Traffic Light Design: Team SPAN

Problem:

Question: How long should a traffic light stay yellow in order to have ideal crossing/stopping time at an intersection? What factors, relating to speed, time, and length, affect this decision?

Assumptions:

- Cars are constantly traveling at the speed limit, unless a decision is made to stop, in which case the car will begin to decelerate.
- A car proceeding through an intersection must make it through the entire intersection before the light turns red, similarly, a car whose driver made the decision to stop safely must be able to decelerate before entering the intersection

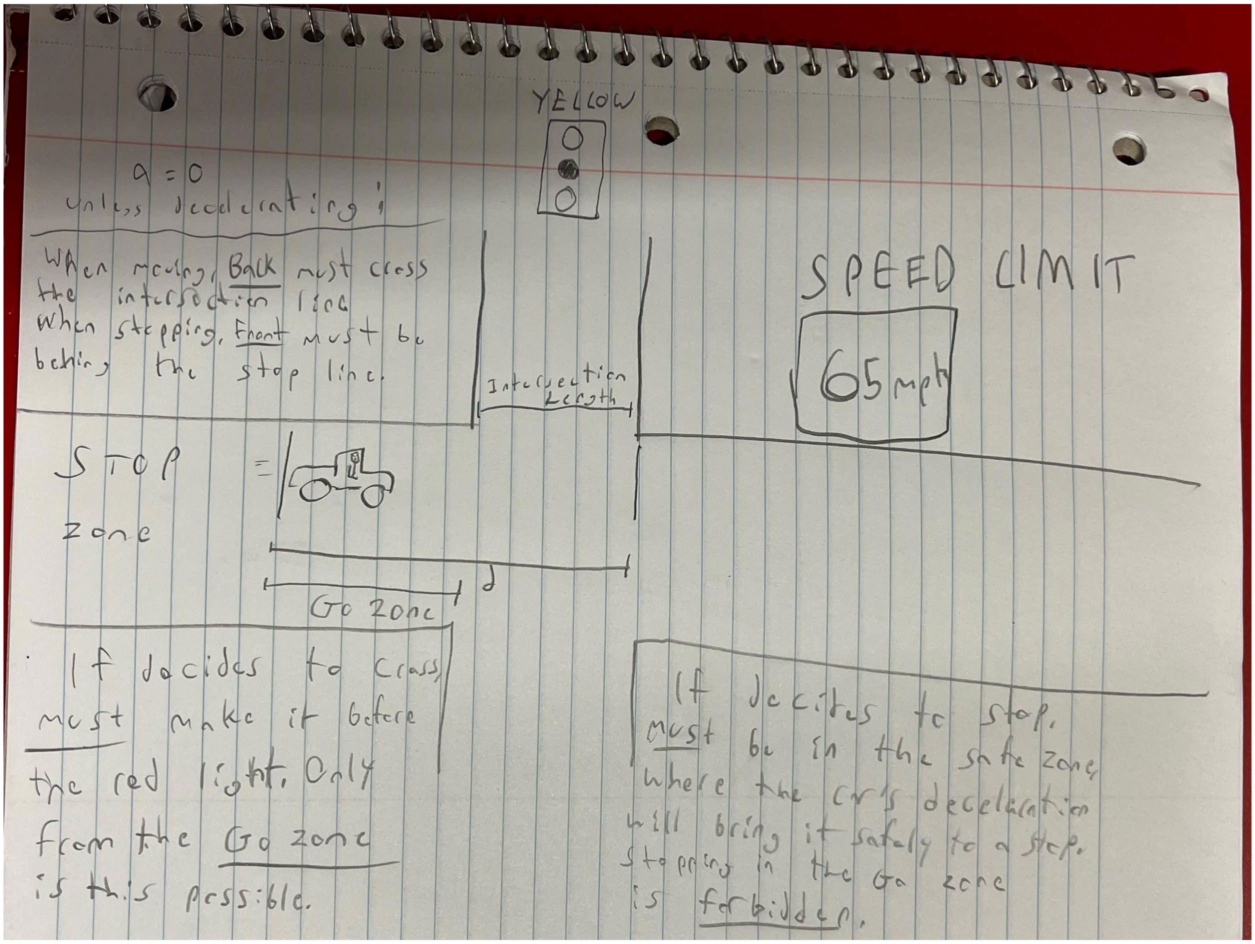
Terminology:

- GO zone means the region in which it is safe for a car to proceed through the yellow light
- STOP zone means the region in which it is not safe for a car to proceed through the yellow light, meaning it will be forced to slow to a stop.

Problem In a Nutshell:

Using the given assumptions, find a formula which expresses the duration a yellow light should be to allow drivers who decide to go through the intersection to make it safely and those who decide to stop to decelerate perfectly.

Model:



Process:

Our assumptions -

- The driver decides whether to stop or go as soon as the light turns from green to yellow
- All drivers at a certain intersection decelerate at the same rate
- If a car can mathematically pass the intersection, the driver WILL decide to go
- A car that decides to go will not accelerate or decelerate
- STOP zone starts where the GO zone ends

Variables -

- Speed Limit (V_L)
- Length of the Intersection (ΔX_I)
- Length of the GO zone (ΔX_G)
    - We created a formula for this using the other variables
- Average Deceleration Rate ($\bar{a}_d$)
- Car length (ΔX_C)

We defined the STOP zone as the entire road directly behind the GO zone. This means that a car must be able to completely stop before the end of the GO zone.

The first variable we decided on was the speed limit. This variable is important in determining the time needed to cross the intersection and to stop. Our next variable was intersection length. The longer this length is, the longer the car will need to cross it at a certain speed. Next, we took into account the average deceleration rate which will aid in understanding how long a car in the STOP zone will take to completely stop. Using these variables, we set up 2 values to solve for: the length of the go zone and the time required to pass the intersection (which is equal to the time that the yellow light must be on for).

To solve for the length of the go zone, we used the kinematic equation that didn't involve t ($v_0^2 + 2a\Delta x = v^2$) and isolated for displacement, keeping in mind that $v = 0$, since the car comes to a complete stop. After substitution, we are left with the following equation:

$$v_0^2 = -2a\Delta x.$$

Isolating displacement above leaves us with:

$$\Delta x = \frac{v_0^2}{-2a}.$$

The total distance that a car that decides to go must travel is the GO zone + the length of the intersection:

$$\frac{v_0^2}{-2a} + \Delta x_i$$

To account for the discrepancy between the go zone and stop zone, which we assumed to be the length of the car itself, we add this value to the numerator also. For the length of the car, we assumed that all cars are the same length, but another approach with the same result is taking the average car length at the intersection.

$$\frac{v_0^2}{-2a} + \Delta x_i + \Delta x_c$$

Another component to consider is the distance traveled from the car's velocity multiplied by the reaction time of the driver to make a decision. This distance is given by the formula $\Delta x = (\frac{v_0+v}{2})t$. In this context, the initial and final velocities are both the speed limit since the driver is moving at their normal speed while making the decision. In this case, we can consider the average to be v_r (reaction time) and derive the displacement to be $v_r t_r$ (both variables have a subscript r, for the reaction time, to distinguish from other instances of the same variable) which we add to the previous equation

$$\frac{v_0^2}{-2a} + \Delta x_i + \Delta x_c + v_r t_r$$

This equation is now divided by v_l (the speed limit) to get us our final equation:

$$t = \frac{\frac{v_l^2}{-2a} + \Delta x_i + \Delta x_c + v_l t_r}{v_l}.$$

Solution:

Our solution is:

Yellow Light Time: $t = \frac{\frac{v_l^2}{-2a} + \Delta x_i + \Delta x_c + v_l t_r}{v_l}$

In order to understand this solution, we need to first intuitively think about what this solution needs to be:

- A formula that gives us the amount of time that…
    - A car can completely pass the intersection from the back of the GO zone
    - A car can decelerate completely from the beginning of the STOP zone

So we know that the **max distance that any car would need to cross completely** is **(**V_L^2)/(-2$\bar{a}_d$). Using this formula in our equation will give the maximum amount of time anyone will need to traverse the intersection.

We add in the **length of the car** to account for the fact that the *back* of the car needs to cross safely. We also considered the **reaction time of the driver**. During this time the driver will have gone at the speed limit only decelerating afterwards. So, we add in $v_l t_r$, the distance a driver will have driven during their reaction time.

The final distance: $\frac{v_l^2}{-2a} + \Delta x_i + \Delta x_c + v_l t_r$

Finally, by dividing the total distance by the speed limit, we can get the maximum time needed to cross the intersection. This is the length of the yellow time. Overall, this shows that our solution is logical and complete for the situation we are modeling.

When making sure our solution is accurate, we should consider what change increasing or decreasing each of these variables has on the overall yellow light time and whether it makes sense. For speed limit, if it increases, the overall time, using the formula, increases because the variable is squared in the numerator but linear in the denominator meaning it increases more in the numerator. This makes sense because if there is a higher speed limit, the car needs more distance to completely stop, which means that any cars in front of that distance need to cross more distance when trying to get through the intersection, leading to a conclusion of a longer yellow light time needed if the speed limit is higher. The reverse can be said for a smaller speed limit: less time would be needed to stop, less distance would need to be covered, less time for the overall yellow light. Then, if acceleration increases (in a magnitude as it is always deceleration), the time would decrease because acceleration is in the denominator. This makes sense because as the magnitude of acceleration increases, the car can come to a stop faster, needing less distance before the intersection to stop. Following a similar logic as above, this means a lower yellow light time needed, matching the formula. If we increase car length or length of intersection, the numerator would increase so the overall yellow light time would increase. This makes sense because each of these variables would increase the total distance needed to cover (car length, because the front of the car would need to get farther past the intersection end line). So, if total distance increases, the amount of time needed to cross would increase, which matches the formula. Finally, reaction time. If reaction time increases, the time would increase because reaction time linearly increases in the numerator. This makes sense because the longer the reaction time, the more distance the driver would drive from when the light first turns yellow to when it makes a decision to keep driving or stop which would increase the total distance driven during the yellow light, therefore increasing the yellow light time.

- Speed Limit (V_L)
- Length of the Intersection (ΔX_I)
- Average Deceleration Rate ($\bar{a}_d$)
- Car length (ΔX_C)
- Reaction time (t_r)

Discussion Questions:

1. What must be true about the STOP and GO zones for the intersection to be safe? Describe what happens at the intersection if the zones do not adhere to this rule.
    - 2 possibilities can be considered safe:

1. The point where the GO zone starts is the point where the STOP zone ends
2. The beginning of the GO zone is before the end of the STOP zone

■ If the STOP zone ends before the GO zone, cars in the space between are left with no safe options. This is unsafe because the car is past the zone where they can safely stop before hitting the intersection, but they are not close enough to the intersection where they can keep driving and cross completely. This would leave drivers with a dangerous situation where they would be stuck in the intersection when the light turns red.

2. How would the following conditions affect the required yellow light time? Which zone would they affect? Via which variable?

We had 2 interpretations of this question. Both are listed!

a. Exceeding the speed limit

■ If the driver exceeds the speed limit, it would increase the required yellow light time. This is because at a higher speed the driver would need more time to slow down before stopping, pushing back the stop zone. This is via the V_L variable in the model formula.

■ There are 2 possibilities depending on circumstances:

1. If the back of the car has crossed into the GO zone, they will easily be able to cross the intersection in the time allotted because they are driving faster than the expected driving speed and d/s = t so if s increases, t decreases as s is in the denominator, so the time needed to cross would be less than the yellow light time.
2. If the car is in the STOP zone, its safety will depend on the decision the driver takes.

a. If the car goes, it may be able to traverse the intersection depending on how much they are exceeding the limit. This does risk an accident with the car in front, assuming they are going at the constant speed limit
b. If the car stops, the car may not be able to stop safely before the light turns red. Yet, this may change if their rate of deceleration is higher

b. Bald Tires
   - We would need to increase the yellow light time via the variable $\bar{a}_{d.}$ (acceleration) This is because the driver will need longer to decelerate due to decreased friction. This means that the threshold for the stop zone will be pushed back.
   - Since a car with a bald tire has less friction, the car will take *longer* to decelerate. Due to this, if this car is at the beginning of the STOP zone, it will not have sufficient time to decelerate before the allotted time.

c. Long vehicles such as a semi-truck
   - Our equation would increase the time via the variable $\Delta X_{C.}$ (car length). A longer vehicle will need longer to fully traverse the intersection safely. The GO zone would increase since the vehicle length increases.
   - Our equation takes into account this idea. The time would not have to change

d. Distracted Driver
   - This would increase the yellow light time via the variable t_r (reaction time). They would take longer to identify that they need to slow down. This would push back the stop zone to give the driver more time to notice that they have to stop.

- This driver, assuming they will go in any situation (regardless of what ZONE they are in), will need a much longer time to traverse the intersection. This affects the length of the GO zone since this driver could be in the STOP zone

3. Would it be a good idea to mark the stop and go zones on the road leading up to the intersection? Why or why not?

    - If this were the real world, meaning the drivers did not automatically know the threshold, marking the STOP and GO zones would be very helpful. This can help drivers decide whether to go or stop. Honestly, mismarking the zones so that the GO zone appears smaller takes into account the time it takes to see this marking and the possibility that a driver doesn't know whether the front or the back of the vehicle should cross this zone.

4. In the 1960s, traffic engineers piloted a system that would display a countdown timer to show how much time was left before the light would turn red. Why do you think the engineers decided against the idea?

    - This is because of reckless drivers, who, when they can see the amount of time left to traverse, will think “I can cross that! I’ll just speed up”. This leads to more accidents and more unsafe conditions. Therefore, this timer may have been taken down.