was to cover January 1, 1901 to December 31, 2099, we began to manually calculate what day of the week January 1, 1901 was. We started from March 1, 2023, which we mentally calculated to be a Wednesday. Through the cycles of 28 years established above, this date would be a Wednesday. We then subtracted 28 until we got as close to 1900 as possible:

March 1 2023 = Wednesday
2023 - (28*4) = 1911
March 1 1911 = Wednesday

Then, using our 28 year cycle, we were able to calculate the day of the week for March 1, 1900.

0, 1, 2, 3 | 5, 6, 0, 1 | **3**, 4, 5, 6 | 1, 2, 3, 4 | 6, 0, 1, **2** | 4, 5, 6, 0 | 2, 3, 4, 5 | 0, 1, …
**Since 1911 is a Wednesday (2) preceding a leap year, we know that the number indicated in the cycle must represent 1911.**
**We know that this value is 1900 from counting backwards from where 1911 is in the cycle.**
Thus, we know that March 1, 1900 is a Thursday (3).

From there, knowing that the range of dates for the solution had to start from 1901, from March 1, 1900, we calculated what day of the week January 1, 1901 would be by adding the number of days of each month between March 1900 (inclusive) and January 1901 (exclusive) together, taking mod 7 of that sum, adding that to the 3 that represents Thursday, March 1, 1900, and taking mod 7 of that, like so:

March 1, 1900 = Thursday (3)
(3 + ((31 + 30 + 31 + 30 + 31 + 31 + 30 + 31 + 30 +31)mod7)mod7
(3 + (306)mod7)mod7
(3 + 5)mod7
8mod7 = 1
Therefore, January 1, 1901 was a Tuesday.
Finding the day that January 1, 1901 landed on was useful as we were now able to use this information to find the days of all other dates between 1901 to 2099. January 1, 1901 was to be used in our final model.

## III. Model Development

Notice that the means by which we reached the specific January 1 day was through adding up all the days of the months to get from the original March 1 date. This was important in constructing a major step in our process. Consider the following two tables below:

Case I: Number of Days Per Month in a Non-Leap Year

| Month | Number of Days in Month | Number of Days Added (to reach this month from previous one) | Cumulative Amount of Days for the Year |
|---|---|---|---|
| Jan | 31 | 0* | 31 |
| Feb | 28 | 31 | 59 |
| Mar | 31 | 28 | 90 |
| Apr | 30 | 31 | 121 |
| May | 31 | 30 | 151 |
| Jun | 30 | 31 | 182 |
| Jul | 31 | 30 | 212 |
| Aug | 31 | 31 | 243 |
| Sep | 30 | 31 | 274 |
| Oct | 31 | 30 | 304 |
| Nov | 30 | 31 | 335 |
| Dec | 31 | 30 | 365 |

Case II: Number of Days Per Month in a Leap Year

| Month | Number of Days to Add | Number of Days Added (to reach this month from previous one) | Cumulative Amount of Days for the Year |
|---|---|---|---|
| Jan | 31 | 0* | 31 |
| Feb | 29 | 31 | 60 |
| Mar | 31 | 29 | 91 |
| Apr | 30 | 31 | 122 |
| May | 31 | 30 | 152 |
| Jun | 30 | 31 | 183 |
| Jul | 31 | 30 | 213 |
| Aug | 31 | 31 | 244 |
| Sep | 30 | 31 | 275 |
| Oct | 31 | 30 | 305 |
| Nov | 30 | 31 | 336 |
| Dec | 31 | 30 | 366 |

* January is the starting month, so we add 0.

Since our starting date was now set to January 1, we could simply add the number of days between January 1 and any given day of that year, using the number of days for each full month, and the remaining number of days for the month in which the date itself was. For example, if we wanted to get from January 1 to September 14, we would add the number of days in the months January through August, and the remaining 14 days in September to get to September 14.

However, for leap years, 29 days would be added because of February's leap day, as opposed to 28 (hence the two tables above). Taking mod 7 of this sum, and adding it to the number that mapped to the day of the week of January 1 for the year, and taking mod 7 of that, would get the day of the week of that given day.

Therefore, the only thing left was to establish a cycle for the days of the week of January 1. To these ends, knowing that the cycle length was 28 years long, the patterns of how the days shift (one day forward for every year, two days forward every fourth), and that January 1, 1901 was a Tuesday, we constructed the following table below:

| Remainder from Mod 28 (i.e. position in cycle) | Day of the Week of January 1 | Remainder from Mod 28 (i.e. position in cycle) | Day of the Week of January 1 |
|---|---|---|---|
| 0 | 1 | 14 | 4 |
| 1 | 2 | 15 | 5 |
| 2 | 3 | 16 | 0 |
| 3 | 4 | 17 | 1 |
| 4 | 6 | 18 | 2 |
| 5 | 0 | 19 | 3 |
| 6 | 1 | 20 | 5 |
| 7 | 2 | 21 | 6 |
| 8 | 4 | 22 | 0 |
| 9 | 5 | 23 | 1 |
| 10 | 6 | 24 | 3 |
| 11 | 0 | 25 | 4 |
| 12 | 2 | 26 | 5 |
| 13 | 3 | 27 | 6 |

From this format, 1901 would map to position 0 in the cycle, 1902 to 1, and so on, until 1929, which would return to position 0 and restart the cycle, up until December 31, 2099. Thus, this established the day of the week of January 1 for any given year. From there, the process of addition and modular arithmetic to get to the specific date and identify its day of the week that was described above could be performed.

Therefore, we established the following formula:
(Jan 1[(Y - 1901)mod 28]+ ($R_D$mod7))mod7,
Where Y is the given year, and $1901 \leq Y \leq 2099$;
$R_D$ is the number of days between the date chosen and January 1 (i.e. the process of summing the days of each month and the remaining days described); and,
Where Jan 1[] represents a function for the day of the week of January 1 of the year chosen, this function deciphering where in the 28-year cycle a year resides and linking that to a day of the week, where Monday = 0 through Sunday = 6.

However, this formula was erroneous. Consider the following example, which shall both show how this model is incorrect, and clarify our overall process and solution.

Example: May 19, 2007. This was a Saturday. Therefore, our formula should produce a 5.
(Jan 1[(Y - 1901)mod 28]+ ($R_D$mod7))mod7

- Plug in 2007 for Y, and (31 + 28 + 31 + 30 + 19) as $R_D$. (31 + 28 + 31 + 30 + 19) gets us from January 1 to May 19.

(Jan 1[(2007 - 1901)mod 28]+ ((31 + 28 + 31 + 30 + 19)mod7))mod7

- Simplify.

(Jan 1[(106)mod 28]+ (139mod7))mod7

- Simplify.

(Jan 1[22]+ 6)mod7

- Now, we know that the day of the week of January 1, 2007 is in position 22 of the 28-year cycle. We can identify what day of the week that is using the table of remainders of mod28 and the corresponding days of the week for January 1, as highlighted below:

| Remainder from Mod 28 (i.e. position in cycle) | Day of the Week of January 1 | Remainder from Mod 28 (i.e. position in cycle) | Day of the Week of January 1 |
|---|---|---|---|
| 0 | 1 | 14 | 4 |
| 1 | 2 | 15 | 5 |
| 2 | 3 | 16 | 0 |
| 3 | 4 | 17 | 1 |
| 4 | 6 | 18 | 2 |
| 5 | 0 | 19 | 3 |
| 6 | 1 | 20 | 5 |
| 7 | 2 | 21 | 6 |
| 8 | 4 | 22 | 0 |
| 9 | 5 | 23 | 1 |
| 10 | 6 | 24 | 3 |
| 11 | 0 | 25 | 4 |
| 12 | 2 | 26 | 5 |
| 13 | 3 | 27 | 6 |

Therefore, we plug in 0 into the equation, as Jan 1[22] is 0:

(0+ 6)mod7
6mod7
6

This maps to Sunday, which is 1 day ahead of the actual day, Saturday. All dates that were tested were consistently one day ahead of what they should have been. This was because our process was inclusive of both January 1 and the date being used, when it in fact should have been exclusive of the date being used. Since every result was consistently one day ahead, we simply shifted our dating scheme back by one weekday, like so:

Old Dating Scheme:

| Day of the Week | Index |
|---|---|
| Monday | 0 |
| Tuesday | 1 |
| Wednesday | 2 |
| Thursday | 3 |
| Friday | 4 |
| Saturday | 5 |
| Sunday | 6 |

New Dating Scheme:

| Day of the Week | Index |
|---|---|
| Sunday | 0 |
| Monday | 1 |
| Tuesday | 2 |
| Wednesday | 3 |
| Thursday | 4 |
| Friday | 5 |
| Saturday | 6 |

Now, since 6 is Saturday, the model gives an accurate answer. Therefore, our final model is the following:

(Jan 1[(Y - 1901)mod 28]+ ($R_D$mod7))mod7, where all variables that were defined above remain the same; but,

Where an output of 0 corresponds to Sunday, 1 to Monday, and so on, until 6, which is Saturday.

**Solution with Instructions**

1. Take the year you want to test, between 1901 and 2099, and subtract 1901.
2. Divide this difference by 28, and find the remainder.
3. Plug this number into table 1 under the "Remainder" column, and find the corresponding value in the same row under the "Starting Day of the Week" column. Note this value.

Table 1:

| Remainder | Starting Day of the Week | Remainder | Starting Day of the Week |
|---|---|---|---|
| 0 | 1 | 14 | 4 |
| 1 | 2 | 15 | 5 |
| 2 | 3 | 16 | 0 |
| 3 | 4 | 17 | 1 |
| 4 | 6 | 18 | 2 |
| 5 | 0 | 19 | 3 |
| 6 | 1 | 20 | 5 |
| 7 | 2 | 21 | 6 |
| 8 | 4 | 22 | 0 |
| 9 | 5 | 23 | 1 |
| 10 | 6 | 24 | 3 |
| 11 | 0 | 25 | 4 |
| 12 | 2 | 26 | 5 |
| 13 | 3 | 27 | 6 |

4. Take note of whether the year you are testing is a leap year.
    a. To do this, take the year you are testing, and divide by 4. If there is a remainder, this year is not a leap year, and if there is not a remainder, it is.
5. Take the month and day that you are testing. Add the number of days of all full months before your date. Then, add the remaining days needed to get to your specific date. Note this value.
    a. To help, table 2 provides the number of days for each month to add in a leap-year or non-leap year. Add the days of all the full months, and then add the number date of the month you are testing.
    b. For example, in a non-leap year, to get to April 14, we would add the 31, 28, and 31 days from the full months January, February and March, and then 14 from April.

Table 2:

| Month | How Much to Add (Non-Leap Year) | How Much to Add (Non-Leap Year) |
|---|---|---|
| Jan | 0 | 0 |
| Feb | 31 | 31 |
| Mar | 59 | 60 |
| Apr | 90 | 91 |
| May | 120 | 121 |
| Jun | 151 | 152 |
| Jul | 181 | 182 |
| Aug | 212 | 213 |
| Sep | 243 | 244 |
| Oct | 273 | 274 |
| Nov | 304 | 305 |
| Dec | 334 | 335 |

6. Divide by the sum found in step 5 by 7, and get the remainder. Note this value.
7. Add the values noted from steps 3 and 6. Divide this number by 7 and find the remainder.
8. Finally, plug this number into table 3 under the "Result" column, and find the corresponding day under the "Day" column. This value should be the day of the week of the date that you are testing!

Table 3

| Result | Day |
|---|---|
| 0 | Sunday |
| 1 | Monday |
| 2 | Tuesday |
| 3 | Wednesday |
| 4 | Thursday |
| 5 | Friday |
| 6 | Saturday |

**Extensions**

Additional Problems to Consider:

- How can one convert between days across the dating systems of other cultures?
  - Ex. Given a date in the Gregorian calendar, find an equivalent date in the Mayan calendar.
- How can the formulas presented be modeled with computer programming?
- How do different methods in calculating the day of the week for any given date compare? Which ones are better or worse, and why?
  - What is the most efficient method in solving this problem?
- Calculating the probability and/or working with the distributions of days of the week for certain dates
  - Ex. How often is Christmas (December 25) a Monday, Tuesday, etc.?

**Calculating Days of the Week for Before 1900 and After 2100**

Process:

For years that are divisible by 100, in order to be a leap year, they must also be divisible by 400. Therefore, 1600, 2000, and 2400 are leap years, but 1700, 1800, 1900, 2100, 2200, and 2300 are not. We know that for any date, every year, it shifts one day forward, and every fourth year, it shifts two days forward. However, for centuries that are not leap years, this does not occur, and the day shifts forward only by one.

In order to model years beyond the bounds of 1901 to 2099, we chose to model the shift in January 1, using 1 from January 1, 1901 as a starting point. We mapped out our 28-year cycle as if all centuries were leap years in addition to mapping out what would actually happen.

For beyond 2099, consider the following:

| Year | January 1 Actual Day | January 1 Day Following Our 28-Year Cycle |
|---|---|---|
| 2097 | 1 | 1 |
| 2098 | 2 | 2 |
| 2099 | 3 | 3 |
| 2100 | 4 | 4 |
| 2101 | 5 | 6 |
| 2102 | 6 | 0 |
| 2103 | 0 | 1 |
| 2104 | 1 | 2 |
| 2105 | 3 | 4 |
| 2106 | 4 | 5 |
| 2107 | 5 | 6 |

Since 2100 is not a leap year, it only adds 1 and not 2. So, it falls behind by one day of the week. For example, in 2104, 2-1 = 1, 2105, 4-3 =1, etc.

Consider what occurs at 2200 and 2300 (columns same as above):

| | | |
|---|---|---|
| 2195 | 3 | 4 |
| 2196 | 4 | 5 |
| 2197 | 6 | 0 |
| 2198 | 0 | 1 |
| 2199 | 1 | 2 |
| 2200 | 2 | 3 |
| 2201 | 3 | 5 |
| 2202 | 4 | 6 |
| 2203 | 5 | 0 |
| 2204 | 6 | 1 |
| 2205 | 1 | 3 |
| 2206 | 2 | 4 |

| | | |
|---|---|---|
| 2295 | 1 | 3 |
| 2296 | 2 | 4 |
| 2297 | 4 | 6 |
| 2298 | 5 | 0 |
| 2299 | 6 | 1 |
| 2300 | 0 | 2 |
| 2301 | 1 | 4 |
| 2302 | 2 | 5 |
| 2303 | 3 | 6 |
| 2304 | 4 | 0 |
| 2305 | 6 | 2 |

Each time, the cycle falls further behind by one day of the week. By the 2300's, the actual cycle is three days behind the cycle following our 28-year pattern.
For example, in 2302, 5-3 = 2, and in 2305, 2-3 = 6 (remember that all of this is in mod7)

However, consider what occurs at 2400:

| | | |
|---|---|---|
| 2392 | 3 | 5 |
| 2393 | 5 | 0 |
| 2394 | 6 | 1 |
| 2395 | 0 | 2 |
| 2396 | 1 | 3 |
| 2397 | 2 | 5 |
| 2398 | 3 | 6 |
| 2399 | 4 | 0 |
| 2400 | 5 | 1 |
| 2401 | 0 | 3 |
| 2402 | 1 | 4 |
| 2403 | 2 | 5 |
| 2404 | 3 | 6 |
| 2405 | 5 | 1 |
| 2406 | 6 | 2 |

Here, the difference remains at 3, because 2400 is a leap year, meaning both cycles will jump by 2. Since they jump by the same quantity, the difference remains the same.
For example, in 2406, 2-3 = 6 still is true.

Now, let us consider what occurs for centuries before 1900:

| | | |
|---|---|---|
| 1886 | 4 | 3 |
| 1887 | 5 | 4 |
| 1888 | 6 | 5 |
| 1889 | 1 | 0 |
| 1890 | 2 | 1 |
| 1891 | 3 | 2 |
| 1892 | 4 | 3 |
| 1893 | 6 | 5 |
| 1894 | 0 | 6 |
| 1895 | 1 | 0 |
| 1896 | 2 | 1 |
| 1897 | 4 | 3 |
| 1898 | 5 | 4 |
| 1899 | 6 | 5 |
| 1900 | 0 | 6 |
| 1901 | 1 | 1 |
| 1902 | 2 | 2 |
| 1903 | 3 | 3 |
| 1904 | 4 | 4 |
| 1905 | 6 | 6 |
| 1906 | 0 | 0 |

The cycles are in sync between 1900 and 2099 because 0 + 1 and 6 +2 are both 1 in mod7, and since 2000 is a leap year, both our 28-year cycle and the actual cycle jumped 2, so the difference there was also the same for that century. However, before 1900, they were out of sync. Since, from 1901 to 1900, our 28-year cycle subtracts 2, but the actual cycle only subtracts 1, making it a day ahead.
For example, in 1892, 3 + 1 = 4.

Consider what occurs for 1800 and 1700:

| | | |
|---|---|---|
| 1792 | 6 | 4 |
| 1793 | 1 | 6 |
| 1794 | 2 | 0 |
| 1795 | 3 | 1 |
| 1796 | 4 | 2 |
| 1797 | 6 | 4 |
| 1798 | 0 | 5 |
| 1799 | 1 | 6 |
| 1800 | 2 | 0 |
| 1801 | 3 | 2 |
| 1802 | 4 | 3 |
| 1803 | 5 | 4 |
| 1804 | 6 | 5 |
| 1805 | 1 | 0 |
| 1806 | 2 | 1 |

| | | |
|---|---|---|
| 1691 | 0 | 4 |
| 1692 | 1 | 5 |
| 1693 | 3 | 0 |
| 1694 | 4 | 1 |
| 1695 | 5 | 2 |
| 1696 | 6 | 3 |
| 1697 | 1 | 5 |
| 1698 | 2 | 6 |
| 1699 | 3 | 0 |
| 1700 | 4 | 1 |
| 1701 | 5 | 3 |
| 1702 | 6 | 4 |
| 1703 | 0 | 5 |
| 1704 | 1 | 6 |
| 1705 | 3 | 1 |
| 1706 | 4 | 2 |
| 1707 | 5 | 3 |
| 1708 | 6 | 4 |

In both cases, the actual cycle subtracts 1, while our 28-year cycle subtracts 2, leading to it being 2 days, then 3 days ahead for between 1700 and 1800 and between 1600 and 1700 respectively. For example, in 1797, 4 + 2 = 6, and in 1698, 5 + 3 = 1

However, 1600 is a leap year, so both systems subtract 2, and the difference remains at 3:

| | | |
|---|---|---|
| 1589 | 6 | 3 |
| 1590 | 0 | 4 |
| 1591 | 1 | 5 |
| 1592 | 2 | 6 |
| 1593 | 4 | 1 |
| 1594 | 5 | 2 |
| 1595 | 6 | 3 |
| 1596 | 0 | 4 |
| 1597 | 2 | 6 |
| 1598 | 3 | 0 |
| 1599 | 4 | 1 |
| 1600 | 5 | 2 |
| 1601 | 0 | 4 |
| 1602 | 1 | 5 |
| 1603 | 2 | 6 |

For example, in 1591, 5 + 3 = 1 is still true.

For all of these examples, the difference between our 28-year model and the actual model increases by 1 every year, except every fourth year, it increases by 0. This is because a century is a leap year only every 400 years.

Given these observations, we provide the conjectures for the following formulas, using the formula for between 1901-2099 as a template:

Case I: Prior to 1900
$(((\text{Jan } 1[(Y - 1901) \bmod 28] + N)) \bmod 7) + (R_D \bmod 7)) \bmod 7$,
Where all previously defined variables have the same meaning;

Where $N = \mathbf{\Sigma}\{k_1 + k_2 + k_3 + k_4 + \ldots + k_n + k_{2n} + k_{3n} + k_{4n}\}$; and,
Where $k_n$ = # of centuries before 1999 and $k_1 = k_2 = k_3 = 1$, $k_4 = 0$.

Case II: Prior to 2100
(((Jan 1[(Y - 1901)mod 28] - N)mod7)+ ($R_D$mod7))mod7,
Where all previously defined variables have the same meaning;
Where $N = \mathbf{\Sigma}\{k_1 + k_2 + k_3 + k_4 + \ldots + k_n + k_{2n} + k_{3n} + k_{4n}\}$; and,
Where $k_n$ = # of centuries after 2000 and $k_1 = k_2 = k_3 = 1$, $k_4 = 0$.

Explanation:
N is used to add up the total difference between our 28-year model and what would actually occur, given $k_n$, the number of centuries before 1999 and after 2000 in cases before 1900 and after 2100 respectively. k is assigned to these specific values because this results in the actual difference between the two cycles that we observed above. Since every fourth century, this difference does not change, k4 and its multiples are 0, while all other “elements” of k in “series” N are 1. We add/subtract this from the number we attained from the position in our 28-year cycle, because that’s how far off the real cycle is from the cycle we created for our original formula. The examples below should provide clarity about these new formulas.

When giving someone directions for a case like this, the directions could be edited like so:
For before 1900:
- After you see the value your remainder of 28 maps to in the “Starting Day of the Week” column, consider the number of centuries before 1900 the date you are testing is.
- For every century, starting from 0, add 1, except for every fourth century, where you are to add 0.
- Add this sum to the value you found from the “Starting Day of the Week” column.

For after 2100:
- After you see the value your remainder of 28 maps to in the “Starting Day of the Week” column, consider the number of centuries after 2000 the date you are testing is.
- For every century, starting from 0, add 1, except for every fourth century, where you are to add 0.
- Subtract this sum from the value you found from the “Starting Day of the Week” column.

Test Cases:
July 4, 1776

(((Jan 1[(Y - 1901)mod 28] + N)mod7)+ ($R_D$mod7))mod7
(((Jan 1[(1776 - 1901)mod 28] + N)mod7)+ ((31+29+31+30+31+30+4)mod7))mod7

$N = \mathbf{\Sigma}\{k_1 + k_2 + k_3 + k_4 + \ldots + k_n + k_{2n} + k_{3n} + k_{4n}\}$

$k_n = 2 \rightarrow$ Since we define $k_n$ to be centuries before 1999, and 1999-1776 = 223, we know that we are 2 centuries from 1999.

$N = \mathbf{\Sigma}\{k_1 + k_2\}$

$N = 1 + 1$

$N = 2$

(((Jan 1[(1776 - 1901)mod 28] + N))mod7)+ ((31+29+31+30+31+30+4)mod7))mod7

(((Jan 1[15] + 2)mod7) + (186mod7))mod7

((5 + 2)mod7+ (186mod7))mod7

(0 + 4)mod7

4; This maps to a Thursday, which is the correct answer.

January 28, 2225

(((Jan 1[(Y - 1901)mod 28] - N)mod7)+ ($R_D$mod7))mod7

(((Jan 1[(2225 - 1901)mod 28] - N)mod7)+ (28mod7))mod7

$N = \mathbf{\Sigma}\{k_1 + k_2 + k_3 + k_4 + \ldots + k_n + k_{2n} + k_{3n} + k_{4n}\}$

$k_n = 2 \rightarrow$ Since we define $k_n$ to be centuries after 2000, and 2225-2000= 225, we know that we are 2 centuries from 2000.

$N = \mathbf{\Sigma}\{k_1 + k_2\}$

$N = 1 + 1$

$N = 2$

(((Jan 1[(324)mod 28] - 2)mod7)+ 0)mod7

(((Jan 1[16] - 2)mod7)+ 0)mod7

((0 - 2)mod7)+ 0)mod7

((-2mod7)+ 0)mod7

(5 + 0)mod7

5mod7

5; This maps to a Friday, which is the correct answer.